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Synchronization control of stochastic delayed neural networks ☆

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Abstract

In this paper, synchronization control of stochastic neural networks with time-varying delays has been considered. A novel control method is given using the Lyapunov functional method and linear matrix inequality (LMI) approach. Several sufficient conditions have been derived to ensure the global asymptotical stability in mean square for the error system, and thus the drive system synchronize with the response system. Also, the estimation gains can be obtained. With these new and effective methods, synchronization can be achieved. Simulation results are given to verify the theoretical analysis in this paper.

Keywords: Synchronization; Time-varying delays; Lyapunov functional; Chaos; LMI approach; Stochastic delayed system

1. Introduction

In recent years, stability of stochastic delayed system [1–8] has been a focal subject for research due to the uncertainties that exist in the real system. Stochastic modelling has come to play an important role in many branches of science and industry. A real system is usually affected by external perturbations which in many cases are of great uncertainties and hence may be treated as random, as fluctuations from the release of neurotransmitters, and other probabilistic causes. Therefore, it is significant and of prime importance to consider stochastic effects to the stability property of the delayed networks.

An area of particular interest has been the automatic control of synchronization of stochastic systems. To the best of our knowledge, however, there are few works about the synchronization of stochastic delayed system. Actually, chaos synchronization control and dynamics of neural networks or complex networks [12–19,24–28] has attracted interesting attention. A chaotic system has complex dynamical behaviors that possess some special features, such as being extremely sensitive to tiny variations of initial conditions, having bounded trajectories in phase space with a positive Lyapunov exponent, and so on. Chaos control and synchronization have seen flurry research activities over a decade. On the other hand, the possibility of

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encoding a message within a chaotic dynamics through tiny perturbations of a control parameter has been recently shown. This suggests to use chaos synchronization to produce secure message communication between a sender and a receiver.

Chaos dynamics has shown interesting features that make it attractive especially for secure communication. However, a certain number of drawbacks have been revealed in the practical implementation of most chaos-based secure communications algorithms due to the uncertainties of the system. Since few works about the synchronization of stochastic delayed system have been investigated due to the uncertainties in the real system, thus, in this paper, we consider synchronization control of stochastic delayed neural networks with time-varying delays.

The rest of the proposed paper is organized as follows: in Section 2, we give formulation and preliminaries for our main results. In Section 3, some sufficient conditions are presented for the synchronization of the delayed drive and response system. Also, some corollaries and remarks are given to show the advantages of this paper. In Section 4, examples are given to show the effectiveness and feasibility of this paper. In Section 5, we give our conclusions.

2. Model formulation and preliminaries

In this section, we will give preliminary knowledge for our main results. Since most of the synchronization methods belong to master–slave (drive–response) type. By one system driving another we mean that the two systems are coupled so that the behavior of the second is influenced by the behavior of the first one, but the behavior of the first is independent of the second. The first system will be called the master system or drive system, and the second system will be the slave system or response system.

In this paper, the object is to design a controller to let the slave system synchronize with the master system.

Now let us consider the following recurrent network:

$$\mathrm{d}x(t) = [-Cx(t) + Af(x(t)) + Bf(x(t-\tau(t)))]\,\mathrm{d}t \tag{1}$$

or

$$\mathrm{d}x_i(t) = \left[-c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau(t)))\right]\mathrm{d}t, \quad i = 1, 2, \ldots, n, \tag{2}$$

where n denotes the number of neurons in the network, $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^{\mathrm{T}} \in R^n$ is the state vector associated with the neurons, $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t)))^{\mathrm{T}} \in R^n$ corresponds to the activation functions of neurons, $\tau(t)$ is the time-varying delay, we suppose $\tau(t)$ is bounded and the initial conditions of (1) are given by $x_i(t) = \phi_i(t) \in \mathscr{C}([-r, 0], R)$ with $r = \max_{t \in R}\{\tau(t)\}$, where $\mathscr{C}([-r, 0], R)$ denotes the set of all continuous functions from $[-r, 0]$ to R. $C = diag(c_1, c_2, \ldots, c_n)$ is a diagonal matrix, $A = (a_{ij})_{n\times n}$ and $B = (b_{ij})_{n\times n}$ are the connection weight matrix and the delayed connection weight matrix, respectively.

