Robust Adaptive Control of Unknown Modified Cohen–Grossberg Neural Networks With Delays

Wenwu Yu, *Student Member, IEEE*, Jinde Cao, and Guanrong Chen, *Fellow, IEEE*

***Abstract*—In this brief, robust adaptive control of unknown modified Cohen–Grossberg neural networks with time delays is considered based on nonsmooth analysis and matrix inequality technique. Several new controllers are designed to ensure the global asymptotical stability of the targeted equilibrium point. The designed controllers are independent of the bounds of the perturbations, system functions and the time delays. One does not need to know the bounds of the unknown parameters, but only needs to know the structures of the modified Cohen–Grossberg neural networks with time delays. Finally, some simulations examples are given to verify the theoretical results.**

***Index Terms*—Global asymptotical stability, Lyapunov functional method, matrix inequality, modified Cohen–Grossberg neural network, nonsmooth analysis, time delay.**

I. Introduction

RECENTLY, many artificial neural networks especially Cohen–Crossberg neural networks [3], which was proposed in 1983, have been a focal subject for research due to their wide applications in signal processing, image processing, pattern recognition and optimization problems. Some of these applications require the knowledge of the dynamical behaviors of the network used, such as the uniqueness and global asymptotical stability of its equilibrium point [8]–[11], [19]–[21]. Thus, the qualitative analysis of these dynamic behaviors is a prerequisite step for the practical design and application of neural networks.

In particular, although neural networks can be implemented by very large-scale integrated (VLSI) electronic circuits, the finite speeds of switching and transmission signals inevitably induce time delays in the interaction between the neurons, which may result in an oscillation phenomenon or network instability. Since neural networks can be implemented by very large-scale integrated (VLSI) electronic circuits, neural networks become more interesting in the electrical engineering. Ye *et al.* [4] introduced delays into a modified Cohen–Grossberg neural network model where the Hopfield neural network can be considered as a special case of this modified Cohen–Grossberg neural network. Cao and Song [26] proposed a delayed Cohen–Grossberg type bidirectional associative memory network and studied its stability. Since then, the modified Cohen–Grossberg neural networks with time delays [5]–[7], [26]–[28] have been widely studied. Therefore, the modified neural network with time delays is further considered in this brief.

It is well known that, by carefully choosing network structures, training methods, and input data, the neural network controllers have been developed to compensate for the effects of nonlinearities and system uncertainties, so that the stability, error convergence, and robustness of the control system can be greatly improved. Clearly, the recurrent neural network (RNN) has capabilities superior to the feedforward neural networks, such as feedback response and the information-storing ability. Since the RNN has a feedback loop, it captures the dynamical response of a system with external feedback. Thus, it is more important to develop some controllers based on the feedback response to ensure the stability of delayed neural networks.

Furthermore, many efforts have been devoted to searching sufficient conditions for the problem of robust stabilization of time-delay systems recently [12]–[18], [25]. By using Lyapunov methods, some robust stabilization techniques for uncertain delay systems were developed. In the research of control for time-delay systems, adaptive control is commonly used [14]–[18]. For example, Kwon and Park [12] designed a memoryless state-feedback controller by using the linear matrix inequality (LMI) technique. In addition, Yu and Cao [13] proposed a memoryless state-feedback controller together with a delayed feedback controller. The authors of [12], [13] derive some conditions to ensure the stability of delayed systems based on LMI technique and almost all the system parameters are involved in the LMI approach. However, as a useful tool, LMI is hard to apply to nonlinear systems with unknown system parameters, yet Lyapunov functional design has been proven to be an effective tool in such controllers design. Thus, some adaptive controllers received wide interests [14]–[18], [22]–[25]. Nonetheless, the designed controllers [14]–[18] are related to the system parameters or some restrictions must be satisfied in the designed controllers. However, in this brief, we propose to design some controllers that are independent of the bounds of the perturbations and system functions and the time delays. We do not need to know the bounds of the unknown parameters, but only need to know the structures of the modified Cohen–Grossberg neural networks.

