Adaptive synchronization of complex networks

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Abstract: In this paper, the adaptive synchronization of coupled complex networks is investigated. Some controllers and adaptive laws are designed to ensure the synchronization of complex networks. With this new effective method, a general complex network can achieve synchronization. Two examples are simulated, using the chaotic Lorenz system and the delayed neural network as the nodes of the dynamical complex network, which demonstrate the feasibility and effectiveness of the proposed method in this paper.

Keywords: Complex network; Diffusively coupled networks; Lyapunov functional; Synchronization; Chaos; Dynamical network

1. INTRODUCTION

Complex networks exist in all fields of sciences and societies, and have been intensively studied over the last few years [**?**]-[**?**]. Among these are computer networks, the World Wide Web, telephone call graphs, food webs, neural networks, electrical power grids and citation networks of scientists. Recently, in [**?**] the authors proposed a new complex network model for reputation computation in virtual organizations and and also investigated its convergence dynamics.

Synchronization of complex networks of dynamical systems has received a great deal of attention [**?**]-[**?**] due to proposition of the diffusively coupled networks in [**?**][**?**], especially about small-world and scale-free dynamical network models. In [**?**][**?**][**?**], the authors linearized the nonlinear dynamical nodes around the synchronization state, which is known as local synchronization. By using the Lyapunov functional method, synchronization manifold and linear matrix inequality (LMI) approach, several sufficient conditions have been derived to ensure the synchronization of complex networks [**?**]-[**?**].

Moreover, one can not guarantee that all the dynamical nodes can synchronize. However, it is very desirable if some controllers are designed to ensure the synchronization of all the nodes in the complex network. Some controllers are intensively studied, such as feedback and delayed feedback controller [**?**], nonlinear adaptive feedback controller [**?**] and so on. Sometimes, it is too hard to design a controller to achieve the synchronization. However, it could be interesting to achieve synchronization for a general complex network model. In [**?**]-[**?**], the authors proposed a new adaptive law to ensure the synchronization of the system, and it is very efficient. By using this new effective method based on Lyapunov functional method and a suitable adaptive law to adjust parameters in this paper, the synchronization of a general complex network can be achieved. In the recent study of synchronization in complex networks, an elementary assumption is that the inner coupling is uniform which means that the inner coupling between arbitrary two linked nodes is the same. It is of great interest if the inner coupling is nonuniform. Thus, the synchronization in complex network with nonuniform inner couplings can also be obtained in this paper, and we will investigate a general complex network model which include almost all the dynamical systems [**?**]-[**?**][**?**]-[**?**].

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The organization of this paper is as follows: In Section 2, we give preliminaries for the synchronization of coupled dynamical networks. In Section 3, main results are given. Several controllers are presented for the synchronization of coupled complex network. Some remarks are also given to show the advantages of the obtained results. In Section 4, some simulation examples including more large-scale Lorenz system and delayed neural network are given to show the effectiveness and feasibility of this paper. In Section 5, we give conclusions and some prospects for our future works.

2. PRELIMINARIES

Consider a general complex dynamical network consisting of N diffusively coupled identical nodes, with each node being a n-dimensional dynamical system, in the following form:

$$\begin{aligned}\dot{x}_i(t) &= f(x_i(t)) + g(x_i(t-\tau_1)) \\ &\quad + h_i(x_1(t), x_2(t), \cdots, x_N(t)) \\ &\quad + l_i(x_1(t-\tau_2), x_2(t-\tau_2), \cdots, x_N(t-\tau_2)), \\ &\quad i = 1, 2, \cdots, N. \end{aligned} \quad (1)$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \cdots, x_{in}(t))^T \in \mathbb{R}^n, i = 1, 2, \cdots, N$ is the state vector representing the state variables of node i, τ_1, τ_2 are the time delays, $f, g : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ are continuously differentiable nonlinear vector functions, and $h_i, l_i : \mathbb{R}^{nN} \longrightarrow \mathbb{R}^n$ are diffusively coupling functions. We assume that the system (1) satisfies the following initial conditions: $x_i(t) = \phi_i(t) \in \mathcal{C}([-r, 0], \mathbb{R}^n) (i = 1, 2, \cdots, N)$ with $r = \max\{\tau_1, \tau_2\}$, where $\mathcal{C}([-r, 0], \mathbb{R}^n)$ denotes the set of all continuous functions from $[-r, 0]$ to $\mathbb{R}^n$.

