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# Adaptive synchronization and lag synchronization of uncertain dynamical system with time delay based on parameter identification ☆

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**Abstract**

In this paper, the adaptive synchronization and lag synchronization are considered for uncertain dynamical system with time delay based on parameter identification and a novel control method is then further given using the Lyapunov functional method. With this new and effective method, parameter identification and lag synchronization can be achieved simultaneously. Simulation results are given to justify the theoretical analysis in this paper.


*Keywords:* Synchronization; Lag synchronization; Parameter identification; Lyapunov functional; Chaos

## 1. Introduction

Dynamical behaviors [1–30] are interesting nonlinear phenomena and have been intensively investigated in many years due to its potential applications in secure communications [6], chemical reactions, biological systems and so on. Stability [1–5,33], bifurcation [13–19] and chaos synchronization [7–12,20–30,34] are studied by many researchers.

It is known that chaotic systems exhibit sensitive dependence on initial conditions. Because of this property, chaotic systems are difficult to be synchronized or controlled. However, important results have been reported on the control and synchronization of chaotic systems [7–12,20–30,34] in recent years. Chaos dynamics have shown interesting features that make it attractive especially for secure communication. However, a certain number of drawbacks have been revealed in the practical implementation of most chaos-based secure communications algorithms. In particular, one of the basic issues of interest is the effect of uncertainties and parameters mismatch on the stability of the process of synchronization of the chaotic oscillators. To overcome these difficulties, various adaptive synchronization schemes have been proposed and investigated [20–30].

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Among these strategies, those based on the identification of system parameters appear to be of great practical interest, especially when the system state is available to external measurements. The main method is to use the possible modulation of a system parameter based on the transmitted message.

In particular, a time delay will occur in the activation between the neurons in electronic implementation of dynamical systems, which will affect the dynamical behaviors of the neuron system. In recent years, a lot of efforts have been made to study the dynamical behaviors of the delayed systems [1–20]. On the other hand, besides time-delayed features of such dynamical systems, there might also be some uncertainties such as perturbations and component variations, which might lead to very complex dynamical behaviors. Therefore, time delays should be considered into the synchronization of dynamical systems.

In this paper, we will study synchronization and lag synchronization of dynamical systems, which is presented based on parameter identification and Lyapunov functional method. The obtained results improve and extend the earlier works. In addition, we proposed a new concept: lag synchronization and considered lag synchronization of dynamical systems with time delay.

The organization of this paper is as follows: In Section 2, preliminaries and main results are given. Several sufficient conditions are presented for the synchronization and lag synchronization of dynamical systems. In Section 3, some remarks and examples are constructed to show the effectiveness and feasibility of this paper. The conclusions are finally drawn in Section 4.

## 2. Preliminaries and main results

In this section, the main results for adaptive synchronization and lag synchronization of uncertain dynamical systems with time delay are proposed. As is known to all, most of the synchronization methods belong to drive–response type, which means that two systems are coupled by one system driving another so that the behavior of the second is dependent on the behavior of the first, but the first in not influenced by the behavior of the second. The first system will be called the drive system and the second will the response system.

In practice, determining some system parameters in advance may be difficult. Furthermore, most parametrical values are characterized by uncertainties related to experimental conditions (temperature, external electric and magnetic field) that can destroy or even break the synchronization. These problems can be tackled to a certain extent through adaptive synchronization.

The drive system considered in this paper is as follows:

$$\dot{x}(t) = f(x(t)) + F(x(t))\theta_1 + G(x(t-\tau(t)))\theta_2, \tag{1}$$

where $\tau(t)$ is a function of time delay, $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^{\mathrm{T}} \in R^n$ is the state vector associated with the neurons, $\theta_1 = (\theta_{11}, \theta_{12}, \ldots, \theta_{1n})^{\mathrm{T}} \in R^n$ and $\theta_2 = (\theta_{21}, \theta_{22}, \ldots, \theta_{2n})^{\mathrm{T}} \in R^n$ are the constant vectors of the system parameters, $f : R^n \longrightarrow R^n$, $F : R^n \longrightarrow R^{n\times n}$, $G : R^n \longrightarrow R^{n\times n}$. The response system is

$$\dot{y}(t) = f(y(t)) + F(y(t))\alpha_1(t) + G(y(t-\tau(t)))\alpha_2(t) + u(t), \tag{2}$$

where $\alpha_1(t) = (\alpha_{11}(t), \alpha_{12}(t), \ldots, \alpha_{1n}(t))^{\mathrm{T}} \in R^n$ and $\alpha_2(t) = (\alpha_{21}(t), \alpha_{22}(t), \ldots, \alpha_{2n}(t))^{\mathrm{T}} \in R^n$ are functions depending on the time $t$, $u$ is the controller. It has the same structure as the drive system but the parameter vectors $\alpha_1(t)$ and $\alpha_2(t)$ are unknown. In practical situation, the output signals (state vector) of the drive system (1) can be received by the response system (2), but the parameter vectors $\theta_1$ and $\theta_2$ of the drive system (1) may not be known a priori, which need to be identified.

The problem is to design an adaptive synchronization algorithm

$$u = u(x, y, \alpha_1, \alpha_2, t), \quad \dot{\alpha}_1 = \alpha_1(x, y, \alpha_1, \alpha_2, t), \quad \dot{\alpha}_2 = \alpha_2(x, y, \alpha_1, \alpha_2, t),$$

where, $\alpha_1$ and $\alpha_2$ are the vectors of parameter estimates of the unknown parameter vectors $\theta_1$ and $\theta_2$. The object of this paper is to design $u$, $\alpha_1$ and $\alpha_2$ to force the state $y(t)$ of the response system (2) to asymptotically synchronize with the state $x(t)$ of the drive system (1), i.e., to archive

$$y(t) - x(t) \longrightarrow 0, \quad t \longrightarrow \infty,$$

$$\alpha_1(t) - \theta_1 \longrightarrow 0, \quad t \longrightarrow \infty,$$

$$\alpha_2(t) - \theta_2 \longrightarrow 0, \quad t \longrightarrow \infty.$$

Let

$$e(t) = y(t) - x(t),$$

then subtracting (1) from (2) yields the synchronization error dynamical system as follows:

$$\dot{e}(t) = f(y(t)) + F(y(t))\alpha_1(t) + G(y(t-\tau(t)))\alpha_2(t) - f(x(t)) - F(x(t))\theta_1 - G(x(t-\tau(t)))\theta_2 + u(t). \quad (3)$$

## 2.1. Adaptive synchronization

**Theorem 1.** *The drive system* (1) *synchronizes with the response system* (2) *if we choose*

$$u = -e(t) + f(x(t)) - f(y(t)) + [F(x(t)) - F(y(t))]\alpha_1 + [G(x(t-\tau(t))) - G(y(t-\tau(t)))]\alpha_2, \quad (4)$$

$$\dot{\alpha}_1(t) = -Q^{-1}F^{\mathrm{T}}(x(t))Pe(t), \quad (5)$$

$$\dot{\alpha}_2(t) = -R^{-1}G^{\mathrm{T}}(x(t-\tau(t)))Pe(t), \quad (6)$$

*where $P$, $Q$ and $R$ are the positive definite matrices, $u$, $\alpha_1$ and $\alpha_2$ are independent of $\theta_1$ and $\theta_2$.*

**Proof.** According to (4), we rewrite the error system (3) as

$$\dot{e}(t) = -e(t) + F(x(t))(\alpha_1(t) - \theta_1) + G(x(t-\tau(t)))(\alpha_2(t) - \theta_2). \quad (7)$$