In this paper, we consider model (1) as the master system. The response system is

$$\mathrm{d}y(t) = [-Cy(t) + Af(y(t)) + Bf(y(t-\tau(t))) + u(t)]\,\mathrm{d}t + \sigma(t, e(t), e(t-\tau(t)))\,\mathrm{d}\omega(t), \tag{3}$$

namely,

$$\begin{aligned}\mathrm{d}y_i(t) = {} & \left[-c_i y_i(t) + \sum_{j=1}^{n} a_{ij} f_j(y_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(y_j(t-\tau(t))) + u_i(t)\right]\mathrm{d}t \\ & + \sum_{j=1}^{n} \sigma_{ij}(t, e(t), e(t-\tau(t)))\,\mathrm{d}\omega_j(t), \quad i = 1, 2, \ldots, n,\end{aligned} \tag{4}$$

where C, A, B are matrices which are the same as (1), $u(t)$ is the controller. It has the same structure as the drive system. $e(t) = y(t) - x(t)$ is the error state, $\omega(t) = (\omega_1(t), \omega_2(t), \ldots, \omega_n(t))$ is a n dimension Brownian motion defined on a complete probability space $(\Omega, \mathscr{F}, \mathbb{P})$ with a natural filtration $\{\mathscr{F}_t\}_{t\geqslant 0}$ generated by $\{\omega(s) : 0 \leqslant s \leqslant t\}$, where we associate Ω with the canonical space generated by $\omega(t)$, and denote $\mathscr{F}$ the associated σ-algebra generated by $\{\omega(t)\}$ with the probability measure $\mathbb{P}$. Here, the white noise $\mathrm{d}\omega_i(t)$ is independent of $d\omega_j(t)$ for mutually different i and j, and $\sigma : \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n \longrightarrow \mathbb{R}^{n\times n}$ is called the noise intensity function matrix. This type of stochastic perturbation can be regarded as a result from the occurrence of random uncertainties of the neural network. Also the initial conditions of (3) are given by $y_i(t) = \psi_i(t) \in \mathscr{C}([-r,0],R)$ with $r = \max_{t\in R}\{\tau(t)\}$, where $\mathscr{C}([-r,0],R)$ denotes the set of all continuous functions from $[-r,0]$ to R. In practical situation, the output signals of the drive system (1) can be received by the response system (3).

To establish our main results, it is necessary to make the following assumptions:

A_1: Each function $f_i : R \to R$ is nondecreasing and globally Lipschitz with a constant $k_i > 0$, i.e.

$$|f_i(u) - f_i(v)| \leqslant k_i|u - v| \quad \forall u,\ v \in R,\ i = 1,2,\ldots,n. \tag{5}$$

A_2: $\sigma : \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n \longrightarrow \mathbb{R}^{n\times n}$ which is locally Lipschitz continuous and satisfies the linear growth condition [9]. Moreover, σ satisfies

$$trace[\sigma^{\mathrm{T}}(t,e(t),e(t-\tau(t))\sigma(t,e(t),e(t-\tau(t))] \leqslant \|M_1e(t)\|^2 + \|M_2e(t-\tau)\|^2, \tag{6}$$

where M_1 and M_2 are matrices with appropriate dimensions.

A_3: $\tau(t)$ is a bounded differential function of time t, and the following conditions are satisfied:

$$r = \max_{t\in R}\{\tau(t)\}, \quad 0 \leqslant \dot{\tau}(t) \leqslant h < 1, \tag{7}$$

where r and h are positive constants.

Let error state be $e(t) = y(t) - x(t)$, subtracting (1) from (3), yields the synchronization error dynamical system as follows:

$$de(t) = [-Ce(t) + Ag(t) + Bg(t-\tau(t)) + u(t)]\,dt + \sigma(t,e(t),e(t-\tau(t)))\,d\omega(t), \tag{8}$$

where $g(t) = f(y(t)) - f(x(t)) = f(x(t)+e(t)) - f(x(t))$.

