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Global robust stability of neural networks with time varying delays☆

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Abstract

In this paper, we study the global robust stability of neural networks with time varying delays using the Lyapunov functional method and matrix inequality technique. Several sufficient conditions are presented to show the existence of equilibrium and global robust stability of neural networks, which is easy to apply. Then we give a simulation to justify the obtained results.
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1. Introduction

In recent years, dynamical behaviors including stability, exponential stability, robust stability, periodic bifurcation and chaos of neural networks [1–35] have been a hot topic. Many researchers focus their attentions on these. A lot of works and efforts about dynamical characteristics have been made. Stability is a basic knowledge for dynamical systems and the stability of neural networks have been studied by many papers [1–16,19,20,23–25,30,31]. There are definitions such as stability, asymptotical stability, exponential stability, robust stability and so on. It is a complex knowledge, but the phenomenon demonstrated in the dynamical systems is useful. There are many things for us to do and we shall continue to study the stability of dynamical systems.

Recently, there has been extensive interest in studying the effect of time delay on the neural networks [20,12,9,6,5]. It is well known that time delay is ubiquitous in most physical, chemical, biological, neural, and other natural system due to finite propagation speeds of signals, finite processing times in synapses, and finite reaction times. Therefore, dynamical analysis of time-delay systems is an important topic in many fields [31,18,25–29,21,4,16,10,15,7,8,11,14].

In neural networks, there are some uncertainties. We cannot have a fixed neural network, it can be perturbed by many factors. According to these, estimation errors are presented. We may study the deviations of coefficients of neural networks which is more realistic. So we consider global robust stability of neural networks. There are some existing works about robust stability [13,30,23,24,20,5–7]. In [24], global robust stability of a delayed interval Hopfield neural network is investigated with respect to the bounded and strictly increasing activation functions. Recently, in

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[23,20], several criteria are presented for global robust stability of neural networks with and without delay such as LMI approach and inequality method. In [13], Cao and Wang study the global asymptotic and robust stability of recurrent neural networks with time delays by using the norm method which are easy to derive, and our paper is an improved and extended work compared to this.

In this paper, we will study the existence of equilibrium point and global robust stability of equilibrium point. The results we obtained improve and extend earlier works. Here, we just want to give some good conditions to ensure the existence of equilibrium point and global robust stability of equilibrium point and these will lead to a clear view of delayed neural network model which may finally lead us to design the real neural network.

The organization of this paper is as follows: in Section 2, we give model formulation and preliminaries for our main results. We proposed a model and some provisions for further study. In Section 3, we give our main results. Several sufficient conditions are presented for the existence of equilibrium point and global robust stability of neural networks. In Section 4, numerical simulations aimed at justifying the theoretical analysis will be reported. In Section 5, we give conclusions.

2. Model formulation and preliminaries

In this paper, we consider the following delayed neural network model:

$$\dot{x}(t) = -Cx(t) + Af(x(t)) + Bf(x(t-\tau(t))) + u \tag{1}$$

or

$$\dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau(t))) + u_i, \quad i = 1, 2, \ldots, n, \tag{2}$$

where n denotes the number of units in a neural network, $\tau(t)$ is time varying delay, $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^{\mathrm{T}} \in R^n$ is the state vector associated with the neurons, $u = (u_1, u_2, \ldots, u_n)^{\mathrm{T}} \in R^n$ is a constant external input vector, $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t)))^{\mathrm{T}} \in R^n$ corresponds to the activation functions of neurons, $f(x(t-\tau(t))) = (f_1(x_1(t-\tau(t))), f_2(x_2(t-\tau(t))), \ldots, f_n(x_n(t-\tau(t))))^{\mathrm{T}} \in R^n$, $C = \mathrm{diag}(c_1, c_2, \ldots, c_n) > 0$ (positive definite diagonal matrix), $A = (a_{ij})_{n\times n}$ and $B = (b_{ij})_{n\times n}$ are the connection weight matrix and the delayed connection weight matrix, respectively.

In this paper, the activation functions and time varying delay are assumed to satisfy the following properties:

A_1: $f_i(x_i)(i = 1, 2, \ldots, n)$ is bounded and monotonically nondecreasing on R;

A_2: The activation function $f_i(x_i)(i = 1, 2, \ldots, n)$ is Lipschitz continues, that is, there exist constant $\mu_i > 0$ such that

$$|f_i(\alpha_1) - f_i(\alpha_2)| \leqslant \mu_i |\alpha_1 - \alpha_2|, \quad \forall \alpha_1, \alpha_2 \in R; \tag{3}$$