Choose the following Lyapunov functional candidate:

$$\begin{aligned} V(e(t), \alpha_1(t), \alpha_2(t)) = {} & \tfrac{1}{2}e^{\mathrm{T}}(t)Pe(t) + \tfrac{1}{2}(\alpha_1(t) - \theta_1)^{\mathrm{T}}Q(\alpha_1(t) - \theta_1) \\ & + \tfrac{1}{2}(\alpha_2(t) - \theta_2)^{\mathrm{T}}R(\alpha_2(t) - \theta_2), \end{aligned} \quad (8)$$

where $P$, $Q$ and $R$ are the positive definite matrices. Differentiating $V$ with respect to time along the solution of (7) yields

$$\begin{aligned} \frac{\mathrm{d}V}{\mathrm{d}t} &= e^{\mathrm{T}}(t)P\dot{e}(t) + \dot{\alpha}_1^{\mathrm{T}}Q(\alpha_1(t) - \theta_1) + \dot{\alpha}_2^{\mathrm{T}}R(\alpha_2(t) - \theta_2) \\ &= e^{\mathrm{T}}(t)P[-e(t) + F(x(t))(\alpha_1(t) - \theta_1) + G(x(t-\tau(t)))(\alpha_2(t) - \theta_2)] + \dot{\alpha}_1^{\mathrm{T}}Q(\alpha_1(t) - \theta_1) \\ &\quad + \dot{\alpha}_2^{\mathrm{T}}R(\alpha_2(t) - \theta_2) \\ &= -e^{\mathrm{T}}(t)Pe(t) + [\dot{\alpha}_1(t) + Q^{-1}F^{\mathrm{T}}(x(t))Pe(t)]^{\mathrm{T}}Q(\alpha_1(t) - \theta_1) \\ &\quad + [\dot{\alpha}_2(t) + R^{-1}G^{\mathrm{T}}(x(t-\tau(t)))Pe(t)]^{\mathrm{T}}R(\alpha_2(t) - \theta_2) \\ &= -e^{\mathrm{T}}(t)Pe(t). \end{aligned} \quad (9)$$

We can therefore conclude that $V$ is a Lyapunov functional of the error system (7) corresponding with (5) and (6), i.e.,

$$y(t) - x(t) \longrightarrow 0, \quad t \longrightarrow \infty,$$

$$\alpha_1(t) - \theta_1 \longrightarrow 0, \quad t \longrightarrow \infty,$$

$$\alpha_2(t) - \theta_2 \longrightarrow 0, \quad t \longrightarrow \infty.$$

This completes the proof. □

**Corollary 1.** *The drive system* (1) *synchronizes with the response system* (2) *if we choose*

$$u = -e(t) + f(x(t)) - f(y(t)) + [F(x(t)) - F(y(t))]\alpha_1 + [G(x(t-\tau(t))) - G(y(t-\tau(t)))]\alpha_2,$$

$$\dot{\alpha}_1(t) = -F^{\mathrm{T}}(x(t))Pe(t),$$

$$\dot{\alpha}_2(t) = -G^{\mathrm{T}}(x(t-\tau(t)))Pe(t),$$

*where $P$ is a positive definite matrix, $u$, $\alpha_1$ and $\alpha_2$ are independent of $\theta_1$ and $\theta_2$.*

**Proof.** Let $Q = R = I$, where $I$ is the identity matrix, we can get Corollary 1 directly from Theorem 1. □

**Corollary 2.** *The drive system* (1) *synchronizes with the response system* (2) *if we choose*

$$u = -e(t) + f(x(t)) - f(y(t)) + [F(x(t)) - F(y(t))]\alpha_1 + [G(x(t-\tau(t))) - G(y(t-\tau(t)))]\alpha_2,$$

$$\dot{\alpha}_1(t) = -F^{\mathrm{T}}(x(t))e(t),$$

$$\dot{\alpha}_2(t) = -G^{\mathrm{T}}(x(t-\tau(t)))e(t),$$

*where* $u$, $\alpha_1$ *and* $\alpha_2$ *are independent of* $\theta_1$ *and* $\theta_2$.

**Proof.** Let $P = Q = R = I$, where $I$ is the identity matrix, we can get Corollary 2 easily from Theorem 1. Here, we omit it. □

### 2.2. Adaptive lag synchronization

In the real systems, there are also transmission delays in the drive system. So in this subsection, we consider another subject: lag synchronization.

**Definition 1.** The drive system (1) is said to lag synchronize with the response system (2) at time $r$ if

$$y(t) - x(t-r) \longrightarrow 0, \quad t \longrightarrow \infty, \tag{10}$$

where $r$ is a given positive time delay.

From (1), we have

$$\dot{x}(t-r) = f(x(t-r)) + F(x(t-r))\theta_1 + G(x(t-r-\tau(t)))\theta_2. \tag{11}$$

Let

$$d(t) = y(t) - x(t-r),$$

subtracting (11) from (2) yields the lag synchronization error dynamical system as follows:

$$\begin{aligned}\dot{d}(t) = {} & f(y(t)) + F(y(t))\alpha_1(t) + G(y(t-\tau(t)))\alpha_2(t) - f(x(t-r)) - F(x(t-r))\theta_1 \\ & - G(x(t-r-\tau(t)))\theta_2 + u(t).\end{aligned} \tag{12}$$

**Theorem 2.** *The drive system* (1) *lag synchronizes with the response system* (2) *at time r if we choose*

$$\begin{aligned}u = {} & -d(t) + f(x(t-r)) - f(y(t)) + [F(x(t-r)) - F(y(t))]\alpha_1 \\ & + [G(x(t-r-\tau(t))) - G(y(t-\tau(t)))]\alpha_2,\end{aligned} \tag{13}$$

$$\dot{\alpha}_1(t) = -Q^{-1}F^{\mathrm{T}}(x(t-r))Pd(t), \tag{14}$$

$$\dot{\alpha}_2(t) = -R^{-1}G^{\mathrm{T}}(x(t-r-\tau(t)))Pd(t), \tag{15}$$

*where* $P$, $Q$ *and* $R$ *are the positive definite matrices,* $u$, $\alpha_1$ *and* $\alpha_2$ *are independent of* $\theta_1$ *and* $\theta_2$.

**Proof.** According to (13), we write error system (12) as

$$\dot{d}(t) = -d(t) + F(x(t-r))(\alpha_1(t) - \theta_1) + G(x(t-r-\tau(t)))(\alpha_2(t) - \theta_2). \tag{16}$$

Choose the following Lyapunov functional:

$$V(d(t), \alpha_1(t), \alpha_2(t)) = \tfrac{1}{2}d^{\mathrm{T}}(t)Pd(t) + \tfrac{1}{2}(\alpha_1(t) - \theta_1)^{\mathrm{T}}Q(\alpha_1(t) - \theta_1) + \tfrac{1}{2}(\alpha_2(t) - \theta_2)^{\mathrm{T}}R(\alpha_2(t) - \theta_2), \tag{17}$$

where $P$, $Q$ and $R$ are the positive definite matrices. Differentiating $V$ with respect to time along the solution of (16), we obtain

$$\begin{aligned}\frac{\mathrm{d}V}{\mathrm{d}t} &= d^{\mathrm{T}}P\dot{d}(t) + \dot{\alpha}_1^{\mathrm{T}}Q(\alpha_1(t)-\theta_1) + \dot{\alpha}_2^{\mathrm{T}}R(\alpha_2(t)-\theta_2)\\ &= d^{\mathrm{T}}P[-d(t) + F(x(t-r))(\alpha_1(t)-\theta_1) + G(x(t-r-\tau(t)))(\alpha_2(t)-\theta_2)] + \dot{\alpha}_1^{\mathrm{T}}Q(\alpha_1(t)-\theta_1)\\ &\quad + \dot{\alpha}_2^{\mathrm{T}}R(\alpha_2(t)-\theta_2)\\ &= -d^{\mathrm{T}}Pd + [\dot{\alpha}_1(t) + Q^{-1}F^{\mathrm{T}}(x(t-r))Pd(t)]^{\mathrm{T}}Q(\alpha_1(t)-\theta_1)\\ &\quad + [\dot{\alpha}_2(t) + R^{-1}G^{\mathrm{T}}(x(t-r-\tau(t)))Pd(t)]^{\mathrm{T}}R(\alpha_2(t)-\theta_2)\\ &= -d^{\mathrm{T}}Pd. \end{aligned} \tag{18}$$

We can therefore conclude that $V$ is a Lyapunov functional of the error system (16) corresponding with (14) and (15), i.e.,

$$y(t) - x(t-r) \longrightarrow 0, \quad t \longrightarrow \infty,$$

$$\alpha_1(t) - \theta_1 \longrightarrow 0, \quad t \longrightarrow \infty,$$

$$\alpha_2(t) - \theta_2 \longrightarrow 0, \quad t \longrightarrow \infty.$$

This completes the proof. □

**Corollary 3.** *The drive system* (1) *lag synchronizes with the response system* (2) *at time $r$ if we choose*

$$u = -d(t) + f(x(t)) - f(y(t-r)) + [F(x(t-r)) - F(y(t))]\alpha_1 + [G(x(t-r-\tau(t))) - G(y(t-\tau(t)))]\alpha_2,$$

$$\dot{\alpha}_1(t) = -F^{\mathrm{T}}P(x(t-r))d(t),$$

$$\dot{\alpha}_2(t) = -G^{\mathrm{T}}P(x(t-r-\tau(t)))d(t),$$

*where $P$ is a positive definite matrix, $u$, $\alpha_1$ and $\alpha_2$ are independent of $\theta_1$ and $\theta_2$.*