In many real applications, we are interested in designing a memoryless state-feedback controller

$$u(t) = Ge(t), \tag{9}$$

where $G \in \mathbb{R}^{n\times n}$ is a constant gain matrix.

For a special case where the information on the size of time-varying delay $\tau(t)$ is available, we also consider a delayed feedback controller of the following form:

$$u(t) = Ge(t) + G_1e(t-\tau(t)). \tag{10}$$

Although a memoryless controller (9) has an advantage of easy implementation, its performance cannot be better than a delayed feedback controller which utilize the available information of the size of time-varying delay. A more general form of a delayed feedback controller is

$$u(t) = Ge(t) + \int_{t-\tau}^{t} G_2e(s)\,ds. \tag{11}$$

However, the task of storing all the previous state $e(\cdot)$ is difficult. In this respect, the controller (10) could be considered as a compromise between the performance improvement and the implementation simplicity.

Let $u(t) = Ge(t) + G_1e(t-\tau(t))$, and substituting this into (8), we obtain

$$de(t) = [(-C+G)e(t) + Ag(t) + G_1e(t-\tau(t)) + Bg(t-\tau(t))]\,dt + \sigma(t,e(t),e(t-\tau(t)))\,d\omega(t). \tag{12}$$

Also, the following definition is needed.

Initial function in (12) is $\varphi(t) = \psi(t) - \phi(t)$, where $\varphi(t) \in L^2_{\mathscr{F}_0}([-r,0];\mathbb{R}^n)$, here $L^2_{\mathscr{F}_0}([-r,0];\mathbb{R}^n)$ denotes the family of $\mathbb{R}^n$-valued stochastic processes $\xi(s), -r \leqslant s \leqslant 0$ such that $\xi(s)$ is $\mathscr{F}_0$-measurable and $\int_{-r}^{0} \mathbf{E}\|\xi(s)\|^2\,ds < \infty$. It is well known that system (12) has a unique solution [9].

Definition 1. System (12) is said to be globally asymptotically stable in mean square if for any given condition such that

$$\lim_{t\longrightarrow\infty} \mathbf{E}\|e(t)\|^2 \longrightarrow 0, \tag{13}$$

where $\mathbf{E}\{\cdot\}$ is the mathematical expectation.

3. Criteria of syncrhonization

In this section, new criteria are presented for the global asymptotical stability of the equilibrium point of the neural network defined by (12), and thus the drive system (1) synchronize with the response system (3). Its proof is based on a new Lyapunov functional method and linear matrix inequality (LMI) approach [23].

Theorem 1. *Under the assumptions* $A_1 - A_3$, *the equilibrium point of model* (12) *is globally asymptotically stable in mean square if there are positive definite diagonal matrix* $D = diag(d_1, d_2, \ldots, d_n) > 0$ *and positive definite matrices* $H = (h_{ij})_{n\times n}$, $P = (p_{ij})_{n\times n}$, $R = (r_{ij})_{n\times n}$, *such that*

$$N = \begin{pmatrix} P(-C+G) + (-C+G)^{\mathrm{T}}P + R + \rho M^{\mathrm{T}}M & PG_1 & PA + KD & PB \\ G_1^{\mathrm{T}}P & \rho M_1^{\mathrm{T}}M_1 - (1-h)R & 0 & 0 \\ A^{\mathrm{T}}P + DK & 0 & H - 2D & 0 \\ B^{\mathrm{T}}P & 0 & 0 & -(1-h)H \end{pmatrix} < 0, \tag{14}$$

$$P \leqslant \rho I. \tag{15}$$

Proof. Consider the Lyapunov functional

$$V(t) = \sum_{i=1}^{i=3} V_i(t), \tag{16}$$

where

$$V_1(t) = e^{\mathrm{T}}(t)Pe(t), \tag{17}$$

$$V_2(t) = \int_{t-\tau(t)}^{t} e^{\mathrm{T}}(s)Re(s)\,\mathrm{d}s, \tag{18}$$

$$V_3(t) = \int_{t-\tau(t)}^{t} g^{\mathrm{T}}(s)Hg(s)\,\mathrm{d}s, \tag{19}$$

where $H = (h_{ij})_{n\times n}$, $R = (r_{ij})_{n\times n}$ and $P = (p_{ij})_{n\times n}$ are positive definite matrices.