Finally, it should be noted that some functions, for instance, the piecewise-linear approximation of a sigmoidal function, are nonsmooth. However, they are of special interest since they are widely used as activation functions neural networks model. In

Manuscript received April 29, 2006; revised September 25, 2006 and November 12, 2006. This work was supported by the National Natural Science Foundation of China under Grant 60574043 and Grant 60373067, by the Natural Science Foundation of Jiangsu Province of China under Grant BK2006093, and by the City University of Hong Kong under the SRG Grant 7001962. This paper was recommended by Associate Editor J. Suykens.

W. Yu and J. Cao are with the Department of Mathematics, Southeast University, Nanjing 210096, China (e-mail: wenwuyu@gmail.com; jdcao@seu.edu.cn).

G. Chen is with the Department of Electronic Engineering, City University of Hong Kong, Hong Kong (e-mail: eegchen@cityu.edu.hk).

Digital Object Identifier 10.1109/TCSII.2007.894427

this sense, nonsmooth analysis plays an paramount role in designing neural networks.

In this brief, a general modified Cohen–Grossberg neural network with uncertainties in parameters and with unknown time delays is considered. Several controllers are designed to ensure the global asymptotical stability of the network. Some simulation examples are constructed to verify the effectiveness of the designed controllers.

II. Model Formulation and Preliminaries

In this brief, a general modified Cohen–Grossberg neural network model with time delays is considered as follows:

$$\dot{x}(t)=-a\left(x(t)\right)\left[b\left(x(t)\right)-Af\left(x(t)\right)-Bf\left(x(t-\tau)\right)+I\right] \tag{1}$$

or

$$\dot{x}_i(t)=-a_i\left(x_i(t)\right)\left[b_i\left(x_i(t)\right)-\sum_{j=1}^{n}a_{ij}f_j\left(x_j(t)\right)-\sum_{j=1}^{n}b_{ij}f_j\left(x_j(t-\tau_{ij})\right)+I_i\right] \tag{2}$$

where $i=1,2,\cdots,n$, n denotes the number of neurons in the network, $x(t)=(x_1(t),x_2(t),\cdots,x_n(t))^T\in R^n$ is the state vector associated with the neurons, $I=(I_1,I_2,\cdots,I_n)^T\in R^n$ is the external input vector, $f(x(t))=(f_1(x_1(t)),f_2(x_2(t)),\cdots,f_n(x_n(t)))^T\in R^n$ corresponds to the activation functions of neurons, $\tau=\tau_{ij}$ $(i,j=1,2,\cdots,n)$ are the time delays, and the initial conditions are given by $x_i(t)=\phi_i(t)\in\mathcal{C}([-r,0],R)$ with $r=\max_{1\le i,j\le n}\{\tau_{ij}\}$, where $\mathcal{C}([-r,0],R)$ denoting the set of all continuous functions from $[-r,0]$ to R. Moreover, $a(x(t))=\operatorname{diag}(a_1(x_1(t)),a_2(x_2(t)),\cdots,a_n(x_n(t)))\in R^{n\times n}$ and $a_i(x_i(t))$ represents an amplification function, $b(x(t))=(b_1(x_1(t)),b_2(x_2(t)),\cdots,b_n(x_n(t)))^T\in R^n$ and $b_i(x_i(t))$ is a behaved function, and $A=(a_{ij})_{n\times n}$ and $B=(b_{ij})_{n\times n}$ are the unknown connection weight matrix and the delayed connection weight matrix, respectively. The matrices A and B represent the uncertainties in the system parameters, respectively, which are possibly time-varying or random. It is assumed that the equilibrium point exists for every well-posed initial condition. Our objective is to design a controller to ensure system (1) be globally asymptotically stable at its equilibrium point.

To establish the main results for model (1), it is necessary to make the following assumptions:

A1) Each function $a_i(\cdot)$ is positive, continuous and bounded, $0<\underline{a}\le a_i(x)\le\overline{a}(\forall x\in R)$, where $\underline{a}$ and $\overline{a}$ are positive constants.

A2) Each function $b_i:R\to R$ is locally Lipschitz and there exist $l_i>0$ such that $b_i'(x)\ge l_i$ for all $x\in R$ in which $b_i(\cdot)$ is continuously differentiable.