A_3: $\tau(t)$ is a bounded differential function of time t, and the following condition satisfied:

$$0 \leqslant \dot{\tau}(t) \leqslant h < 1.$$

It is known that bounded activation functions always guarantee the existence of an equilibrium point for system (1). In the real neural networks, the values of weight coefficients depend on the resistance and capacitance which are subject to the uncertainties. This may lead to some deviations in the values of c_i, a_{ij} and b_{ij}. So it is useful for us to study the global robust stability of neural network through such parameter deviations [13]. Since these deviations are bounded in practice, the value of c_i, a_{ij} and b_{ij} can be stated as follows:

$$\begin{cases} C_I = \{C = \mathrm{diag}(c_i) : \underline{C} \leqslant C \leqslant \overline{C}, \text{ i.e., } \underline{c}_i \leqslant c_i \leqslant \overline{c}_i, \ i = 1, 2, \ldots, n\}, \\ A_I = \{A = (a_{ij})_{n\times n} : \underline{A} \leqslant A \leqslant \overline{A}, \text{ i.e., } \underline{a}_{ij} \leqslant a_{ij} \leqslant \overline{a}_{ij}, \ i = 1, 2, \ldots, n\}, \\ B_I = \{B = (b_{ij})_{n\times n} : \underline{B} \leqslant B \leqslant \overline{B}, \text{ i.e., } \underline{b}_{ij} \leqslant b_{ij} \leqslant \overline{b}_{ij}, \ i = 1, 2, \ldots, n\}. \end{cases} \tag{4}$$

We assume that the model (1) has an equilibrium $x^* = (x_1^*, x_2^*, \ldots, x_n^*)$ for a given u. To simplify the proofs, we will shift the equilibrium point x^* of (1) to the origin by using the following transformation:

$$y(t) = x(t) - x^*, \quad y(t-\tau(t)) = x(t-\tau(t)) - x^*.$$

Then the model (1) can be transformed into the following form:

$$\dot{y}(t) = -Cy(t) + Ag(y(t)) + Bg(y(t-\tau(t))), \tag{5}$$

namely,

$$\dot{y}_i(t) = -c_i y_i(t) + \sum_{j=1}^{n} a_{ij} g_j(y_j(t)) + \sum_{j=1}^{n} b_{ij} g_j(y_j(t-\tau(t))), \quad i = 1, 2, \ldots, n, \tag{6}$$

where $g(y(t)) = (g_1(y_1(t)), g_2(y_2(t)), \ldots, g_n(y_n(t)))^{\mathrm{T}} \in R^n$ with $g_i(y_i(t)) = f_i(y_i(t) + x_i^*) - f_i(x_i^*)$ and $g(0) = 0$. Moreover, from (3), we know that

$$|g_i(y_i)| \leqslant \mu_i |y_i| \quad \forall y_i \in R, \quad i = 1, 2, \ldots, n. \tag{7}$$

Let $\| \bullet \|$ denote the Euclidean norm in the Euclidean space R^n. If W is a symmetric matrix with $\lambda_{\max}(W)$ and $\lambda_{\min}(W)$ as its largest and smallest eigenvalue, respectively. Then its norm is defined by

$$\|W\|_2 = sup\{\|Wx\| : \|x\| = 1\} = \sqrt{\lambda_{\max}(W^{\mathrm{T}}W)}.$$

For $A^*, A_*, B^*, B_* \in R^n \times n$, where A_*, B_* are nonnegative matrices, we use the notations $[A^* \pm A_*]$, $[B^* \pm B_*]$ to denote the interval matrices $[A^* - A_*, A^* + A_*]$, $[B^* - B_*, B^* + B_*]$, respectively. In fact, any interval matrices $[\underline{A}, \overline{A}]$, $[\underline{B}, \overline{B}]$ have a unique representation of the form $[A^* - A_*, A^* + A_*]$, $[B^* - B_*, B^* + B_*]$, respectively, where $A^* = (\overline{A} + \underline{A})/2$, $A_* = (\overline{A} - \underline{A})/2$, $B^* = (\overline{B} + \underline{B})/2$, $B_* = (\overline{B} - \underline{B})/2$. Thus, we denote $A = A^* + \triangle A$, $B = B^* + \triangle B$, where $\triangle A \in [-A_*, A_*]$, $\triangle B \in [-B_*, B_*]$.

Definition. The neural network model given by (1) or (2) with the parameter ranges defined by (4) is globally robust stable if the unique equilibrium point $x^* = (x_1^*, x_2^*, \ldots, x_n^*)$ of the model is globally asymptotically stable for all $C \in C_I$, $A \in A_I$, $B \in B_I$.