The weak infinitesimal operator $\mathscr{L}$ of the stochastic process $\{x_t = x(t+s), t \geqslant 0, -r \leqslant s \leqslant 0\}$ is given by [1,10]

$$\begin{aligned} \mathscr{L}V_1(t) = {} & 2e^{\mathrm{T}}(t)P[(-C+G)e(t) + Ag(t) + G_1 e(t-\tau(t)) + Bg(t-\tau(t))] \\ & + trace[\sigma^{\mathrm{T}}(t, e(t), e(t-\tau(t))P\sigma(t, e(t), e(t-\tau(t))]. \end{aligned} \tag{20}$$

By (6) and (15), we have

$$\begin{aligned} & trace[\sigma^{\mathrm{T}}(t, e(t), e(t-\tau(t))P\sigma(t, e(t), e(t-\tau(t))] \\ & \quad \leqslant \rho\, trace[\sigma^{\mathrm{T}}(t, e(t), e(t-\tau(t))\sigma(t, e(t), e(t-\tau(t))] \\ & \quad = \rho[e^{\mathrm{T}}(t)M^{\mathrm{T}}Me(t) + e^{\mathrm{T}}(t-\tau(t))M_1^{\mathrm{T}}M_1 e(t-\tau(t))], \end{aligned} \tag{21}$$

$$\mathscr{L}V_2(t) = e^{\mathrm{T}}(t)Re(t) - (1-\dot{\tau}(t))e^{\mathrm{T}}(t-\tau(t))Re(t-\tau(t)), \tag{22}$$

$$\mathscr{L}V_3(t) = g^{\mathrm{T}}(t)Hg(t) - (1-\dot{\tau}(t))g^{\mathrm{T}}(t-\tau(t))Hg(t-\tau(t)). \tag{23}$$

From Assumption A_1, it is obvious that

$$g^{\mathrm{T}}(t)DKe(t) = \sum_{i=1}^{n} g_i(t)d_i k_i e_i(t) \geqslant \sum_{i=1}^{n} d_i g_i^2(t) = g^{\mathrm{T}}(t)Dg(t), \tag{24}$$

where $D = diag(d_1, d_2, \ldots, d_n)$ and $K = diag(k_1, k_2, \ldots, k_n)$ are positive definite diagonal matrices.

Therefore, combining (20)–(24) we have

$$\begin{aligned}\mathscr{L}V(t) &= e^{\mathrm{T}}(t)[2P(-C+G)+R+\rho M^{\mathrm{T}}M]e(t)+2e^{\mathrm{T}}(t)PG_1e(t-\tau(t))+2e^{\mathrm{T}}(t)PAg(t)\\&\quad+2e^{\mathrm{T}}(t)PBg(t-\tau(t))+e^{\mathrm{T}}(t-\tau(t))[\rho M_1^{\mathrm{T}}M_1-(1-\dot{\tau}(t))R]e(t-\tau(t))+g^{\mathrm{T}}(t)Hg(t)\\&\quad-(1-\dot{\tau}(t))g^{\mathrm{T}}(t-\tau(t))Hg(t-\tau(t))\\&= e^{\mathrm{T}}(t)[2P(-C+G)+R+\rho M^{\mathrm{T}}M]e(t)+2e^{\mathrm{T}}(t)PG_1e(t-\tau(t))+2e^{\mathrm{T}}(t)PAg(t)\\&\quad+2e^{\mathrm{T}}(t)PBg(t-\tau(t))+e^{\mathrm{T}}(t-\tau(t))[\rho M_1^{\mathrm{T}}M_1-(1-h)R]e(t-\tau(t))+g^{\mathrm{T}}(t)Hg(t)\\&\quad-(1-h)g^{\mathrm{T}}(t-\tau(t))Hg(t-\tau(t))+2[g^{\mathrm{T}}(t)DKe(t)-g^{\mathrm{T}}(t)Dg(t)]\\&= e^{\mathrm{T}}(t)[P(-C+G)+(-C+G)^{\mathrm{T}}P+R+\rho M^{\mathrm{T}}M]e(t)+2e^{\mathrm{T}}(t)PG_1e(t-\tau(t))\\&\quad+2e^{\mathrm{T}}(t)[PA+KD]g(t)+2e^{\mathrm{T}}(t)PBg(t-\tau(t))+e^{\mathrm{T}}(t-\tau(t))[\rho M_1^{\mathrm{T}}M_1-(1-h)R]e(t-\tau(t))\\&\quad+g^{\mathrm{T}}(t)(H-2D)g(t)-(1-h)g^{\mathrm{T}}(t-\tau(t))Hg(t-\tau(t))\\&= (e^{\mathrm{T}}(t)\ \ e^{\mathrm{T}}(t-\tau(t))\ \ g^{\mathrm{T}}(t)\ \ g^{\mathrm{T}}(t-\tau(t)))N\begin{pmatrix}e(t)\\ e(t-\tau(t))\\ g(t)\\ g(t-\tau(t))\end{pmatrix}.\end{aligned}\tag{25}$$