A3) Each function $f_i:R\to R$ is nondecreasing and globally Lipschitz with constants $k_i>0$, i.e.

$$|f_i(u)-f_i(v)|\le k_i|u-v| \qquad \forall u,v\in R, \qquad i=1,2,\cdots,n. \tag{3}$$

A4) The unknown matrices A and B are norm bounded

$$\|A\|\le\sqrt{\rho_A};\quad \|B\|\le\sqrt{\rho_B} \tag{4}$$

where ρ_A and ρ_B are positive constants.

Next, some notations are introduced, which will be used later for convenience.

For any vector $v=(v_1,v_2,\cdots,v_n)$, define $|v|=(|v_1|,|v_2|,\cdots,|v_n|)$, $\|v\|^2=v^Tv$. Similarly, for any matrix $W=(w_{ij})_{n\times n}$, define $|W|=(|w_{ij}|)_{n\times n}$. Let $\|\cdot\|$ denote the Euclidean norm in R^n. If W is a symmetric matrix with $\lambda_{\max}(W)$ and $\lambda_{\min}(W)$ as its largest and smallest eigenvalue, respectively, then its norm is defined by $\|W\|=sup\{\|Wx\|:\|x\|=1\}=\sqrt{\lambda_{\max}(W^TW)}$. The notation $W>0(W<0)$ means that W is positive definite (negative definite). Let also $L=\operatorname{diag}(l_1,l_2,\cdots,l_n)$, $K=\operatorname{diag}(k_1,k_2,\cdots,k_n)$, $l=\min_{1\le i\le n}\{l_i\}$ and $k=\max_{1\le i\le n}\{k_i\}$.

Finally, we introduce the definition of the Generalized Jacobian, which are essential for nonsmooth analysis on Lipschitz continuous functions. Let the function $F:R^n\to R^n$ be locally Lipschitz continuous. According to Rademacher's theorem [1, Th. 9. 60], F is differentiable almost everywhere. Let D_F denote the set of those points at where F is differentiable and $F'(x)$ denote the Jacobian of F at $x\in D_F$. Then, the set D_F is dense in R^n. For any given $x\in R^n$, define

$$\operatorname{Lip}_x F:=\sup_{y\to x,x\ne y\in R^n}\frac{\|F(y)-F(x)\|}{\|y-x\|}. \tag{5}$$

Since F is locally Lipschitz continuous, the constant Lip_xF is finite and $\|F'(x)\|\le\operatorname{Lip}_xF$ for all $x\in D_F$. Now, we are ready to define the generalized Jacobian in the sense of Clarke [2]:

Definition 1: For any $x\in R^n$, let $\partial F(x)$ be the set of the following collection of matrices:

$$\partial F(x)=\operatorname{co}\left\{W\middle|\text{ there exists a sequence of }\{x^k\}\subset D_F \text{ with }\lim_{x^k\to x}F'(x^k)=W\right\}$$

where $\operatorname{co}\Omega$ denotes the convex hull of the set Ω. We call $\partial F(x)$ as the generalized Jacobian.

It is easy to see that the above definition is well defined and $\|W\|\le\operatorname{Lip}_xF$ for all $W\in\partial F(x)$. We say that $\partial F(x)$ is invertible if every element W in $\partial F(x)$ is nonsingular. The generalized Jacobian $\partial F(x)$ have many nice properties, but only a few of them need to be singled out for our purpose. For one thing, the collection $\partial F(x)$ reduces to a singleton $\{F'(x)\}$ whenever F is continuously differentiable at x. We stress that $\partial F(x)$ may contain other elements if F is only differentiable at x.

Lemma 1 (Lebourg Theorem) [2, p. 41]: For any given $x,y\in R^n$, there exists an element W in the union $\bigcup_{z\in[x,y]}\partial F(z)$ such that

$$F(y)-F(x)=W(y-x) \tag{6}$$

where $[x,y]$ denotes the segment connecting x and y.

For more discussions on the generalized Jacobian and its applications, please refer to books [1], [2].