Note that the robust stability actually becomes the classical Lyapunov stability if $\underline{C} = C = \overline{C}$, $\underline{A} = A = \overline{A}$, $\underline{B} = B = \overline{B}$. To obtain our main results, we need the following three elementary lemmas:

Lemma 1. *For any vectors $x, y \in R^n$ and positive definite matrix $G \in R^{n\times n}$, the following matrix inequality holds*:

$$2x^{\mathrm{T}}y \leqslant x^{\mathrm{T}}Gx + y^{\mathrm{T}}G^{-1}y.$$

Lemma 2 (*Cao and Wang [13]*). *If $V, W \in R^{n\times n}$ are two matrices with property that $|V| = (|v_{ij}|_{n\times n}) \leqslant W = (w_{ij})_{n\times n}$, i.e., $|v_{ij}| \leqslant w_{ij}$, then $\|V\|_2 \leqslant \|W\|_2$.*

Lemma 3 (*Cao and Wang [13]*). *For $\forall A \in [\underline{A}, \overline{A}]$, $B \in [\underline{B}, \overline{B}]$, we have*

$$\|A\|_2 \leqslant \|A^*\|_2 + \|A_*\|_2, \quad \|B\|_2 \leqslant \|B^*\|_2 + \|B_*\|_2.$$

3. Global robust stability criteria

In this section, new criteria are presented for the global robust stability of the equilibrium point of the neural network defined by (5). Its proof is based on a new Lyapunov functional method and matrix inequalities approach.

Theorem 1. *Under the assumptions A_1–A_3, the equilibrium point of model (5) is globally robust stable if there are positive definite diagonal matrix $P = \mathrm{diag}(p_1, p_2, \ldots, p_n)$ and positive definite matrix $H = (h_{ij})_{n\times n}$ such that*

$$2P\underline{C}\Lambda^{-1} - \bar{F} - \|\widehat{A}\|_2 I - H - \frac{1}{1-h}\|P\|^2 \|H^{-1}\|(\|B_*\|_2 + \|B^*\|_2)^2 I > 0, \tag{8}$$

where I is the identity matrix, $B^* = (\overline{B} + \underline{B})/2$, $B_* = (\overline{B} - \underline{B})/2$, $\Lambda = \mathrm{diag}(\mu_1, \mu_2, \ldots, \mu_n)$, $\bar{F} = \mathrm{diag}(2p_1\bar{a}_{11}, 2p_2\bar{a}_{22}, \ldots, 2p_n\bar{a}_{nn})$, $\underline{C} = \mathrm{diag}(\underline{c}_1, \underline{c}_2, \ldots, \underline{c}_n)$ *and*

$$\widehat{A} = \begin{cases} 0, & i = j, \\ \max(|p_i\bar{a}_{ij} + p_j\bar{a}_{ji}|, |p_i\underline{a}_{ij} + p_j\underline{a}_{ji}|) \triangleq \widehat{a}_{ij}, & i \neq j. \end{cases} \tag{9}$$

Proof. We shall prove this theorem in two steps. First, we prove the uniqueness of the equilibrium point, and then we establish its global robust stability.

Step 1: We will prove the uniqueness of the equilibrium point using the method of contradiction. Consider the equilibrium equation of (5)

$$Cy^* - Ag(y^*) - Bg(y^*) = 0. \tag{10}$$

It is evident to see that if $g(y^*) = 0$, then $y^* = 0$. Now let $g(y^*) \neq 0$. Multiplying both sides of (10) by $2g^{\mathrm{T}}(y^*)P$, yields

$$2g^{\mathrm{T}}(y^*)PCy^* - 2g^{\mathrm{T}}(y^*)PAg(y^*) - 2g^{\mathrm{T}}(y^*)PBg(y^*) = 0. \tag{11}$$

Eq. (11) can be rewritten as

$$2g^{\mathrm{T}}(y^*)PCy^* - g^{\mathrm{T}}(y^*)PAg(y^*) - g^{\mathrm{T}}(y^*)A^{\mathrm{T}}Pg(y^*) - 2g^{\mathrm{T}}(y^*)PBg(y^*) = 0. \tag{12}$$

From (7) and the assumptions A_1 and A_2, we get

$$y_i g_i(y_i) \geqslant 0 \quad \forall y_i \in R, \quad i = 1, 2, \ldots, n \tag{13}$$

and

$$g_i^2(y_i) \leqslant \mu_i y_i g_i(y_i) \quad \forall y_i \in R, \quad i = 1, 2, \ldots, n. \tag{14}$$

According to (13) and (14), we can obtain the following inequality

$$\begin{aligned} g^{\mathrm{T}}(y^*)PCy^* &= \sum_{i=1}^{n} g_i(y_i^*) p_i c_i y_i^* \\ &\geqslant \sum_{i=1}^{n} \frac{p_i c_i}{\mu_i} g_i^2(y_i^*) \\ &\geqslant \sum_{i=1}^{n} \frac{p_i \underline{c}_i}{\mu_i} g_i^2(y_i^*) \\ &= g^{\mathrm{T}}(y^*)P\underline{C}\Lambda^{-1}g(y^*), \end{aligned} \tag{15}$$

where $\Lambda = \mathrm{diag}(\mu_1, \mu_2, \ldots, \mu_n)$.