From (25) and Itô formula, it is obvious to see that

$$\mathbf{E}V(t)-\mathbf{E}V(t_0)=\mathbf{E}\int_{t_0}^{t}\mathscr{L}V(s)\,\mathrm{d}s.\tag{26}$$

From the definition of $V(t)$ in (16), there exist positive constant λ_1 such that

$$\lambda_1\mathbf{E}\|e(t)\|^2\leqslant\mathbf{E}V(t)\leqslant\mathbf{E}V(t_0)+\mathbf{E}\int_{t_0}^{t}\mathscr{L}V(s)\,\mathrm{d}s\leqslant\mathbf{E}V(t_0)+\lambda_{max}\mathbf{E}\int_{t_0}^{t}\|e(s)\|^2\,\mathrm{d}s,\tag{27}$$

where λ_{max} is the maximal eigenvalue of N and it is negative.

Therefore, from (27) and the discussion in Ref. [11], we know that the equilibrium of (12) is globally asymptotically stable in mean square. This completes the proof. □

Corollary 1. *Under the assumptions A_1-A_3, the equilibrium point of model* (12) *is globally asymptotically stable in mean square if there are positive definite diagonal matrix $D=diag(d_1,d_2,\ldots,d_n)>0$, positive definite matrices $H=(h_{ij})_{n\times n}$, $R=(r_{ij})_{n\times n}$, such that*

$$N=\begin{pmatrix}\rho(-C+G)+\rho(-C+G)^{\mathrm{T}}+R+\rho M^{\mathrm{T}}M & \rho G_1 & \rho A+KD & \rho B\\ \rho G_1^{\mathrm{T}} & \rho M_1^{\mathrm{T}}M_1-(1-h)R & 0 & 0\\ \rho A^{\mathrm{T}}+DK & 0 & H-2D & 0\\ \rho B^{\mathrm{T}} & 0 & 0 & -(1-h)H\end{pmatrix}<0.\tag{28}$$

Proof. Let $P=\rho I$, where I is the identity matrix. We can obtain Corollary 1 directly form Theorem 1. □

In order to show the design of estimate gain matrix G and G_1, a simple transformation is made to derive the following theorem:

Theorem 2. *Under the assumptions A_1-A_3, the equilibrium point of model* (12) *is globally asymptotically stable in mean square if there are positive definite diagonal matrix $D=diag(d_1,d_2,\ldots,d_n)>0$ and positive definite* *matrices* $H=(h_{ij})_{n\times n}$, $P=(p_{ij})_{n\times n}$, $R=(r_{ij})_{n\times n}$, *such that*

$$N=\begin{pmatrix} -PC+G'-C^{\mathrm{T}}P+G'^{\mathrm{T}}+R+\rho M^{\mathrm{T}}M & G'_1 & PA+KD & PB \\ G_1'^{\mathrm{T}} & \rho M_1^{\mathrm{T}}M_1-(1-h)R & 0 & 0 \\ A^{\mathrm{T}}P+DK & 0 & H-2D & 0 \\ B^{\mathrm{T}}P & 0 & 0 & -(1-h)H \end{pmatrix}<0. \tag{29}$$

$$P\leqslant \rho I, \tag{30}$$

Moreover, the estimation gain $G=P^{-1}G'$ and $G_1=P^{-1}G'_1$.