**Proof.** Let $Q = R = I$, where $I$ is the identity matrix, we can obtain Corollary 3 from Theorem 2. □

**Corollary 4.** *The drive system* (1) *lag synchronizes with the response system* (2) *at time $r$ if we choose*

$$\begin{aligned}u = &-d(t) + f(x(t)) - f(y(t-r)) + [F(x(t-r)) - F(y(t))]\alpha_1 + [G(x(t-r-\tau(t)))\\ &- G(y(t-\tau(t)))]\alpha_2,\end{aligned}$$

$$\dot{\alpha}_1(t) = -F^{\mathrm{T}}(x(t-r))d(t),$$

$$\dot{\alpha}_2(t) = -G^{\mathrm{T}}(x(t-r-\tau(t)))d(t),$$

*where $u$, $\alpha_1$ and $\alpha_2$ are independent of $\theta_1$ and $\theta_2$.*

**Proof.** Let $P = Q = R = I$, where $I$ is the identity matrix, we can get Corollary 4 easily from Theorem 2. We omit it. □

**Definition 2.** The drive system (1) is said to lag synchronize with the response system (2) at time $r$ in time interval $[T_1, T_2]$ if

$$y(t) - x(t-r) \longrightarrow 0, \quad t \in [T_1, T_2], \tag{19}$$

where $r$ is a given positive time delay.

Let

$$\tilde{e}(t) = y(t) - x(t - \tilde{r}(t)),$$

where $\tilde{r}(t)$ is a function of time $t$ and $\tilde{r}(t) \geqslant 0$ for all time $t$.

Combining Theorems 1 and 2, we have the following theorem:

**Theorem 3.** *The drive system* (1) *synchronizes with the response system* (2) *in time interval* $[0, T/2]$ *and lag synchronizes with the response system* (2) *at time r in time interval* $[T/2, T]$ *if we choose*

$$u = -\widetilde{e}(t) + f(x(t-\widetilde{r}(t))) - f(y(t)) + [F(x(t-\widetilde{r}(t))) - F(y(t))]\alpha_1 + [G(x(t-\widetilde{r}(t)-\tau(t))) - G(y(t-\tau(t)))]\alpha_2, \tag{20}$$

$$\dot{\alpha}_1(t) = -Q^{-1}F^{\mathrm{T}}(x(t-\widetilde{r}(t)))P\widetilde{e}(t), \tag{21}$$

$$\dot{\alpha}_2(t) = -R^{-1}G^{\mathrm{T}}(x(t-\widetilde{r}(t)-\tau(t)))P\widetilde{e}(t), \tag{22}$$

$$\widetilde{r}(t) = \begin{cases} 0, & t \in [0, T/2], \\ r, & t \in [T/2, T], \end{cases} \tag{23}$$

*where P, Q and R are the positive definite matrices, u,* $\alpha_1$ *and* $\alpha_2$ *are independent of* $\theta_1$ *and* $\theta_2$*. T is a sufficient large real value.*

**Proof.** It can be directly obtained form Theorems 1 and 2. Here we omit it. □

**Corollary 5.** *The drive system* (1) *synchronizes with the response system* (2) *in time interval* $[0, T/2]$ *and lag synchronizes with the response system* (2) *at time r in time interval* $[T/2, T]$ *if we choose*

$$u = -\widetilde{e}(t) + f(x(t-\widetilde{r}(t))) - f(y(t)) + [F(x(t-\widetilde{r}(t))) - F(y(t))]\alpha_1 + [G(x(t-\widetilde{r}(t)-\tau(t))) - G(y(t-\tau(t)))]\alpha_2,$$

$$\dot{\alpha}_1(t) = -F^{\mathrm{T}}(x(t-\widetilde{r}(t)))P\widetilde{e}(t),$$

$$\dot{\alpha}_2(t) = -G^{\mathrm{T}}(x(t-\widetilde{r}(t)-\tau(t)))P\widetilde{e}(t),$$

$$\widetilde{r}(t) = \begin{cases} 0, & t \in [0, T/2], \\ r, & t \in [T/2, T], \end{cases}$$

*where P is a positive definite matrix, u,* $\alpha_1$ *and* $\alpha_2$ *are independent of* $\theta_1$ *and* $\theta_2$*. T is a sufficient large real value.* □

**Proof.** Let $Q = R = I$ in Theorem 3. □

**Corollary 6.** *The drive system* (1) *synchronizes with the response system* (2) *in time interval* $[0, T/2]$ *and lag synchronizes with the response system* (2) *at time r in time interval* $[T/2, T]$ *if we choose*

$$u = -\widetilde{e}(t) + f(x(t-\widetilde{r}(t))) - f(y(t)) + [F(x(t-\widetilde{r}(t))) - F(y(t))]\alpha_1 + [G(x(t-\widetilde{r}(t)-\tau(t))) - G(y(t-\tau(t)))]\alpha_2,$$

$$\dot{\alpha}_1(t) = -F^{\mathrm{T}}(x(t-\widetilde{r}(t)))\widetilde{e}(t),$$

$$\dot{\alpha}_2(t) = -G^{\mathrm{T}}(x(t-\widetilde{r}(t)-\tau(t)))\widetilde{e}(t),$$

$$\widetilde{r}(t) = \begin{cases} 0, & t \in [0, T/2], \\ r, & t \in [T/2, T], \end{cases}$$

*where u,* $\alpha_1$ *and* $\alpha_2$ *are independent of* $\theta_1$ *and* $\theta_2$*. T is a sufficient large real value.*

**Proof.** Let $P = Q = R = I$ in Theorem 3. □

Finally, we consider a general situation that the delay $\widetilde{r}(t)$ is a more general function.

**Definition 3.** The drive system (1) is said to lag synchronize with the response system (2) at time $\widetilde{r}(t)$ if

$$y(t) - x(t - \widetilde{r}(t)) \longrightarrow 0, \quad t \longrightarrow \infty, \tag{24}$$

where $\widetilde{r}(t)$ is a function of time $t$, $\widetilde{r}(t) \geqslant 0$ for all time $t$.

From (1), we have

$$\dot{x}(t - \widetilde{r}(t)) = (1 - \dot{\widetilde{r}}(t))[f(x(t - \widetilde{r}(t))) + F(x(t - \widetilde{r}(t)))\theta_1 + G(x(t - \widetilde{r}(t) - \tau(t)))\theta_2]. \tag{25}$$

Subtracting (25) from (2) yields the lag synchronization error dynamical system as follows:

$$\begin{aligned}\dot{\widetilde{e}}(t) = {} & f(y(t)) + F(y(t))\alpha_1(t) + G(y(t - \tau(t)))\alpha_2(t) - (1 - \dot{\widetilde{r}}(t))[f(x(t - \widetilde{r}(t))) + F(x(t - \widetilde{r}(t)))\theta_1 \\ & + G(x(t - \widetilde{r}(t) - \tau(t)))\theta_2] + u(t).\end{aligned} \tag{26}$$

**Theorem 4.** *The drive system* (1) *lag synchronizes with the response system* (2) *at time* $\widetilde{r}(t)$ *if we choose*

$$\begin{aligned}u = {} & -\widetilde{e}(t) + (1 - \dot{\widetilde{r}}(t))f(x(t - \widetilde{r}(t))) - f(y(t)) + [(1 - \dot{\widetilde{r}}(t))F(x(t - \widetilde{r}(t))) - F(y(t))]\alpha_1 \\ & + [(1 - \dot{\widetilde{r}}(t))G(x(t - \widetilde{r}(t) - \tau(t))) - G(y(t - \tau(t)))]\alpha_2,\end{aligned} \tag{27}$$

$$\dot{\alpha}_1(t) = -(1 - \dot{\widetilde{r}}(t))Q^{-1}F^{\mathrm{T}}(x(t - \widetilde{r}(t)))P\widetilde{e}(t), \tag{28}$$

$$\dot{\alpha}_2(t) = -(1 - \dot{\widetilde{r}}(t))R^{-1}G^{\mathrm{T}}(x(t - \widetilde{r}(t) - \tau(t)))P\widetilde{e}(t), \tag{29}$$

*where $P$, $Q$ and $R$ are the positive definite matrices, $u$, $\alpha_1$ and $\alpha_2$ are independent of $\theta_1$ and $\theta_2$, $\widetilde{r}(t)$ is a function of time $t$, $\widetilde{r}(t) \geqslant 0$ for all time $t$.*