We know that

$$PA + A^{\mathrm{T}}P = \begin{cases} 2p_i a_{ii}, & i = j, \\ p_i a_{ij} + p_j a_{ji} \triangleq \widetilde{a}_{ij}, & i \neq j. \end{cases} \tag{16}$$

Let

$$F = \mathrm{diag}(2p_1 a_{11}, 2p_2 a_{22}, \ldots, 2p_n a_{nn}), \tag{17}$$

$$\bar{F} = \mathrm{diag}(2p_1\bar{a}_{11}, 2p_2\bar{a}_{22}, \ldots, 2p_n\bar{a}_{nn}), \tag{18}$$

$$\widetilde{A}=\begin{cases}0, & i=j,\\ p_i a_{ij}+p_j a_{ji}\triangleq\widetilde{a}_{ij}, & i\neq j,\end{cases} \tag{19}$$

$$\widehat{A}=\begin{cases}0, & i=j,\\ \max(|p_i\bar{a}_{ij}+p_j\bar{a}_{ji}|,|p_i\underline{a}_{ij}+p_j\underline{a}_{ji}|)\triangleq\widehat{a}_{ij}, & i\neq j.\end{cases} \tag{20}$$

It is easy to see that

$$|\widetilde{a}_{ij}|\leqslant\widehat{a}_{ij},\quad i\neq j. \tag{21}$$

From (21), according to Lemma 2, we obtain that

$$\|\widetilde{A}\|_2\leqslant\|\widehat{A}\|_2, \tag{22}$$

then we obtain

$$\begin{aligned}&PA+A^{\mathrm{T}}P\\ &\quad=F+\widetilde{A}\\ &\quad=F+\|\widetilde{A}\|_2 I-(\|\widetilde{A}\|_2 I-\widetilde{A})\\ &\quad=F+\|\widehat{A}\|_2 I-(\|\widehat{A}\|_2-\|\widetilde{A}\|_2)I-(\|\widetilde{A}\|_2 I-\widetilde{A})\\ &\quad=\bar{F}-(\bar{F}-F)+\|\widehat{A}\|_2 I-(\|\widehat{A}\|_2-\|\widetilde{A}\|_2)I-(\|\widetilde{A}\|_2 I-\widetilde{A})\\ &\quad\leqslant\bar{F}+\|\widehat{A}\|_2 I.\end{aligned} \tag{23}$$

According to Lemma 1, we have

$$\begin{aligned}2g^{\mathrm{T}}(y^*)PBg(y^*)&\leqslant g^{\mathrm{T}}(y^*)Hg(y^*)+g^{\mathrm{T}}(y^*)PBH^{-1}B^{\mathrm{T}}Pg(y^*)\\ &\leqslant g^{\mathrm{T}}(y^*)Hg(y^*)+\frac{1}{1-h}g^{\mathrm{T}}(y^*)PBH^{-1}B^{\mathrm{T}}Pg(y^*).\end{aligned} \tag{24}$$

Also, with Lemma 3, we obtain

$$\begin{aligned}&PBH^{-1}B^{\mathrm{T}}P\\ &\quad=\|P\|^2\|H^{-1}\|\|B\|_2^2 I-[\|P\|^2\|H^{-1}\|\|B\|_2^2 I-PBH^{-1}B^{\mathrm{T}}P]\\ &\quad\leqslant\|P\|^2\|H^{-1}\|(\|B_*\|_2+\|B^*\|_2)^2 I-[\|P\|^2\|H^{-1}\|\|B\|_2^2 I-PBH^{-1}B^{\mathrm{T}}P]\\ &\quad\leqslant\|P\|^2\|H^{-1}\|(\|B_*\|_2+\|B^*\|_2)^2 I.\end{aligned} \tag{25}$$

Substituting (15), (23), (24) and (25) into (12), we get

$$\begin{aligned}0&=2g^{\mathrm{T}}(y^*)PCy^*-g^{\mathrm{T}}(y^*)PAg(y^*)-g^{\mathrm{T}}(y^*)A^{\mathrm{T}}Pg(y^*)-2g^{\mathrm{T}}(y^*)PBg(y^*)\\ &\geqslant g^{\mathrm{T}}(y^*)[2P\underline{C}\Lambda^{-1}-\bar{F}-\|\widehat{A}\|_2 I-H-\frac{1}{1-h}\|P\|^2\|H^{-1}\|(\|B_*\|_2+\|B^*\|_2)^2 I]g(y^*).\end{aligned} \tag{26}$$

From condition (8), we have

$$g^{\mathrm{T}}(y^*)[2P\underline{C}\Lambda^{-1}-\bar{F}-\|\widehat{A}\|_2 I-H-\frac{1}{1-h}\|P\|^2\|H^{-1}\|(\|B_*\|_2+\|B^*\|_2)^2 I]g(y^*)>0. \tag{27}$$

Obviously, (26) contradicts with (27) which in turn implies that at the equilibrium point $g(y^*)=0$, as well as $y^*=0$. Thus, we proved that the origin of model (5) is a unique equilibrium point.