Note that the complex network model (1) is general. It can be almost all the dynamical systems [**?**]-[**?**][**?**]-[**?**]. Also, the diffusively coupling functions h_i and l_i are general. First, it can be chosen as linear combination of the states of the nodes, i.e., $h_i = c\sum_{j=1}^{N} a_{ij}\Gamma x_j$ [**?**][**?**][**?**]-[**?**], where c is the coupling strength, Γ is the inner coupling matrix, and $A = (a_{ij})$ is the coupling configuration matrix which satisfies $\sum_{j=1}^{N} a_{ij} = 0$. Second, it can be chosen as delayed coupling, i.e., $l_i = c\sum_{j=1}^{N} a_{ij}\Gamma x_j(t-\tau)$ [**?**][**?**], where τ is a time delay. Also, it can be distributed delay $l_i = \sum\limits_{j=1}^{N} a_{ij}\int_{-\infty}^{t} K(t-s)x_j(s)ds$ [**?**], where $K(\cdot)$ is the weight matrix function. Actually, h_i and l_i can be the combination of nonlinear functions, namely, $h_i = c\sum_{j=1}^{N} a_{ij}\Gamma H(x_j)$ and $l_i = c\sum_{j=1}^{N} a_{ij}\Gamma L(x_j(t-\tau))$, where $H(\cdot)$ and $L(\cdot)$ are the inner coupling functions between two nodes.

In this paper, we consider the synchronization of complex network model (1). When the complex network (1) achieves synchronization, namely, the states $x_1 = x_2 = \cdots = x_N \to s(t)$, as $t \to \infty$, where $s(t)$ is a solution of an isolated node which satisfies

$$\dot{s}(t) = f(s(t)) + g(s(t-\tau_1)). \tag{2}$$

Here, $s(t)$ can be an equilibrium point, a periodic orbit, or even a chaotic attractor.

In order to investigate the synchronization of complex network (1), we add some simple controllers to the nodes of the complex network (1). Then the controlled complex network is described by

$$\begin{aligned}\dot{x}_i(t) &= f(x_i(t)) + g(x_i(t-\tau_1)) \\ &\quad + h_i(x_1(t), x_2(t), \cdots, x_N(t)) \\ &\quad + l_i(x_1(t-\tau_2), x_2(t-\tau_2), \cdots, x_N(t-\tau_2)) \\ &\quad + u_i, i = 1, 2, \cdots, N,\end{aligned} \tag{3}$$

where $u_i \in \mathbb{R}^n$ are the feedback controllers. The objective of control here is to find some controllers such that the solutions of controlled complex network (3) synchronize with the solution of (2), in the sense that

$$\lim_{t\to\infty} \|x_i(t) - s(t)\| = 0,\ i = 1, 2, \cdots, N. \tag{4}$$

When the complex network (3) achieves synchronization, the coupling functions and the control inputs should vanish, i.e., $h_i(x_1(t), x_2(t), \cdots, x_N(t)) = 0$, $l_i(x_1(t-\tau_2), x_2(t-\tau_2), \cdots, x_N(t-\tau_2)) = 0$, and $u_i = 0$. This ensures any solution $x_i(t)$ of a single isolated node is also a solution of synchronized coupled complex network.

In order to give our main results, it is necessary to make the following assumptions:

A_1: For $i = 1, 2, \cdots, N$, there exist non-negative constants α_i such that

$$\|f(x_i(t)) - f(s(t))\| \le \alpha_i \|x_i(t) - s(t)\|. \tag{5}$$

A_2: For $i = 1, 2, \cdots, N$, there exist non-negative constants βi such that

$$\|g(x_i(t)) - g(s(t))\| \le \beta_i \|x_i(t) - s(t)\|. \tag{6}$$

A_3: For $i = 1, 2, \cdots, N$, there exist non-negative constants γ_{ij} $(j = 1, 2, \cdots, N)$ such that

$$\begin{aligned}&\|h_i(x_1, x_2, \cdots, x_N) - h_i(s, s, \cdots, s)\| \\ &\le \sum_{j=1}^{N} \gamma_{ij}\|x_j - s\|.\end{aligned} \tag{7}$$

A_4: For $i = 1, 2, \cdots, N$, there exist non-negative constants η_{ij} $(j = 1, 2, \cdots, N)$ such that

$$\begin{aligned}&\|l_i(x_1, x_2, \cdots, x_N) - l_i(s, s, \cdots, s)\| \\ &\le \sum_{j=1}^{N} \eta_{ij}\|x_j - s\|.\end{aligned} \tag{8}$$

Note that the above assumptions are very loose, for example, Assumption A_1 is satisfied as long as $\partial f_k/\partial x_{ij}(k, j = 1, 2, \cdots, n, i = 1, 2, \cdots, N)$ are bounded. Therefore, the complex network (1) can include all well-known dynamical complex systems.