**Proof.** According to (27), we write error system (26) as

$$\dot{\widetilde{e}}(t) = -\widetilde{e}(t) + (1 - \dot{\widetilde{r}}(t))F(x(t - \widetilde{r}(t)))(\alpha_1(t) - \theta_1) + (1 - \dot{\widetilde{r}}(t))G(x(t - \widetilde{r}(t) - \tau(t)))(\alpha_2(t) - \theta_2). \tag{30}$$

Choose the following Lyapunov functional:

$$V(\widetilde{e}(t), \alpha_1(t), \alpha_2(t)) = \tfrac{1}{2}\widetilde{e}^{\mathrm{T}}(t)P\widetilde{e}(t) + \tfrac{1}{2}(\alpha_1(t) - \theta_1)^{\mathrm{T}}Q(\alpha_1(t) - \theta_1) + \tfrac{1}{2}(\alpha_2(t) - \theta_2)^{\mathrm{T}}R(\alpha_2(t) - \theta_2), \tag{31}$$

where $P$, $Q$ and $R$ are the positive definite matrices. Differentiating $V$ with respect to time along the solution of (30), we obtain

$$\begin{aligned}\frac{\mathrm{d}V}{\mathrm{d}t} & = \widetilde{e}^{\mathrm{T}}P\dot{\widetilde{e}}(t) + \dot{\alpha}_1^{\mathrm{T}}Q(\alpha_1(t) - \theta_1) + \dot{\alpha}_2^{\mathrm{T}}R(\alpha_2(t) - \theta_2) \\ & = \widetilde{e}^{\mathrm{T}}P[-\widetilde{e}(t) + (1 - \dot{\widetilde{r}}(t))F(x(t - \widetilde{r}(t)))(\alpha_1(t) - \theta_1) + (1 - \dot{\widetilde{r}}(t))G(x(t - \widetilde{r}(t) - \tau(t)))(\alpha_2(t) - \theta_2)] \\ & \quad + \dot{\alpha}_1^{\mathrm{T}}Q(\alpha_1(t) - \theta_1) + \dot{\alpha}_2^{\mathrm{T}}R(\alpha_2(t) - \theta_2) \\ & = -\widetilde{e}^{\mathrm{T}}P\widetilde{e} + [\dot{\alpha}_1(t) + (1 - \dot{\widetilde{r}}(t))Q^{-1}F^{\mathrm{T}}(x(t - \widetilde{r}(t)))P\widetilde{e}(t)]^{\mathrm{T}}Q(\alpha_1(t) - \theta_1) \\ & \quad + [\dot{\alpha}_2(t) + (1 - \dot{\widetilde{r}}(t))R^{-1}G^{\mathrm{T}}(x(t - \widetilde{r}(t) - \tau(t)))P\widetilde{e}(t)]^{\mathrm{T}}R(\alpha_2(t) - \theta_2) \\ & = -\widetilde{e}^{\mathrm{T}}P\widetilde{e}.\end{aligned} \tag{32}$$

We can therefore conclude that $V$ is a Lyapunov functional of the error system (30) corresponding with (28) and (29), i.e.,

$$y(t) - x(t - \widetilde{r}(t)) \longrightarrow 0, \quad t \longrightarrow \infty,$$

$$\alpha_1(t) - \theta_1 \longrightarrow 0, \quad t \longrightarrow \infty,$$

$$\alpha_2(t) - \theta_2 \longrightarrow 0, \quad t \longrightarrow \infty.$$

This completes the proof. □

**Corollary 7.** *The drive system* (1) *lag synchronizes with the response system* (2) *at time* $\tilde{r}(t)$ *if we choose*

$$u = -\tilde{e}(t) + (1 - \dot{\tilde{r}}(t))f(x(t - \tilde{r}(t))) - f(y(t)) + [(1 - \dot{\tilde{r}}(t))F(x(t - \tilde{r}(t))) - F(y(t))]\alpha_1 + [(1 - \dot{\tilde{r}}(t))G(x(t - \tilde{r}(t) - \tau(t))) - G(y(t - \tau(t)))]\alpha_2,$$

$$\dot{\alpha}_1(t) = -(1 - \dot{\tilde{r}}(t))F^{\mathrm{T}}(x(t - \tilde{r}(t)))P\tilde{e}(t),$$

$$\dot{\alpha}_2(t) = -(1 - \dot{\tilde{r}}(t))G^{\mathrm{T}}(x(t - \tilde{r}(t) - \tau(t)))P\tilde{e}(t),$$

*where P is a positive definite matrix*, $u$, $\alpha_1$ *and* $\alpha_2$ *are independent of* $\theta_1$ *and* $\theta_2$, $\tilde{r}(t)$ *is a function of time t*, $\tilde{r}(t) \geqslant 0$ *for all time t*.

**Proof.** Let $Q = R = I$, where $I$ is the identity matrix, we can obtain Corollary 7 from Theorem 4. □

**Corollary 8.** *The drive system* (1) *lag synchronizes with the response system* (2) *at time* $\tilde{r}(t)$ *if we choose*

$$u = -\tilde{e}(t) + (1 - \dot{\tilde{r}}(t))f(x(t - \tilde{r}(t))) - f(y(t)) + [(1 - \dot{\tilde{r}}(t))F(x(t - \tilde{r}(t))) - F(y(t))]\alpha_1 + [(1 - \dot{\tilde{r}}(t))G(x(t - \tilde{r}(t) - \tau(t))) - G(y(t - \tau(t)))]\alpha_2,$$

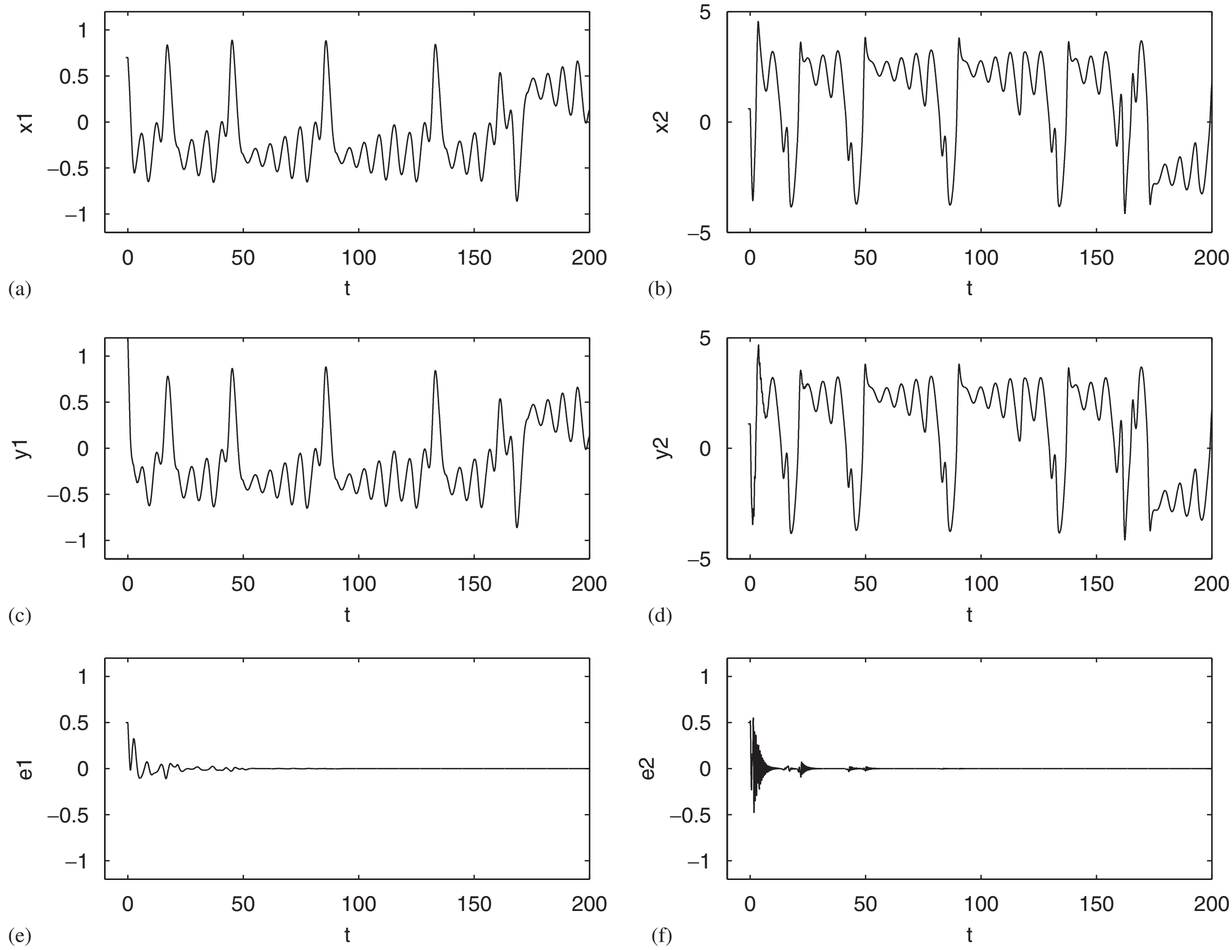


Fig. 1. State trajectories of drive, response and error systems: (a) $x_1(t)$; (b) $x_2(t)$; (c) $y_1(t)$; (d) $y_2(t)$; (e) $e_1(t)$; and (f) $e_2(t)$.