Step 2: We now prove that the condition given in (8) also imply the global stability of the origin of (5). Consider the Lyapunov functional

$$V(y(t))=2\sum_{i=1}^{n}p_i\int_0^{y_i(t)}g_i(s)ds+\int_{t-\tau(t)}^{t}g^{\mathrm{T}}(y(s))Hg(y(s)), \tag{28}$$

where $P=\mathrm{diag}(p_1,p_2,\ldots,p_n)$ is a positive definite diagonal matrix and $H=(h_{ij})_{n\times n}$ is a positive definite matrix.

Taking the derivative of $V(y)$ along the trajectories of (5), we obtain

$$\begin{aligned}\dot{V}(y(t))|_{(5)} &= 2g^{\mathrm{T}}(y(t))P\dot{y}(t) + g^{\mathrm{T}}(y(t))Hg(y(t)) - (1-\dot{\tau}(t))g^{\mathrm{T}}(y(t-\tau(t)))Hg(y(t-\tau(t))) \\ &= -2g^{\mathrm{T}}(y(t))PCy(t) + 2g^{\mathrm{T}}(y(t))PAg(y(t)) + 2g^{\mathrm{T}}(y(t))PBg(y(t-\tau(t))) \\ &\quad + g^{\mathrm{T}}(y(t))Hg(y(t)) - (1-\dot{\tau}(t))g^{\mathrm{T}}(y(t-\tau(t)))Hg(y(t-\tau(t))) \\ &= -2g^{\mathrm{T}}(y(t))PCy(t) + g^{\mathrm{T}}(y(t))PAg(y(t)) + g^{\mathrm{T}}(y(t))A^{\mathrm{T}}Pg(y(t)) \\ &\quad + 2g^{\mathrm{T}}(y(t))PBg(y(t-\tau(t))) + g^{\mathrm{T}}(y(t))Hg(y(t)) \\ &\quad - (1-\dot{\tau}(t))g^{\mathrm{T}}(y(t-\tau(t)))Hg(y(t-\tau(t))). \end{aligned} \tag{29}$$

Applying Lemma 1, we get

$$\begin{aligned}2g^{\mathrm{T}}(y(t))PBg(y(t-\tau(t))) &\leqslant (1-\dot{\tau}(t))g^{\mathrm{T}}(y(t-\tau(t)))Hg(y(t-\tau(t))) \\ &\quad + \frac{1}{(1-\dot{\tau}(t))}g^{\mathrm{T}}(y(t))PBH^{-1}B^{\mathrm{T}}Pg(y(t)),\end{aligned} \tag{30}$$

then similarly using (15) and (30), we obtain

$$\begin{aligned}\dot{V}(y(t))|_{(5)} &\leqslant -2g^{\mathrm{T}}(y(t))P\underline{C}\Lambda^{-1}g(y(t)) + g^{\mathrm{T}}(y(t))PAg(y(t)) + g^{\mathrm{T}}(y(t))A^{\mathrm{T}}Pg(y(t)) \\ &\quad + \frac{1}{(1-\dot{\tau}(t))}g^{\mathrm{T}}(y(t))PBH^{-1}B^{\mathrm{T}}Pg(y(t)) + g^{\mathrm{T}}(y(t))Hg(y(t)) \\ &\leqslant -g^{\mathrm{T}}(y(t))[2P\underline{C}\Lambda^{-1} - PA - A^{\mathrm{T}}P - \frac{1}{1-h}PBH^{-1}B^{\mathrm{T}}P - H]g(y(t)).\end{aligned} \tag{31}$$

Similarly to (26), we have

$$\begin{aligned}&2P\underline{C}\Lambda^{-1} - PA - A^{\mathrm{T}}P - PBH^{-1}B^{\mathrm{T}}P - H \\ &\quad \geqslant 2P\underline{C}\Lambda^{-1} - \bar{F} - \|\widehat{A}\|_2 I - H - \frac{1}{1-h}\|P\|^2\|H^{-1}\|(\|B_*\|_2 + \|B^*\|_2)^2 I > 0.\end{aligned} \tag{32}$$

Therefore, under the given condition (8), $\dot{V}(y(t)) = 0$ if and only if $y(t) = g(y(t)) = g(y(t-\tau(t))) = 0$, otherwise $\dot{V}(y(t)) \leqslant 0$. Moreover, on the other hand, $V(y)$ is radially unbounded since $V(y(t)) \to \infty$ as $\|y(t)\| \to \infty$. We have proved that the equilibrium of (5) is globally asymptotically stable. This completes the proof. □