Now, we analyze system (1) from the viewpoint of nonsmooth analysis. We recall that a state $x^* \in R^n$ is called an equilibrium point of system (1) if it satisfies

$$-a(x^*)\left[b(x^*) - (A+B)f(x^*) + I\right] = 0. \tag{7}$$

Notice that $a_i(x_i(t))$ are positive, so (7) is equivalent to

$$b(x^*) - (A+B)f(x^*) + I = 0. \tag{8}$$

Assume that system (1) has an equilibrium $x^* = (x_1^*, x_2^*, \cdots, x_n^*)$ for a given I. To simplify the following proofs, we will shift the equilibrium point x^* of system (1) to the origin by using the following transformation:

$$y(t) = x(t) - x^*, \qquad y(t-\tau) = x(t-\tau) - x^*. \tag{9}$$

Then system (1) can be transformed into the following form:

$$\dot{y}(t) = -\overline{a}\left(y(t)\right)\left[\overline{b}\left(y(t)\right) - Ag\left(y(t)\right) - Bg\left(y(t-\tau)\right)\right] \tag{10}$$

or

$$\dot{y}_i(t) = -\overline{a}_i\left(y_i(t)\right)\left[\overline{b}_i\left(y_i(t)\right) - \sum_{j=1}^{n} a_{ij} g_j\left(y_j(t)\right) - \sum_{j=1}^{n} b_{ij} g_j\left(y_j(t-\tau_{ij})\right)\right], \qquad i = 1, 2, \cdots, n \tag{11}$$

where

$$\begin{aligned}
y(t) &= \left(y_1(t), y_2(t), \cdots, y_n(t)\right)^T \in R^n\\
\overline{a}\left(y(t)\right) &= \mathrm{diag}\left(\overline{a}_1\left(y_1(t)\right), \overline{a}_2\left(y_2(t)\right), \cdots \overline{a}_n\left(y_n(t)\right)\right) \in R^{n\times n}\\
\overline{b}\left(y(t)\right) &= \left(\overline{b}_1\left(y_1(t)\right), \overline{b}_2\left(y_2(t)\right), \cdots, \overline{b}_n\left(y_n(t)\right)\right)^T \in R^n\\
g\left(y(t)\right) &= \left(g_1\left(y_1(t)\right), g_2\left(y_2(t)\right), \cdots, g_n\left(y_n(t)\right)\right)^T \in R^n\\
\overline{a}_i\left(y_i(t)\right) &= a_i\left(y_i(t) + x_i^*\right)\\
\overline{b}_i\left(y_i(t)\right) &= b_i\left(y_i(t) + x_i^*\right) - b_i\left(x_i^*\right)\\
g_i\left(y_i(t)\right) &= f_i\left(y_i(t) + x_i^*\right) - f_i\left(x_i^*\right).
\end{aligned}$$

It is easy to see that

$$b_i(0) = 0, \qquad g_i(0) = 0 \qquad \forall i = 1, 2, \cdots, n.$$

Moreover, from (3), we know that

$$|g_i(y_i)| \le k_i |y_i| \qquad \forall y_i \in R, \qquad i = 1, 2, \cdots, n \tag{12}$$

and

$$\left\|g\left(y(t)\right)\right\|^2 \le y^T(t) K g\left(y(t)\right) \le y^T(t) K^2 y(t) \le k^2 y^T(t) y(t) \tag{13}$$

where $K = \mathrm{diag}(k_1, k_2, \cdots, k_n) > 0$ and $k = \max_{1\le i\le n}\{k_i\}$.

We add a controller u to system (10) to ensure the system be globally asymptotically stable about the origin and consider the following system:

$$\dot{y}(t) = -\overline{a}\left(y(t)\right)\left[\overline{b}\left(y(t)\right) - Ag\left(y(t)\right) - Bg\left(y(t-\tau)\right)\right] + u\left(y(t)\right) \tag{14}$$

or

$$\dot{y}_i(t) = -\overline{a}_i\left(y_i(t)\right)\left[\overline{b}_i\left(y_i(t)\right) - \sum_{j=1}^{n} a_{ij} g_j\left(y_j(t)\right) - \sum_{j=1}^{n} b_{ij} g_j\left(y_j(t-\tau_{ij})\right)\right] + u_i\left(y_i(t)\right) \tag{15}$$

where $i = 1, 2, \cdots, n$ and u is a function of the state vector. Note that if we use the controller u to stabilize system (14), then the goal of ensuring the asymptotical stability of system (1) can be obtained by the transformation (9). Thus, in this brief we just consider the asymptotical stability of system (14).