$$\dot{\alpha}_1(t) = -(1 - \dot{\widetilde{r}}(t))F^{\mathrm{T}}(x(t - \widetilde{r}(t)))\widetilde{e}(t),$$

$$\dot{\alpha}_2(t) = -(1 - \dot{\widetilde{r}}(t))G^{\mathrm{T}}(x(t - \widetilde{r}(t) - \tau(t)))\widetilde{e}(t),$$

*where* $u$, $\alpha_1$ *and* $\alpha_2$ *are independent of* $\theta_1$ *and* $\theta_2$, $\widetilde{r}(t)$ *is a function of time* $t$, $\widetilde{r}(t) \geqslant 0$ *for all time* $t$.

**Proof.** Let $P = Q = R = I$, where $I$ is the identity matrix, we can get Corollary 8 easily from Theorem 4. We omit it. □

**Remark 1.** In Refs. [22,26], Park has studied the adaptive synchronization of hyperchaotic Chen system with uncertain parameters and Chen et al. considered adaptive synchronization of uncertain Rössler hyperchaotic system based on parameter identification. However, they both discussed a specially simple model and could not be carried to a general model. In this paper, we discussed a general model.

**Remark 2.** In Ref. [24], Chen and Lü considered parameters identification and synchronization of chaotic systems based upon adaptive control. The method is proposed for a class of chaotic systems dependent linearly on unknown parameters based upon adaptive control, which just considered a class of chaotic systems and it is also a special case in our paper.

**Remark 3.** In Refs. [21,23], the adaptive synchronization of uncertain chaotic systems based on parameter identification has been considered by Fotsin, Woafo and Daafouz. However, in this paper we considered the adaptive synchronization of uncertain dynamical systems with time delays. We have introduced time delays in the dynamical systems, it is more common in a real system for time delays.

**Remark 4.** The value of this paper is that we introduce time delays in the adaptive synchronization and we also proposed a new concept: lag synchronization. Since the drive system and the response system may not synchronize, but with a time delay, they can synchronize. In addition, the time delay may be a constant or a function of time $t$. Furthermore, the drive system may sometimes synchronize with the response system and sometimes lag synchronizes with the response system.

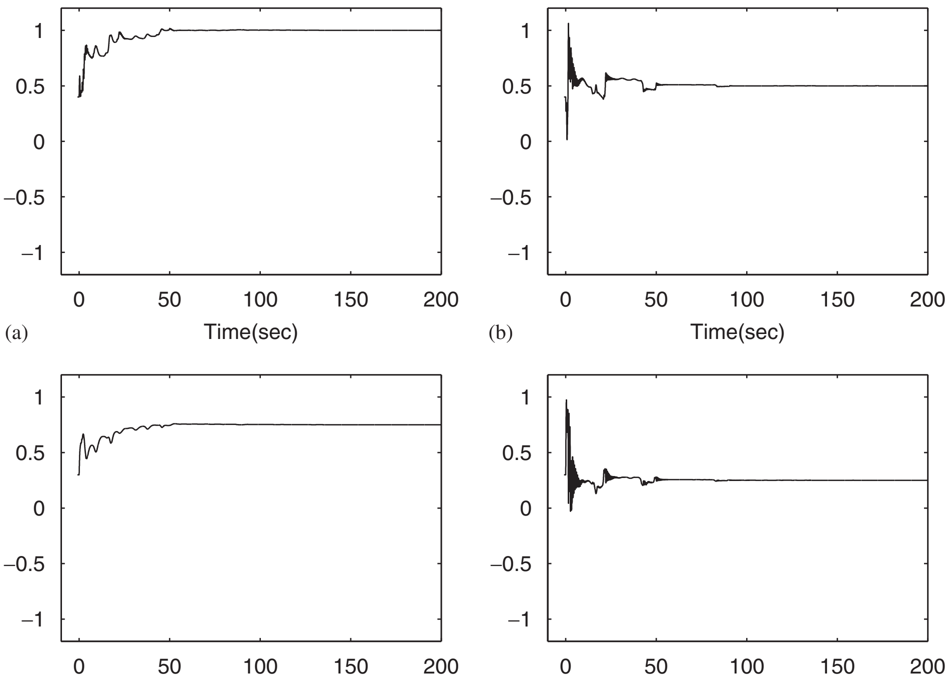


Fig. 2. Changing parameters of response system: (a) $\alpha_{11}(t)$; (b) $\alpha_{12}(t)$; (c) $\alpha_{21}(t)$; and (d) $\alpha_{22}(t)$.

**Remark 5.** Recently, many researchers [7,21–24] studied adaptive synchronization and parameter identification based on Lyapunov method. However, we can only obtain the stability of $\alpha_1$ and $\alpha_2$ in (5) and (6) from (9) as the same in the papers [7,21–24]. Here, we just use asymptotical stability instead of stability for simplicity. In [31], Li et al. found that this method was not effective for the stable system, and may be not good for estimating parameters. However, in this paper, we mainly interested in chaotic systems since their wide application in many research fields. In [32], Yu and Wu proposed that the demonstration of Li et al. is not accurate, and this method can be used to identify the parameters of periodic or chaotic systems.

## 3. Numerical examples

In this section, some simulation examples are constructed to show the effectiveness of the proposed method in this paper.

First, we consider the adaptive synchronization.