Proof. Let $G=P^{-1}G'$ and $G^1=P^{-1}G'_1$ in Theorem 1, it is obvious to see. □

Corollary 2. *Under the assumptions* A_1-A_3, *the equilibrium point of model* (12) *is globally asymptotically stable in mean square if there are positive definite diagonal matrix* $D=diag(d_1,d_2,\dots,d_n)>0$ *and positive definite matrices* $H=(h_{ij})_{n\times n}$, $R=(r_{ij})_{n\times n}$, *such that*

$$N=\begin{pmatrix} -\rho C+G'-\rho C^{\mathrm{T}}+G'^{\mathrm{T}}+R+\rho M^{\mathrm{T}}M & G'_1 & \rho A+KD & \rho B \\ G_1'^{\mathrm{T}} & \rho M_1^{\mathrm{T}}M_1-(1-h)R & 0 & 0 \\ \rho A^{\mathrm{T}}+DK & 0 & H-2D & 0 \\ \rho B^{\mathrm{T}} & 0 & 0 & -(1-h)H \end{pmatrix}<0. \tag{31}$$

Moreover, the estimation gain $G=\rho^{-1}G'$ *and* $G_1=\rho^{-1}G'_1$.

Proof. Let $P=\rho I$ in Theorem 2, where I is the identity matrix and it is obvious to see. □

Remark 1. Stability of stochastic delayed system [1–8] have been a focal subject for research due to random uncertainties exist in the real system. However, there are few works about the synchronization of stochastic delayed system. In this paper, we consider the synchronization of stochastic delayed neural networks with time-varying delays.

Remark 2. Chaos synchronization has been a hot topic in nonlinear science and has attracted more attention in many fields such as physics, secure communication, automatical control, artificial neural networks, etc. Many recent works [12–19,24–27] have been added to it. However, few studies about chaos synchronization of stochastic delayed system are considered. If there is no stochastic phenomenon, the existing results are special cases in our paper.

Remark 3. In Theorem 2, we give an approach to choose the estimation gain matrices G and G_1 and it is helpful for the design of the controller to let the drive system synchronize with the response system.

4. Numerical example

In this section, we will give an example to justify Theorem 2 obtained above.

Example 1. Consider the drive system (1) of a typical delayed Hopfield neural network as follows:

$$\mathrm{d}x(t)=[-Cx(t)+Ag(x(t))+Bg(x(t-\tau(t)))]\,\mathrm{d}t,$$

where

$$C=\begin{pmatrix}1&0\\0&1\end{pmatrix},\quad A=\begin{pmatrix}2.0&-0.1\\-5.0&2.8\end{pmatrix},\quad B=\begin{pmatrix}-1.6&-0.1\\-0.3&-2.5\end{pmatrix},\quad \tau(t)=1,\quad g(x)=\begin{pmatrix}\tanh x_1\\ \tanh x_2\end{pmatrix}.$$

The corresponding response system can be

$$\mathrm{d}y(t)=[-Cy(t)+Ag(y(t))+Bg(y(t-\tau(t)))]\,\mathrm{d}t+\sigma(t,e(t),e(t-\tau(t)))\,\mathrm{d}\omega(t),$$

where

$$\sigma(t, e(t), e(t-\tau(t))) = \begin{pmatrix} \|e(t)\| & 0 \\ 0 & \|e(t-\tau(t))\| \end{pmatrix}.$$