To obtain the main results, we furthermore need the following elementary lemma:

Lemma 2 [3]: For any vectors $x, y \in R^n$ and positive definite matrix $G \in R^{n\times n}$, the following matrix inequality holds:

$$2x^T y \le x^T G x + y^T G^{-1} y.$$

III. Robust Adaptive Controller Design

In this section, a simple adaptive controller is designed to stabilize system (14).

Theorem 1: Under the assumptions A1)–A4), the origin of model (14) is globally asymptotically stable if we choose the controller

$$u(y) = -\frac{1}{2}\underline{a}\widehat{l}p^{-1}y(t) \tag{16}$$

$$\dot{\widehat{l}} = y^T(t)y(t) \tag{17}$$

where $p > 0$ is a positive constant.

Proof: Consider the Lyapunov functional

$$V\left(y(t)\right) = p\sum_{i=1}^{n}\int_0^{y_i(t)} \frac{2s}{\overline{a}_i(s)}ds + \gamma\int_{t-\tau}^{t} g^T\left(y(s)\right)g\left(y(s)\right)ds + \frac{1}{2}\left[\widehat{l} - p(\varepsilon_A + \varepsilon_B) - p\left(\rho_A\varepsilon_A^{-1} + \rho_B\varepsilon_B^{-1}\right)k^2\right]^2 \tag{18}$$

where $p, \gamma, \varepsilon_A, \varepsilon_B$ are positive constants.

Taking the derivative of $V(y)$ along the trajectories of (14), we obtain

$$\begin{aligned}\dot V(y(t))\Big|_{(14)} &= p\sum_{i=1}^{n}\frac{2y_i(t)}{\overline{a}_i(y_i(t))}\dot y_i(t)+\gamma g^T(y(t))g(y(t))\\ &\quad-\gamma g^T(y(t-\tau))g(y(t-\tau))\\ &\quad+\left[\widehat{l}-p(\varepsilon_A+\varepsilon_B)-p\left(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1}\right)k^2\right]\dot{\widehat{l}}\\ &=-2py^T(t)\overline{b}(y(t))+2py^T(t)Ag(y(t))\\ &\quad+2py^T(t)Bg(y(t-\tau))+2py^T(t)\overline{a}(y(t))^{-1}u(y(t))\\ &\quad+\gamma g^T(y(t))g(y(t))-\gamma g^T(y(t-\tau))g(y(t-\tau))\\ &\quad+\left[\widehat{l}-p(\varepsilon_A+\varepsilon_B)-p\left(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1}\right)k^2\right]\dot{\widehat{l}}.\end{aligned}\tag{19}$$

From Lemma 1, we have

$$\begin{aligned}\overline{b}(y(t))&=b(y(t)+x^*)-b(x^*)\\ &=\widehat{M}y(t),\qquad \widehat{M}\in\bigcup_{z\in[x^*,y+x^*]}\partial b(z)\end{aligned}\tag{20}$$

where $\widehat{M}=\mathrm{diag}(\widehat{m}_1,\widehat{m}_2,\cdots,\widehat{m}_n)$. It is obvious that $\widehat{m}_i\geq l_i$ for $i=1,2,\cdots,n$. We thus obtain

$$y^T(t)\overline{b}(y(t))=\sum_{i=1}^{n}y_i(t)\widehat{m}_iy_i(t)\geq\sum_{i=1}^{n}l_iy_i^2(t)=y^T(t)Ly(t).\tag{21}$$