Corollary 1. *Under the assumptions* A_1–A_3, *the equilibrium point of model* (5) *is globally robust stable if there is positive definite matrix* $H = (h_{ij})_{n\times n}$ *such that*

$$2\underline{C}\Lambda^{-1} - \bar{F} - \|\widehat{A}\|_2 I - H - \frac{1}{1-h}\|H^{-1}\|(\|B_*\|_2 + \|B^*\|_2)^2 I > 0,$$

where I *is the identity matrix*, $B^* = (\overline{B} + \underline{B})/2$, $B_* = (\overline{B} - \underline{B})/2$, $\Lambda = \mathrm{diag}(\mu_1, \mu_2, \ldots, \mu_n)$, $\bar{F} = \mathrm{diag}(2\bar{a}_{11}, 2\bar{a}_{22}, \ldots, 2\bar{a}_{nn})$, $\underline{C} = \mathrm{diag}(\underline{c}_1, \underline{c}_2, \ldots, \underline{c}_n)$ *and*

$$\widehat{A} = \begin{cases} 0, & i = j, \\ \max(|\bar{a}_{ij} + \bar{a}_{ji}|, |\underline{a}_{ij} + \underline{a}_{ji}|) \triangleq \widehat{a}_{ij}, & i \neq j. \end{cases}$$

Proof. In Theorem 1, let $P = I$, its proof is obvious and here we omitted. □

Corollary 2. *Under the assumptions* A_1–A_3, *the equilibrium point of model* (5) *is globally robust stable if*

$$2\underline{C}\Lambda^{-1} - \bar{F} - \|\widehat{A}\|_2 I - I - \frac{1}{1-h}(\|B_*\|_2 + \|B^*\|_2)^2 I > 0,$$

where I is the identity matrix, $B^*=(\overline{B}+\underline{B})/2$, $B_*=(\overline{B}-\underline{B})/2$, $\Lambda=\text{diag}(\mu_1, \mu_2, \ldots, \mu_n)$, $\bar{F}=\text{diag}(2\bar{a}_{11}, 2\bar{a}_{22}, \ldots, 2\bar{a}_{nn})$, $\underline{C}=\text{diag}(\underline{c}_1, \underline{c}_2, \ldots, \underline{c}_n)$ *and*

$$\widehat{A}=\begin{cases}0, & i=j,\\ \max(|\bar{a}_{ij}+\bar{a}_{ji}|, |\underline{a}_{ij}+\underline{a}_{ji}|)\triangleq\widehat{a}_{ij}, & i\neq j.\end{cases}$$

Proof. In Theorem 1, let $P=H=I$, Corollary 2 is a direct result of Theorem 1, here we omitted. □

Corollary 3. *Under the assumptions* A_1–A_3, *the equilibrium of model* (5) *is globally robust stable if there is positive definite matrix* $H=(h_{ij})_{n\times n}$ *such that*

$$(\|B_*\|_2+\|B^*\|_2)^2<(1-h)\lambda_{\min}[2\underline{C}\Lambda^{-1}-\bar{F}-\|\widehat{A}\|_2 I-H]/\|H^{-1}\|,$$

where I is the identity matrix, $B^*=(\overline{B}+\underline{B})/2$, $B_*=(\overline{B}-\underline{B})/2$, $\Lambda=\text{diag}(\mu_1, \mu_2, \ldots, \mu_n)$, $\bar{F}=\text{diag}(2\bar{a}_{11}, 2\bar{a}_{22}, \ldots, 2\bar{a}_{nn})$, $\underline{C}=\text{diag}(\underline{c}_1, \underline{c}_2, \ldots, \underline{c}_n)$ *and*

$$\widehat{A}=\begin{cases}0, & i=j,\\ \max(|\bar{a}_{ij}+\bar{a}_{ji}|, |\underline{a}_{ij}+\underline{a}_{ji}|)\triangleq\widehat{a}_{ij}, & i\neq j.\end{cases}$$

Proof. Corollary 3 is a direct result of Corollary 1, here we omitted. □

Remark. First, we give a theorem studied in [13]:

Theorem 2. *Under the assumptions* A_1 *and* A_2, *the equilibrium point of model* (5) *is globally robust stable if there are positive definite diagonal matrix* $P=\text{diag}(p_1, p_2, \ldots, p_n)$ *and positive definite matrix* $H=(h_{ij})_{n\times n}$ *such that the following conditions hold*:

(i) *The symmetric matrix* $S=(s_{ij})_{n\times n}$ *is positive definite.*
(ii)

$$2r-\|H\|_2-\|P\|^2\|H^{-1}\|(\|B_*\|_2+\|B^*\|_2)^2>0,$$

where $r=\min_i[p_i\underline{c}_i/\mu_i]$, $B^*=(\overline{B}+\underline{B})/2$, $B_*=(\overline{B}-\underline{B})/2$, *and*

$$s_{ij}=\begin{cases}-2p_i\bar{a}_{ii}, & i=j,\\ -\max(|p_i\bar{a}_{ij}+p_j\bar{a}_{ji}|, |p_i\underline{a}_{ij}+p_j\underline{a}_{ji}|)\triangleq\widehat{a}_{ij}, & i\neq j.\end{cases}$$

In this paper, we do not need the two condition (i) and (ii) are both satisfied, we only have one condition (8). Also, we have considered time varying delays in this paper, if $\dot{\tau}(t)=0$, i.e. $h=0$, it is constant time delay in [13]. We will show our merit of this paper through an example stated below.