It is easy to see $K = M = M_1 = I$, where I is the identity matrix. From Theorem 2 and using LMI toolbox in Matlab, we can obtain the following feasible solutions:

$$P = \begin{pmatrix} 0.4594 & 0.1681 \\ 0.1681 & 0.1916 \end{pmatrix}, \quad D = \begin{pmatrix} 2.0155 & 0 \\ 0 & 2.2435 \end{pmatrix}, \quad R = \begin{pmatrix} 3.8186 & 0.0541 \\ 0.0541 & 3.8764 \end{pmatrix}, \quad H = \begin{pmatrix} 1.7108 & 0.1360 \\ 0.1360 & 1.6403 \end{pmatrix},$$

$$G' = \begin{pmatrix} -4.8626 & -0.1211 \\ -0.1211 & -5.3828 \end{pmatrix}, \quad G_1' = \begin{pmatrix} -1.0599 & -0.0189 \\ -0.0189 & -1.0900 \end{pmatrix}, \quad G = P^{-1}G' = \begin{pmatrix} -15.2473 & 14.7498 \\ 12.7445 & -41.0364 \end{pmatrix},$$

$$G_1 = P^{-1}G_1' = \begin{pmatrix} -3.445 & 3.0045 \\ 2.8352 & -8.3251 \end{pmatrix}, \quad \rho = 1.8775.$$