Subtracting (2) from (3) gives the error dynamical system

$$\begin{aligned}\dot{e}_i(t) &= f(x_i(t)) - f(s(t)) \\ &\quad + g(x_i(t-\tau_1)) - g(s(t-\tau_1)) \\ &\quad + h_i(x_1(t), x_2(t), \cdots, x_N(t)) - h_i(s, s, \cdots, s) \\ &\quad + l_i(x_1(t-\tau_2), x_2(t-\tau_2), \cdots, x_N(t-\tau_2)) \\ &\quad - l_i(s(t-\tau_2), s(t-\tau_2), \cdots, s(t-\tau_2)) + u_i, \\ &\quad i = 1, 2, \cdots, N,\end{aligned} \tag{9}$$

where $e_i(t) = x_i(t) - s(t)$.

3. Synchronization in Complex Networks

In this section, we give our main results. Some controllers are designed to ensure the synchronization of system (3).

Theorem 1: Under the Assumptions A_1-A_4, the complex network (3) is synchronized if we choose the following linear feedback controllers

$$u_i = \theta_i(x_i(t) - s(t)), \tag{10}$$

where $\theta_i = diag(\theta_{i1}, \theta_{i2}, \cdots, \theta_{in})(i = 1, 2, \cdots, N)$ are the feedback gain matrices. The element θ_{ij} of matrix θ_i can be adjusted by

$$\dot{\theta}_{ij} = -\varepsilon_{ij} e_{ij}^2, \tag{11}$$

for $i = 1, 2, \cdots, N, j = 1, 2, \cdots, n$, where ε_{ij} are positive.

Proof: Consider the following Lyapunov functional

$$\begin{aligned}V(t) &= \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)e_i(t) + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{n}\frac{1}{\varepsilon_{ij}}(\theta_{ij} + p)^2 \\ &\quad + q\sum_{i=1}^{N}\int_{t-\tau_1}^{t} e_i^T(s)e_i(s)ds \\ &\quad + m\sum_{i=1}^{N}\int_{t-\tau_2}^{t} e_i^T(s)e_i(s)ds,\end{aligned} \tag{12}$$

where p, q, m are large positive constants.

Taking the derivative of $V(t)$ along the trajectories of (9), we obtain

$$\begin{aligned}\dot{V}(t)|_{(9)} &= \sum_{i=1}^{N} e_i^T(t)\dot{e}_i(t) - \sum_{i=1}^{N}\sum_{j=1}^{n}(\theta_{ij}+p)e_{ij}^2 \\ &+q\sum_{i=1}^{N}(e_i^T(t)e_i(t) - e_i^T(t-\tau_1)e_i(t-\tau_1)) \\ &+m\sum_{i=1}^{N}(e_i^T(t)e_i(t) - e_i^T(t-\tau_2)e_i(t-\tau_2)) \\ &= \sum_{i=1}^{N} e_i^T(t)[f(x_i(t)) - f(s(t)) + g(x_i(t-\tau_1)) \\ &-g(s(t-\tau_1)) + h_i(x_1(t), x_2(t), \cdots, x_N(t)) \\ &-h_i(s, s, \cdots, s) \\ &+l_i(x_1(t-\tau_2), x_2(t-\tau_2), \cdots, x_N(t-\tau_2)) \\ &-l_i(s(t-\tau_2), s(t-\tau_2), \cdots, s(t-\tau_2)) \\ &+\theta_i e_i] - \sum_{i=1}^{N}\sum_{j=1}^{n}(\theta_{ij}+p)e_{ij}^2 + (q+m) \\ &\times\sum_{i=1}^{N} e_i^T(t)e_i(t) - \sum_{i=1}^{N}[qe_i^T(t-\tau_1)e_i(t-\tau_1) \\ &+me_i^T(t-\tau_2)e_i(t-\tau_2)]. \qquad (13)\end{aligned}$$