4. Numerical example

In this section, we will give an example to justify Theorem 1 obtained above.

Consider a delayed neural network (5) with the connection weight matrices

$$\overline{A}=\underline{A}=\begin{pmatrix}0.2 & 0.1\\ 0.1 & 0.15\end{pmatrix},\quad \overline{B}=\underline{B}=\begin{pmatrix}0.1 & 0.2\\ 0.2 & 0.1\end{pmatrix},\quad \overline{C}=\underline{C}=I,$$

where $f_i(x)=(|x+1|-|x-1|)/2$ $(i=1,2)$, this implies that $\Lambda=I$. Now let $P=H=I$ in Corollary 2,

$$2\underline{C}\Lambda^{-1}-\bar{F}-\|\widehat{A}\|_2 I-I-(\|B_*\|_2+\|B^*\|_2)^2 I=\begin{pmatrix}0.04 & 0\\ 0 & 0.14\end{pmatrix}>0,$$

according to Theorem 1, the equilibrium of model (5) is globally robust stable. But in Theorem 2, we have

$$S=\begin{pmatrix}-0.4 & -0.2\\ -0.2 & -0.3\end{pmatrix}.$$

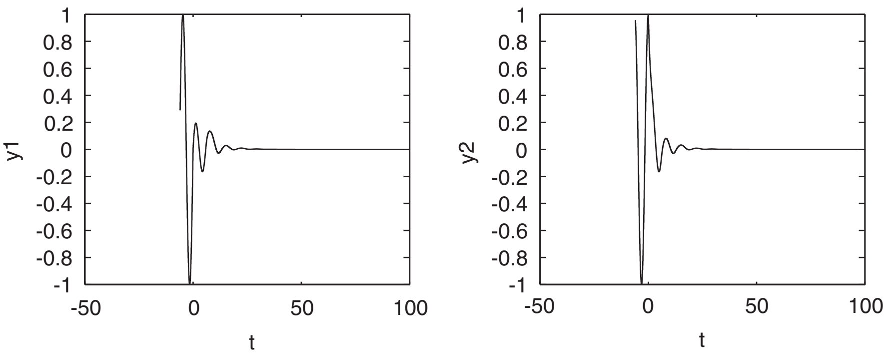

Fig. 1. Trajectories of the state variables of neural networks.

It is not a positive definite matrix, thus the condition (i) is not satisfied. So we cannot use Theorem 2 to solve this problem, but Theorem 1 in this paper can work. Next, we give a simulation to justify Theorem 1, which is shown in Fig. 1. We can know that the equilibrium of model (5) is globally robust stable.

5. Conclusions

In this paper, we have studied global robust stability of neural networks with time varying delays. Due to uncertainties of neural networks, we study the robust stability of neural networks and it is more realistic in the neural network allowing deviations of coefficients. We use Lyapunov method and matrices inequality technique to solve this problem. Several sufficient conditions have been derived to ensure the existence of unique equilibrium point and global robust stability for the delayed neural networks. The obtained results improve and extend the earlier works. It is easy to apply these sufficient conditions to the real networks. Finally, we give a simulation to show the effectiveness of the obtained results.

Dynamical behaviors, such as stability, bifurcation and chaos have been studied by many researchers. Neural networks with time delays have been a hot topic in recent years. There are still many things for us to do, which are studied by investigating the essence of dynamical systems.

References

[1] S. Arik, V. Tavsanoglu, On the global asymptotic stability of delayed cellular neural networks, IEEE Trans. Circuits Systems I, Fundamental Theory Appl. 47 (4) (2000) 571–574.
[2] K.L. Babcock, R.M. Westervelt, Dynamics of simple electronic neural networks, Physica D 28 (1987) 305–316.
[3] P. Baldi, A. Atiya, How delays affect neural dynamics and learning, IEEE Trans. Neural Networks 5 (1994) 610–621.
[4] J. Cao, A set of stability criteria for delayed cellular neural networks, IEEE Trans. Circuits Systems—I 48 (4) (2001) 494–498.
[5] J. Cao, T. Chen, Globally exponentially robust stability and periodicity of delayed neural networks, Chaos, Solitons Fractals 22 (4) (2004) 957–963.
[6] J. Cao, D. Huang, Y. Qu, Global robust stability of delayed recurrent neural networks, Chaos, Solitons Fractals 23 (1) (2005) 221–229.
[7] J. Cao, H. Li, L. Han, Novel results concerning global robust stability of delayed neural networks, Nonlinear Anal. Ser. B 7 (3) (2006) 458–469.
[8] J. Cao, X. Li, Stability in delayed Cohen–Grossberg neural networks: LMI optimization approach, Physica D 212 (1–2) (2005) 54–65.
[9] J. Cao, J. Liang, Boundedness and stability for Cohen–Grossberg neural networks with time-varying delays, J. Math. Anal. Appl. 296 (2) (2004) 665–685.
[10] J. Cao, Q. Song, Stability in Cohen–Grossberg type BAM neural networks with time-varying delays, Nonlinearity 19 (7) (2006) 1601–1617.
[11] J. Cao, J. Wang, Global asymptotic stability of a general class of recurrent neural networks with time-varying delays, IEEE Trans. Circuits Systems—I 50 (1) (2003) 34–44.
[12] J. Cao, J. Wang, Absolute exponential stability of recurrent neural networks with time delays and Lipschitz-continuous activation functions, Neural Networks 17 (3) (2004) 379–390.
[13] J. Cao, J. Wang, Global asymptotic and robust stability of recurrent neural networks with time delays, IEEE Trans. Circuits Systems I, Regular Papers 52 (2) (2005) 417–426.
[14] J. Cao, L. Wang, Exponential stability and periodic oscillatory solution in BAM networks with delays, IEEE Trans. Neural Networks 13 (2) (2002) 457–463.
[15] J. Cao, K. Yuan, D.W.C. Ho, J. Lam, Global point dissipativity of neural networks with mixed time-varying delays, Chaos 16 (013105) (2006).