**Example 1.** Consider a typical delayed dynamical system [30] as follows:

$$\dot{x}(t) = f(x(t)) + F(x(t))\theta_1 + G(x(t-\tau(t)))\theta_2,$$

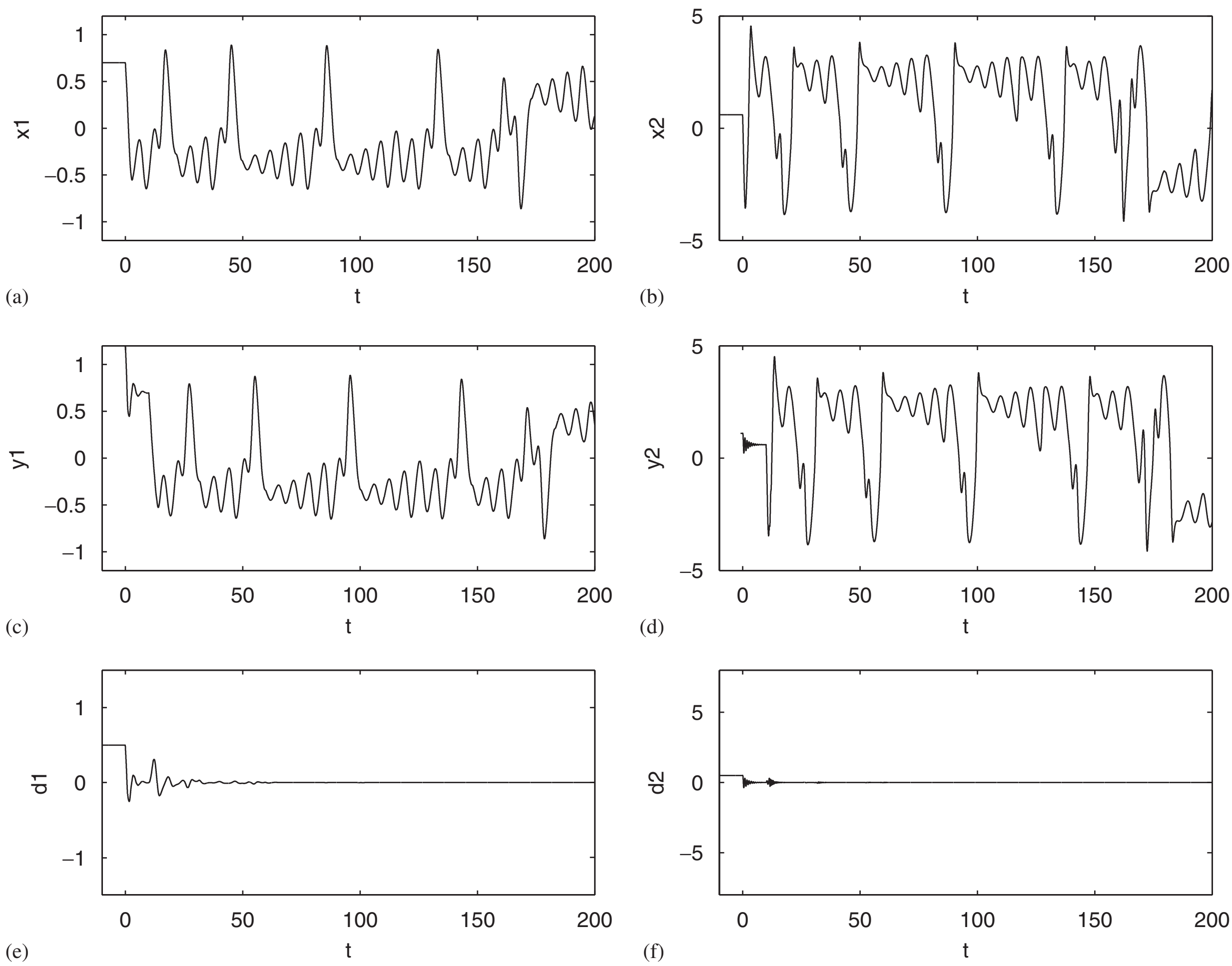


Fig. 3. State trajectories of drive, response and error systems: (a) $x_1(t)$; (b) $x_2(t)$; (c) $y_1(t)$; (d) $y_2(t)$; (e) $d_1(t)$; and (f) $d_2(t)$.

where

$$f(x) = -\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad \theta_1 = \begin{pmatrix} 1 \\ 0.5 \end{pmatrix}, \quad \theta_2 = \begin{pmatrix} 0.75 \\ 0.25 \end{pmatrix}, \quad F(x) = \begin{pmatrix} 2\tanh x_1 & -0.2\tanh x_2 \\ -5\tanh x_1 & 6\tanh x_2 \end{pmatrix},$$

$$G(x(t-\tau(t))) = \begin{pmatrix} -2\tanh x_1(t-\tau(t)) & -0.4\tanh x_2(t-\tau(t)) \\ -0.4\tanh x_1(t-\tau(t)) & -10\tanh x_2(t-\tau(t)) \end{pmatrix}, \quad \tau(t) = 1.$$

By Corollary 2, we obtain

$$\begin{cases} \dot{y}(t) = -(y-x) + f(x(t)) + F(x(t))\alpha_1 + G(x(t-\tau(t)))\alpha_2, \\ \dot{\alpha}_1(t) = -F^{\mathrm{T}}(x(t))(y-x), \\ \dot{\alpha}_2(t) = -G^{\mathrm{T}}(x(t-\tau(t)))(y-x). \end{cases}$$

The above equations are independent of $\theta_1$ and $\theta_2$ which are unknown to us. The trajectories of the drive, response and error systems are shown in Fig. 1. The parameter estimation trajectories of the response system are shown in Fig. 2. From Figs. 1 and 2, we see that the drive system synchronizes with the response system and the parameter estimation in response system tends to converge to the object we want. That is,

$$y(t) - x(t) \longrightarrow 0, \quad t \longrightarrow \infty,$$

$$\alpha_1(t) - \theta_1 \longrightarrow 0, \quad t \longrightarrow \infty,$$

$$\alpha_2(t) - \theta_2 \longrightarrow 0, \quad t \longrightarrow \infty.$$

Next, we consider the adaptive lag synchronization.

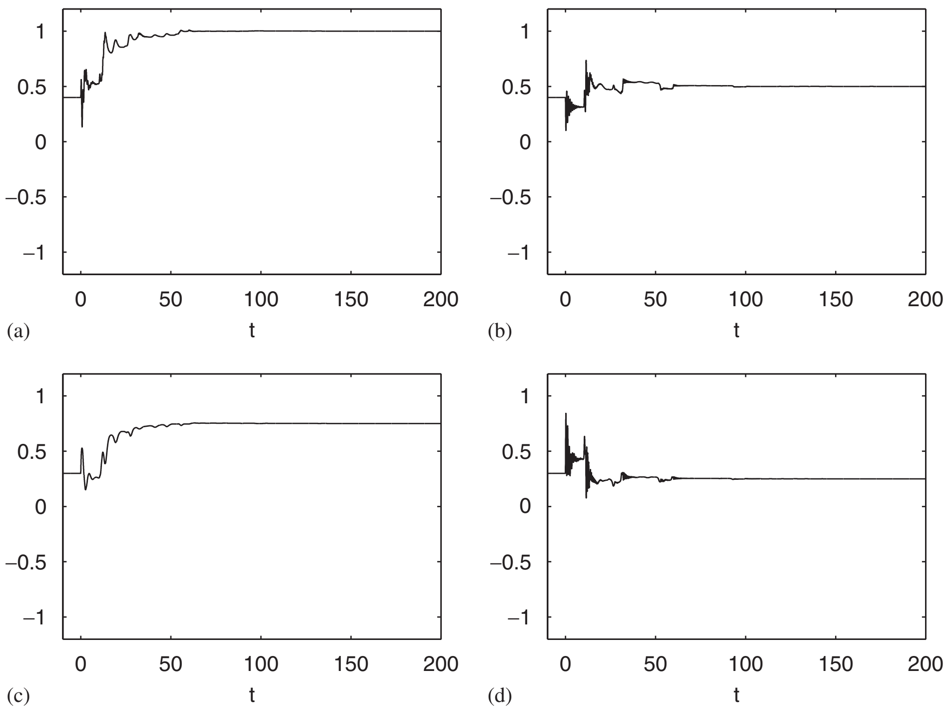


Fig. 4. Changing parameters of response system: (a) $\alpha_{11}(t)$; (b) $\alpha_{12}(t)$; (c) $\alpha_{21}(t)$; and (d) $\alpha_{22}(t)$.

**Example 2.** Consider a typical delayed dynamical system [30] the same as Example 1:

$$\dot{x}(t) = f(x(t)) + F(x(t))\theta_1 + G(x(t-\tau(t)))\theta_2,$$

where

$$f(x) = -\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad \theta_1 = \begin{pmatrix} 1 \\ 0.5 \end{pmatrix}, \quad \theta_2 = \begin{pmatrix} 0.75 \\ 0.25 \end{pmatrix}, \quad F(x) = \begin{pmatrix} 2\tanh x_1 & -0.2\tanh x_2 \\ -5\tanh x_1 & 6\tanh x_2 \end{pmatrix},$$

$$G(x(t-\tau(t))) = \begin{pmatrix} -2\tanh x_1(t-\tau(t)) & -0.4\tanh x_2(t-\tau(t)) \\ -0.4\tanh x_1(t-\tau(t)) & -10\tanh x_2(t-\tau(t)) \end{pmatrix}, \quad \tau(t) = 1,$$

but here we choose $r = 10$.

By Corollary 4, we obtain

$$\begin{cases} \dot{y}(t) = -[y(t) - x(t-r)] + f(x(t-r)) + F(x(t-r))\alpha_1 + G(x(t-r-\tau(t)))\alpha_2, \\ \dot{\alpha}_1(t) = -F^{\mathrm{T}}(x(t-r))[y(t) - x(t-r)], \\ \dot{\alpha}_2(t) = -G^{\mathrm{T}}(x(t-r-\tau(t)))[y(t) - x(t-r)]. \end{cases}$$

The above equations are independent of $\theta_1$ and $\theta_2$ which are unknown to us. The trajectories of the drive, response and error systems are shown in Fig. 3. The parameter estimation trajectories of the response system