Let $\alpha = \max_{1\le i\le N}\{\alpha_i\}$, $\beta = \max_{1\le i\le N}\{\beta_i\}$, $\gamma = \max_{1\le i\le N, 1\le j\le n}\{\gamma_{ij}\}$, $\eta = \max_{1\le i\le N, 1\le j\le n}\{\eta_{ij}\}$, and $\sum\limits_{j=1}^{n} e_{ij}^2 = \|e_i\|^2$, where $e = (e_1^T, e_2^T, \cdots, e_N^T)^T$. According to Assumption A_1-A_4, we can have

$$\begin{aligned}\dot{V}(t)|_{(9)} &\le \sum_{i=1}^{N}\|e_i(t)\|[\alpha_i\|e_i(t)\| + \beta_i\|e_i(t-\tau_1)\| \\ &+\sum_{j=1}^{N}\gamma_{ij}\|e_j(t)\| + \sum_{j=1}^{N}\eta_{ij}\|e_j(t-\tau_2)\|] \\ &-p\sum_{i=1}^{N}\sum_{j=1}^{N}e_{ij}^2 + (q+m)\sum_{i=1}^{N}e_i^T(t)e_i(t) \\ &-\sum_{i=1}^{N}[q\|e_i(t-\tau_1)\|^2 + m\|e_i(t-\tau_2)\|^2] \\ &\le (\alpha+q+m-p)\sum_{i=1}^{N}\|e_i(t)\|^2 \\ &+\frac{\beta}{2}\sum_{i=1}^{N}[\|e_i(t)\|^2\| + e_i(t-\tau_1)\|^2] \\ &+\frac{\gamma}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}[\|e_i(t)\|^2 + \|e_j(t)\|^2] \\ &+\frac{\eta}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}[\|e_i(t)\|^2 + \|e_j(t-\tau_2)\|^2] \\ &-\sum_{i=1}^{N}[q\|e_i(t-\tau_1)\|^2 + m\|e_i(t-\tau_2)\|^2]. \\ &\qquad (14)\end{aligned}$$

Therefore, we obtain

$$\begin{aligned}\dot{V}(t)|_{(9)} &\le (\alpha+q+m+\frac{\beta}{2}+N\gamma+\frac{N\eta}{2}-p) \\ &\times\sum_{i=1}^{N}\|e_i(t)\|^2 + (\frac{\beta}{2}-q)\sum_{i=1}^{N}\|e_i(t-\tau_1)\|^2 \\ &+(\frac{N\eta}{2}-m)\sum_{i=1}^{N}\|e_i(t-\tau_2)\|^2. \qquad (15)\end{aligned}$$

If we choose $p = \alpha+q+m+\frac{\beta}{2}+N\gamma+\frac{N\eta}{2}+1$, $q = \frac{\beta}{2}$, and $m = \frac{N\eta}{2}$, then we have

$$\dot{V}(t)|_{(9)} \le -\sum_{i=1}^{N}\|e_i(t)\|^2. \qquad (16)$$

It is obvious that $\dot{V}(t) = 0$ if and only if $e_{ij} = 0$ for $i = 1, 2, \cdots, N, j = 1, 2, \cdots, n$. Therefore, according to the well-known LaSalle invariance principle [**?**], the proof is completed.

Remark 1: Complex networks have become a focal topic recently, and a lot of studies considered the diffusively coupled complex networks [**?**]-[**?**]. Some simple results were investigated in [**?**][**?**]. However, in this paper, we try to study a general model. It is easy to see that it can be almost all systems, such as Lorenz system, Chen system, delayed neural network and so on. Also, time delays are introduced since delays are inevitable in the dynamical systems.

Remark 2: In fact, the coupling functions h_i and l_i can be nonuniform inner coupling functions. For example, there exist two integers i_0 and $j_0(i_0 \neq j_0)$ such that $h_{i_0} = c\sum_{j=1}^{N} a_{i_0 j}\Gamma x_j$ and $h_{j_0} = c\sum_{j=1}^{N} a_{j_0 j}\Gamma H(x_j)$ where $H(\cdot)$ can be arbitrary nonlinear functions.

4. Numerical Examples

In this section, some simulation examples are given to show the feasibility and effectiveness of the proposed adaptive method.

Example 1: Consider the following famous Lorenz system

$$\begin{cases}\dot{x}_1 = a(x_2 - x_1) \\ \dot{x}_2 = cx_1 - x_2 - x_1x_3 \\ \dot{x}_3 = -bx_3 + x_1x_2,\end{cases} \qquad (17)$$

which is chaotic when parameters $a = 10, b = 8/3, c = 28$. System (17) can exhibit chaotic phenomenon which is illustrated in Fig. 1.