[16] T. Chen, Global exponential stability of delayed Hopfield neural networks, Neural Networks 14 (8) (2001) 977–980.
[17] K. Gopalsamy, I. Leung, Convergence under dynamical thresholds with delays, IEEE Trans. Neural Networks 8 (1997) 341–348.
[18] S. Guo, L. Huang, Linear stability and Hopf bifurcation in a two-neuron network with three delays, Internat. J. Bifurc. Chaos 14 (8) (2004) 2790–2810.
[19] J.J. Hopfield, Neurons with graded response have collective computational properties like those of two-state neurons, Proc. Natl. Acad. Sci. USA 81 (1984) 3088–3092.
[20] S.Q. Hu, J. Wang, Global exponential stability of continuous-time interval neural networks, Phys. Rev. E 65 (2002) 1–9.
[21] X. Liao, K. Wong, Z. Wu, Hopf bifurcation and stability of periodic solutions for van der Pol equation with distributed delay, Nonlinear Dynamics 26 (2001) 23–44.
[22] X. Liao, Z. Wu, J. Yu, Stability swithes and bifurcation analysis of a neural network with continuously delay, IEEE Trans. Systems, Man Cybern.—Part A 29 (1999) 692–696.
[23] X.F. Liao, K.W. Wong, Z. Wu, G. Chen, Novel robust stability criteria for interval-delayed Hopfield neural networks, IEEE Trans. Circuits Systems I, Fundamental Theory Appl. 48 (11) (2001) 1355–1359.
[24] X.F. Liao, J.B. Yu, Robust stability for interval Hopfield neural networks with time delay, IEEE Trans. Neural Networks 9 (5) (1998) 1042–1045.
[25] W. Lu, L. Rong, T. Chen, Global convergence of delayed neural network systems, Internat. J. Neural Systems 13 (3) (2003) 193–204.
[26] S. Ruan, J. Wei, On the zeros of a third degree exponential polynomial with applications to a delayed model for the control of testosterone secretion, IMA J. Math. Appl. Med. Biol. 18 (2001) 41–52.
[27] Y. Song, M. Han, J. Wei, Stability and Hopf bifurcation analysis on a simplified BAM neural network with delays, Physica D 200 (2005) 185–204.
[28] Y. Song, J. Wei, Local Hopf bifurcation and global periodic solutions in a delayed predator-prey system, J. Math. Anal. Appl. 301 (2005) 1–21.
[29] J. Wei, S. Ruan, Stability and bifurcation in a neural network model with two delays, Physica D 130 (1999) 255–272.
[30] H. Ye, A.N. Michel, Robust stability of nonlinear time-delay systems with applications to neural networks, IEEE Trans. Circuits Systems I, Fundamental Theory Appl. 43 (7) (1996) 532–543.
[31] W. Yu, J. Cao, Stability and Hopf bifurcation analysis on a four-neuron BAM neural network with time delays, Phys. Lett. A 351 (1–2) (2006) 64–78.
[32] J. Cao, W. Yu, Y. Qu, A new complex network model and convergence dynamics for reputation computation in virtual organizations, Phys. Lett. A 356 (6) (2006) 414–425.
[33] W. Yu, J. Cao, Adaptive Q-S (lag, anticipated, and complete) time-varying synchronization and parameters identification of uncertain delayed neural networks, Chaos, 16 (023119) (2006).
[34] W. Yu, J. Cao, Synchronization control of stochastic delayed neural networks, Physica A, doi: 10.1016/j.physa.2006.04.105, in press.
[35] W. Yu, A LMI-based approach to global asymptotic stability of neural networks with time varying delays, Nonlinear Dynamics, doi: 10.1007/s11071-006-9080-6, in press.