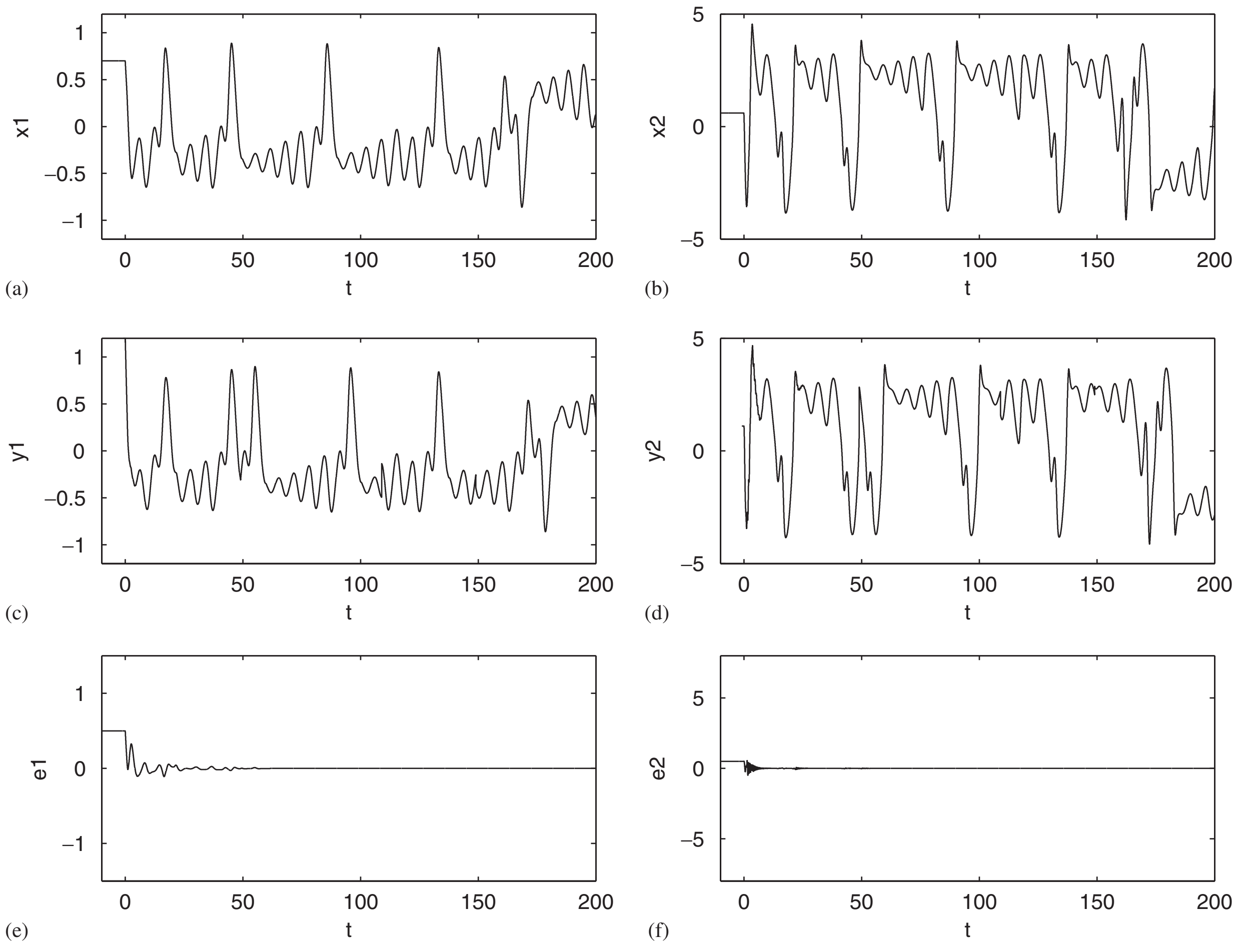


Fig. 5. State trajectories of drive, response and error systems: (a) $x_1(t)$; (b) $x_2(t)$; (c) $y_1(t)$; (d) $y_2(t)$; (e) $\widetilde{e}_1(t)$; and (f) $\widetilde{e}_2(t)$.

are shown in Fig. 4. From Figs. 3 and 4, we see that the drive system lag synchronizes with the response system at time $r$ and the parameter estimation in response system tends to converge to the object we want. That is,

$$y(t) - x(t-r) \longrightarrow 0, \quad t \longrightarrow \infty,$$

$$\alpha_1(t) - \theta_1 \longrightarrow 0, \quad t \longrightarrow \infty,$$

$$\alpha_2(t) - \theta_2 \longrightarrow 0, \quad t \longrightarrow \infty.$$

The drive system can lag synchronize with the response system at time 10, see Fig. 3, where $d(t) = y(t) - x(t-r)$.

**Example 3.** Consider a typical delayed dynamical system [30] the same as Example 1:

$$\dot{x}(t) = f(x(t)) + F(x(t))\theta_1 + G(x(t-\tau(t)))\theta_2,$$

here we choose

$$\widetilde{r}(t) = \begin{cases} 0, & t \in [kT, (k+1/2)T], \\ 10, & t \in [(k+1/2)T, (k+1)T], \end{cases}$$

where $k$ is a arbitrary integer and $T$ is a sufficient large value, here we choose $T = 100$. By Corollary 6, we obtain

$$\begin{cases} \dot{y}(t) = -[y(t) - x(t-\widetilde{r}(t))] + f(x(t-\widetilde{r}(t))) + F(x(t-\widetilde{r}(t)))\alpha_1 + G(x(t-\widetilde{r}(t)-\tau(t)))\alpha_2, \\ \dot{\alpha}_1(t) = -F^{\mathrm{T}}(x(t-\widetilde{r}(t)))\,[y(t) - x(t-\widetilde{r}(t))], \\ \dot{\alpha}_2(t) = -G^{\mathrm{T}}(x(t-\widetilde{r}(t)-\tau(t)))\,[y(t) - x(t-\widetilde{r}(t))]. \end{cases}$$

The above equations are independent of $\theta_1$ and $\theta_2$ which are unknown to us. The trajectories of the drive, response and error systems are shown in Fig. 5. The parameter estimation trajectories of the response system are shown in Fig. 6. From Figs. 5 and 6, we see that the drive system synchronizes with the response system in

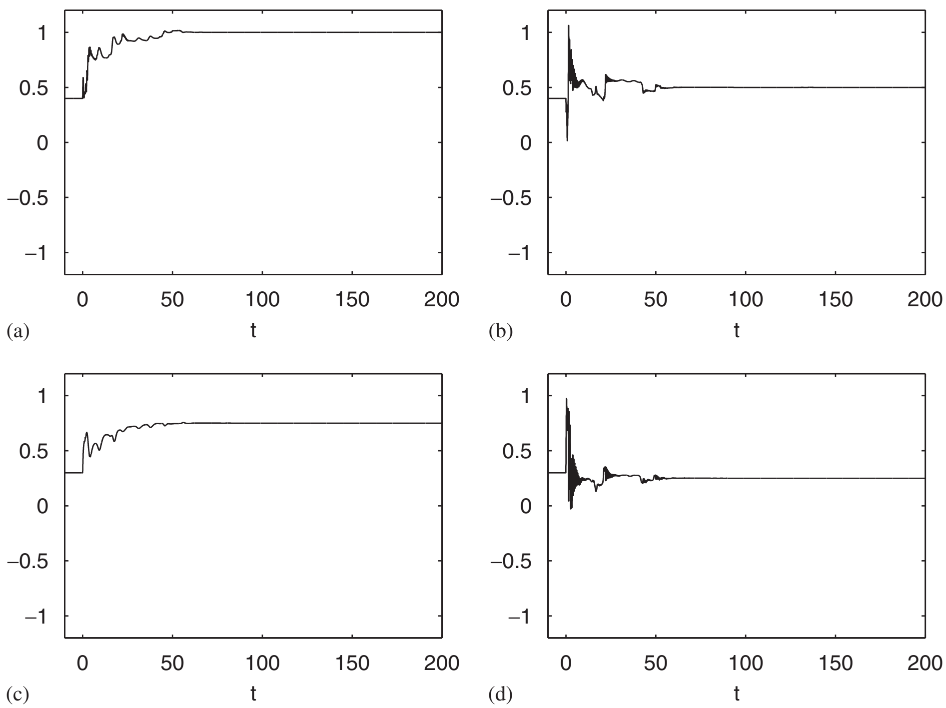


Fig. 6. Changing parameters of response system: (a) $\alpha_{11}(t)$; (b) $\alpha_{12}(t)$; (c) $\alpha_{21}(t)$; and (d) $\alpha_{22}(t)$.