Consider the complex network (3)

$$\begin{cases}\dot{x}_{i1} = a(x_{i2} - x_{i1}) \\ \qquad +c\sum_{j=1}^{N} a_{ij}H_i^1(x_{j1}) + u_{i1} \\ \dot{x}_{i2} = cx_{i1} - x_{i2} - x_{i1}x_{i3} \\ \qquad +c\sum_{j=1}^{N} a_{ij}H_i^2(x_{j2}) + u_{i2} \\ \dot{x}_{i3} = -bx_{i3} + x_{i1}x_{i2} \\ \qquad +c\sum_{j=1}^{N} a_{ij}H_i^3(x_{j3}) + u_{i3},\end{cases} \qquad (18)$$

where $c = 1, H_i^1(x_{j1}) = \sin(x_{j1}), H_i^2(x_{j2}) = \cos(x_{j2}), H_i^3(x_{j3}) = x_{j3}$, $\varepsilon_{ij} = 1$. Suppose that the

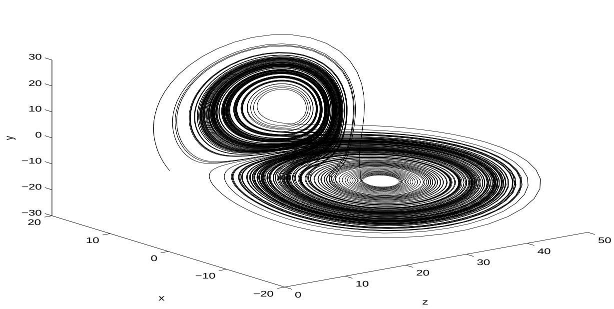

Fig. 1 Trajectories of single node in Lorenz system.

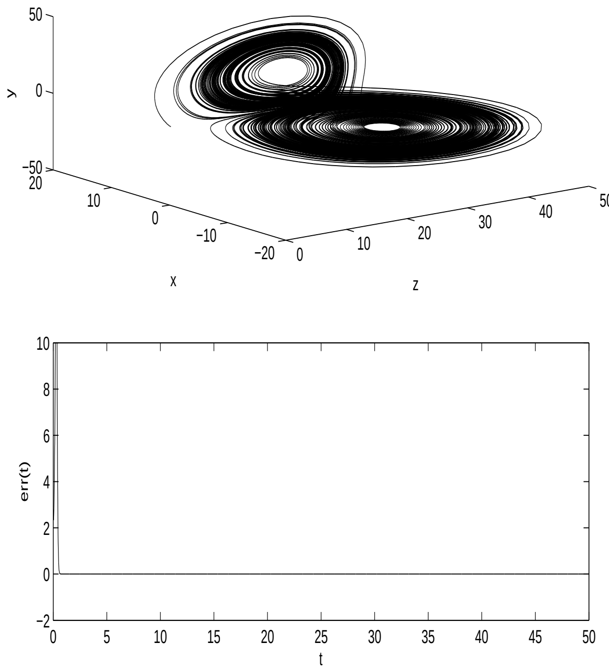

Fig. 2 Trajectories of one node and error distance in the coupled Lorenz complex network.

network is connected in the nearest neighbor coupling, i.e.,

$$(a_{ij})_{N\times N}=\begin{pmatrix} -2 & 1 & 0 & 0 & \cdots & 0 & 1\\ 1 & -2 & 1 & 0 & \cdots & 0 & 0\\ 0 & 1 & -2 & 1 & \cdots & 0 & 0\\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & 0 & 0 & \cdots & -2 & 1\\ 1 & 0 & 0 & 0 & \cdots & 1 & -2 \end{pmatrix},$$

where $N=10$.

The error distance among the nodes of trajectories in the coupled networks are

$$err(t)=\sum_{i=1}^{3}\sqrt{\sum_{j=1}^{10}[x_{1i}(t)-x_{ji}(t)]^2}\ .$$

The trajectories of one node and error distance are illustrated in Fig. 2. It is easy to see that the coupled complex Lorenz system (18) is synchronized.

Example 2: Consider the following 2-dimensional delayed neural network model as follows:

$$\dot{x}(t)=-Cx(t)+Af(x(t))+Bf(x(t-\tau_1))+I(t), \quad (19)$$

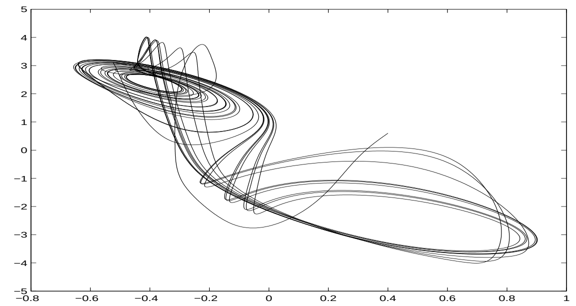

Fig. 3 Trajectories of single node in the delayed neural network.