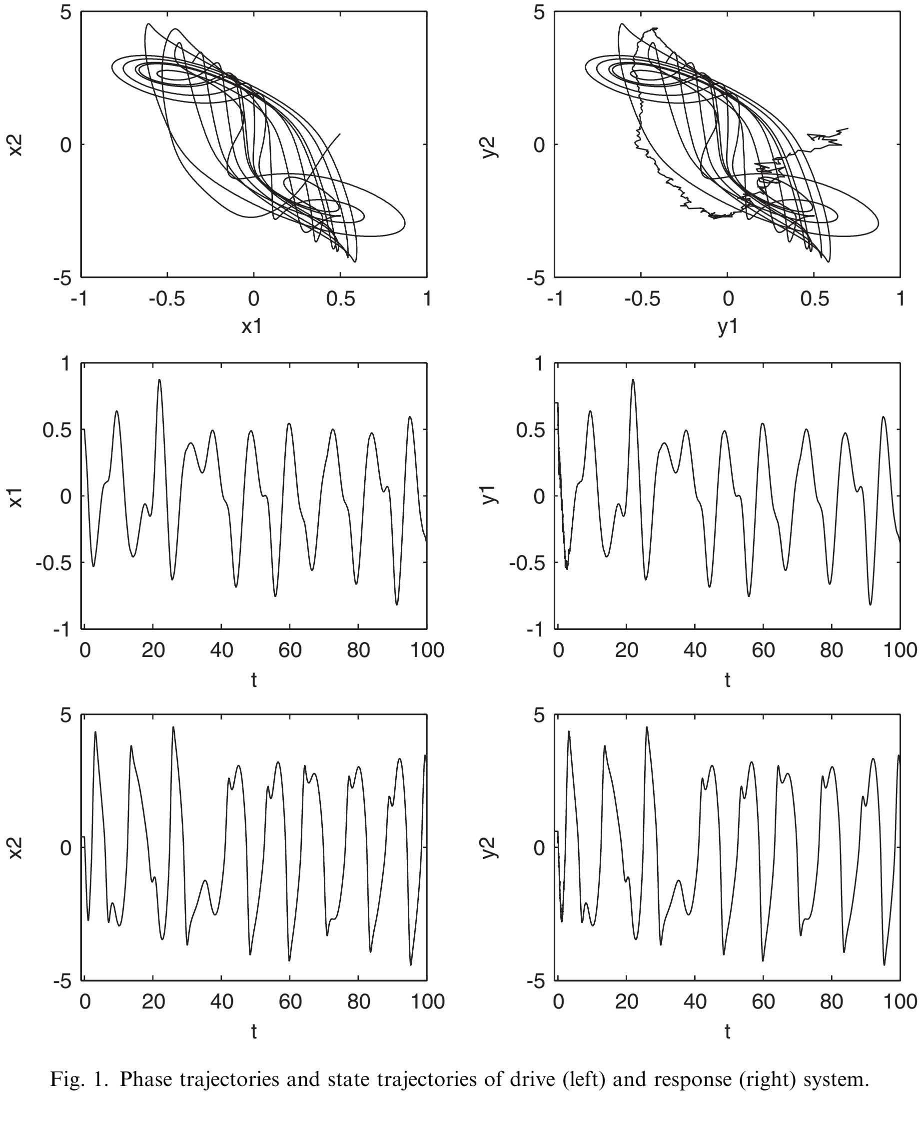

Fig. 1. Phase trajectories and state trajectories of drive (left) and response (right) system.

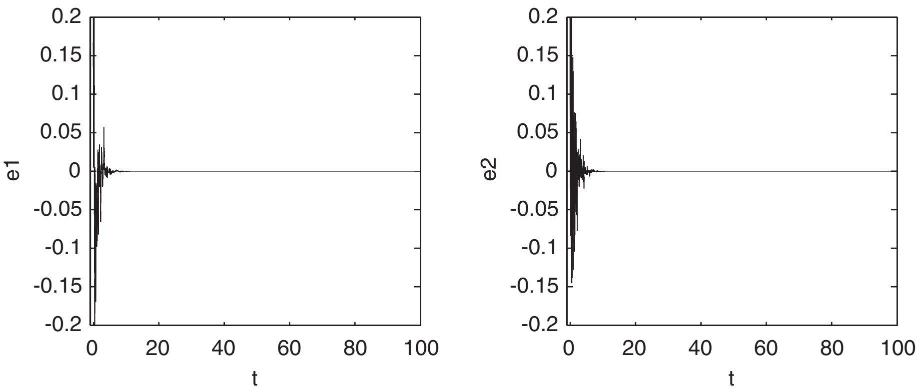

Fig. 2. State trajectories of error system.

Some works [20–22] about numerical simulations of stochastic delayed differential equations have been investigated. In this paper, we adopt the so-called Euler–Maruyama numerical scheme [20] to simulate the drive system (1), the response system (3) and the error system (12). It is noted that the numerical solution given by Euler–Maruyama numerical scheme will converge to true solution of the equilibrium point of the system in an expectation sense as the sampling time step size tends to zero. The phase trajectories of the drive, response are shown in Fig. 1. The trajectories of error system (12) are shown in Fig. 2. From Figs. 1 and 2, we see that the drive system synchronize with the response system.

Remark 4. If we choose a memoryless state-feedback controller (9) to stabilize system in Example 1, i.e. $G_1 = 0$. From Theorem 2 ($G_1' = 0$) and using LMI toolbox in Matlab, we can obtain the feasible gain matrix

$$G = \begin{pmatrix} -15.4632 & 14.2180 \\ 12.5043 & -37.7964 \end{pmatrix}.$$

In fact, the memoryless state-feedback controller (9) can also used to stabilize system (8). However, there has been extensive interest in studying the effect of time delay on the feedback systems. It is well known that time delay is ubiquitous in most physical, chemical, biological, neural, and other natural system due to finite propagation speeds of signals, finite processing times in synapses, and finite reaction times. Therefore, we consider delay-dependent feedback controller (10). If we choose $G_1 = 0$, it is the memoryless state-feedback controller (9). So (9) is a special case of (10). Actually, both the memoryless state-feedback controller (9) and the delay-dependent feedback controller (10) can be used to stabilize system (8). However, delay is ubiquitous in the real system and (10) is a more general controller. Thus in this paper, we consider the delay-dependent feedback controller (10).

5. Conclusions

In this paper, we considered synchronization control of stochastic neural networks with time-varying delays. We use Lyapunov functional method and linear matrix inequality (LMI) technique to solve this problem. Several sufficient conditions have been derived to ensure the global asymptotical stability for the error system, and thus the drive system synchronizes with the response system. Also, the estimation gains can be obtained. The obtained results are novel since there are few works about the synchronization of delayed system. It is easy to apply these sufficient conditions to the real networks. Finally, we give a numerical simulation to verify the theoretical results.

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