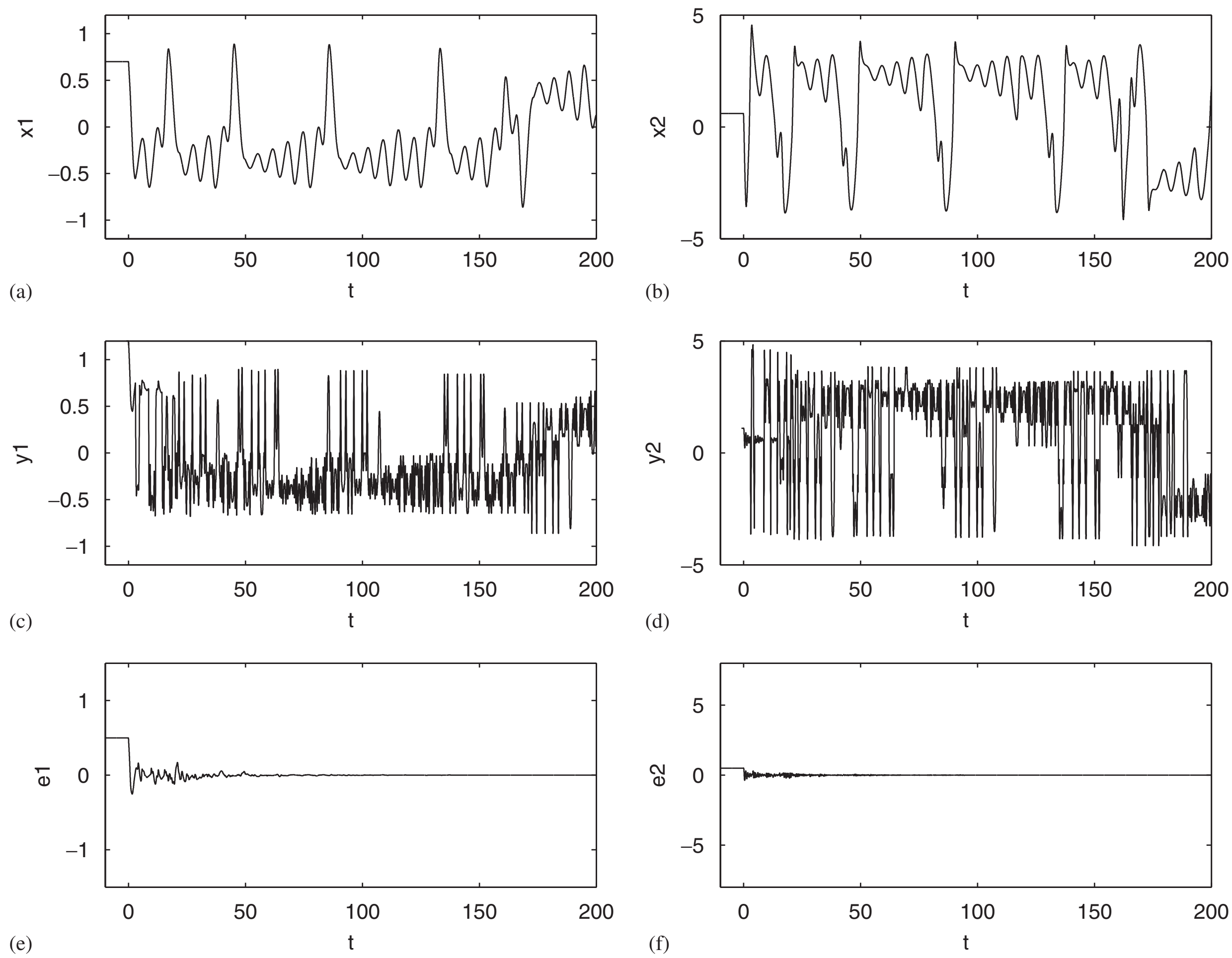


Fig. 7. State trajectories of drive, response and error systems: (a) $x_1(t)$; (b) $x_2(t)$; (c) $y_1(t)$; (d) $y_2(t)$; (e) $\widetilde{e}_1(t)$; and (f) $\widetilde{e}_2(t)$.

time interval $[0, 50] \cup [100, 150]$, lag synchronizes with the response system at time 10 in time interval $[50, 100] \cup [150, 200]$ and the parameter estimation in response system tends to converge to the object we want. That is,

$$y(t) - x(t - \widetilde{r}(t)) \longrightarrow 0,$$

$$\alpha_1(t) - \theta_1 \longrightarrow 0, \quad t \longrightarrow \infty,$$

$$\alpha_2(t) - \theta_2 \longrightarrow 0, \quad t \longrightarrow \infty.$$

**Example 4.** Consider a typical delayed dynamical system [30] the same as Example 1:

$$\dot{x}(t) = f(x(t)) + F(x(t))\theta_1 + G(x(t - \tau(t)))\theta_2,$$

here we choose

$$\widetilde{r}(t) = 10(1 + \sin t).$$

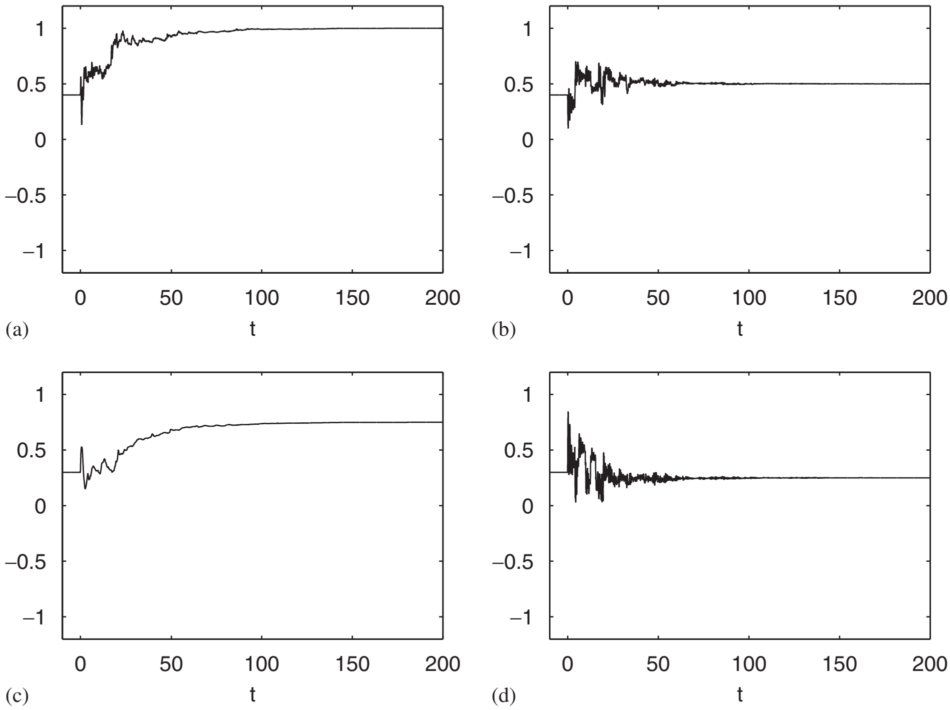


Fig. 8. Changing parameters of response system: (a) $\alpha_{11}(t)$; (b) $\alpha_{12}(t)$; (c) $\alpha_{21}(t)$; and (d) $\alpha_{22}(t)$.

By Corollary 8, we obtain

$$\begin{cases} \dot{y}(t) = -[y(t) - x(t - \tilde{r}(t))] + (1 - \dot{\tilde{r}}(t)) f(x(t - \tilde{r}(t))) + (1 - \dot{\tilde{r}}(t)) F(x(t - \tilde{r}(t)))\alpha_1 \\ \qquad + (1 - \dot{\tilde{r}}(t)) G(x(t - \tilde{r}(t) - \tau(t)))\alpha_2, \\ \dot{\alpha}_1(t) = -(1 - \dot{\tilde{r}}(t)) F^{\mathrm{T}}(x(t - \tilde{r}(t)))[y(t) - x(t - \tilde{r}(t))], \\ \dot{\alpha}_2(t) = -(1 - \dot{\tilde{r}}(t)) G^{\mathrm{T}}(x(t - \tilde{r}(t) - \tau(t)))[y(t) - x(t - \tilde{r}(t))]. \end{cases}$$

The above equations are independent of $\theta_1$ and $\theta_2$ which are unknown to us. The trajectories of the drive, response and error systems are shown in Fig. 7. The parameter estimation trajectories of the response system are shown in Fig. 8. From Figs. 7 and 8, we see that the drive system lag synchronizes with the response system at time $\tilde{r}(t)$ and the parameter estimation in response system tends to converge to the object we want. That is,

$$y(t) - x(t - \tilde{r}(t)) \longrightarrow 0,$$

$$\alpha_1(t) - \theta_1 \longrightarrow 0, \quad t \longrightarrow \infty,$$

$$\alpha_2(t) - \theta_2 \longrightarrow 0, \quad t \longrightarrow \infty.$$

## 4. Conclusion

In this paper, the problem of synchronization and lag synchronization of uncertain dynamical system has been considered. The theorems for synchronization and lag synchronization are derived in this paper. It is easy and convenient to use this method by adopting an adaptive law. Numerical simulations are given to show the effectiveness and feasibility of the developed method.

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