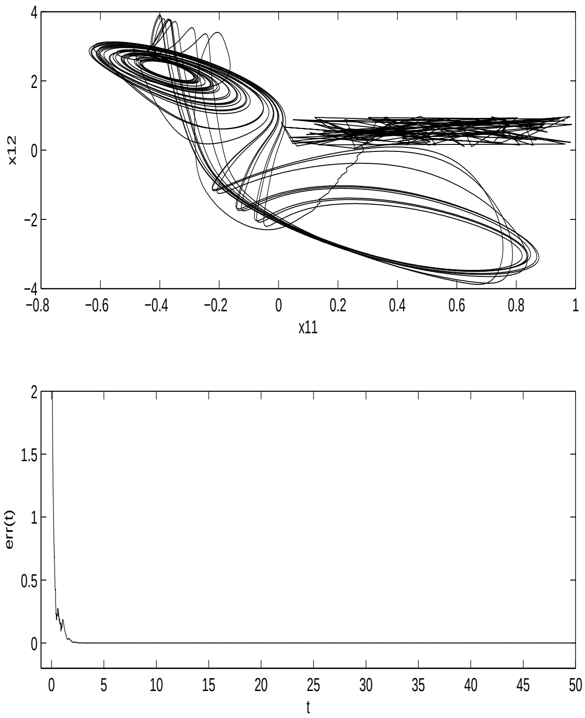

Fig. 4 Trajectories of one node and error distance in the coupled complex neural network.

where $x(t)=(x_1(t),x_2(t))^T, f(x(t))=(\tanh(x_1(t)),\tanh(x_2(t)))^T$, $I(t)=(0,0)^T$, $C=\begin{pmatrix}1&0\\0&1\end{pmatrix}$, $A=\begin{pmatrix}2.0&-0.1\\-5.0&3.0\end{pmatrix}$, $B=\begin{pmatrix}-1.5&-0.1\\-0.2&-2.5\end{pmatrix}$, $\tau_1=1$. Choose the initial conditions:

$$x_1(s)=0.4, x_2(s)=0.6, \forall s\in[-1,0],$$

system (**??**) can exhibit chaotic behaviors as shown in Fig. 3.

Next we consider the linearly coupled and linearly delayed coupled neural network:

$$\begin{aligned}\dot{x}_i(t) &= -Cx_i(t)+Af(x_i(t))+Bf(x_i(t-\tau_1))+I(t)\\ &\quad +\sum_{j=1}^{N}G_{ij}Dx_j(t)+\sum_{j=1}^{N}G_{ij}D_\tau x_j(t-\tau_2)+u_i,\end{aligned} \quad (20)$$

where $D = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix}$, $D_\tau = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix}$, $\tau_2 = 0.5$, $\varepsilon_{ij} = 1$, and suppose that the network is fully connected, i.e.,

$$G = \begin{pmatrix} -(N-1) & 1 & 1 & \cdots & 1 \\ 1 & -(N-1) & 1 & \cdots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & -(N-1) \end{pmatrix},$$

where $N = 20$.

The error distance among the nodes of trajectories in the coupled networks are

$$err(t) = \sum_{i=1}^{2} \sqrt{\sum_{j=1}^{20} [x_{1i}(t) - x_{ji}(t)]^2} \ .$$

The trajectories of one node and error distance are illustrated in Fig. 4. In Fig. 4. The initial conditions are random functions in $[0, 1]$. It is easy to see that the coupled complex neural network (**??**) is synchronized.

5. Conclusions

Recently, complex networks has become a hot topic, and a lot of attentions have been made on it. In this paper, we have investigated the adaptive synchronization of coupled complex networks. Some controllers and adaptive laws are given to ensure the synchronization of complex networks based on Lyapunov functional method. With this new effective method, a general complex network can achieve synchronization.

However, the proposed method is simple. To the best of our knowledge, the theoretical results about the synchronization of coupled complex networks are still lacking and too conservative. Synchronization control of complex network is a new subject recently, and less results have been studied about it. Also, there must be some perturbations in the real complex networks, and few works have investigated this. Actually, there are a lot of works for us to do in the complex network. We will investigate some good theoretical results and applications.

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