From Lemma 2 and Assumption A3), we obtain

$$\begin{aligned}2y^T(t)Ag(y(t))&\leq\varepsilon_Ay^T(t)y(t)+\varepsilon_A^{-1}g^T(y(t))A^TAg(y(t))\\ &\leq\varepsilon_Ay^T(t)y(t)+\rho_A\varepsilon_A^{-1}g^T(y(t))g(y(t))\end{aligned}\tag{22}$$

and

$$\begin{aligned}2y^T(t)Bg(y(t-\tau))&\leq\varepsilon_By^T(t)y(t)+\varepsilon_B^{-1}g^T(y(t-\tau))B^TBg(y(t-\tau))\\ &\leq\varepsilon_By^T(t)y(t)+\rho_B\varepsilon_B^{-1}g^T(y(t-\tau))g(y(t-\tau)).\end{aligned}\tag{23}$$

In addition, substituting (21), (22), and (23) into (19), we have

$$\begin{aligned}\dot V(y(t))\Big|_{(14)}&\leq-2py^T(t)Ly(t)+2py^T(t)\overline{a}(y(t))^{-1}u(y(t))\\ &\quad+p(\varepsilon_A+\varepsilon_B)y^T(t)y(t)+\left[p\rho_A\varepsilon_A^{-1}+\gamma\right]g^T(y(t))g(y(t))\\ &\quad+\left[p\rho_B\varepsilon_B^{-1}-\gamma\right]g^T(y(t-\tau))g(y(t-\tau))\\ &\quad+\left[\widehat{l}-p(\varepsilon_A+\varepsilon_B)-p\left(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1}\right)k^2\right]\dot{\widehat{l}}.\end{aligned}\tag{24}$$

If we choose $\gamma=p\rho_B\varepsilon_B^{-1}$, then it follows that

$$\begin{aligned}\dot V(y(t))\Big|_{(14)}&\leq-2py^T(t)Ly(t)+2py^T(t)\overline{a}(y(t))^{-1}u(y(t))\\ &\quad+p(\varepsilon_A+\varepsilon_B)y^T(t)y(t)\\ &\quad+\left[p\rho_A\varepsilon_A^{-1}+p\rho_B\varepsilon_B^{-1}\right]g^T(y(t))g(y(t))\\ &\quad+\left[\widehat{l}-p(\varepsilon_A+\varepsilon_B)-p\left(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1}\right)k^2\right]\dot{\widehat{l}}.\end{aligned}\tag{25}$$

From (13), we have

$$\begin{aligned}\dot V(y(t))\Big|_{(14)}&\leq-2py^T(t)Ly(t)+2py^T(t)\overline{a}(y(t))^{-1}u(y(t))\\ &\quad+p(\varepsilon_A+\varepsilon_B)y^T(t)y(t)+p\left(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1}\right)k^2y^T(t)y(t)\\ &\quad+\left[\widehat{l}-p(\varepsilon_A+\varepsilon_B)-p\left(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1}\right)k^2\right]\dot{\widehat{l}}\\ &\leq-2py^T(t)Ly(t)-\widehat{l}y^T(t)y(t)\\ &\quad+p\left[(\varepsilon_A+\varepsilon_B)+\left(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1}\right)k^2\right]y^T(t)y(t)\\ &\quad+\left[\widehat{l}-p(\varepsilon_A+\varepsilon_B)-p\left(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1}\right)k^2\right]\dot{\widehat{l}}\\ &=-2py^T(t)Ly(t)\\ &\quad+\left[\widehat{l}-p(\varepsilon_A+\varepsilon_B)-p\left(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1}\right)k^2\right]\left[\dot{\widehat{l}}-y^T(t)y(t)\right]\\ &=-2py^T(t)Ly(t).\end{aligned}\tag{26}$$

Therefore, with the designed controller (16) and (17), $\dot V(y(t))=0$ if and only if $y(t)=0$; otherwise $\dot V(y(t))\leq 0$. Moreover, $V(y)$ is radially unbounded since $V(y(t))\to\infty$ as $\|y(t)\|\to\infty$. We have thus proved that the equilibrium of (14) is globally asymptotically stable. This completes the proof.

Corollary 1: Under the assumptions A1)–A4), the origin of model (14) is globally asymptotically stable if we choose the controller

$$u(y)=-\widehat{l}y(t)\tag{27}$$

$$\dot{\widehat{l}}=y^T(t)y(t).\tag{28}$$

Proof: We choose $p=(1/2)\underline{a}$ to obtain Corollary 1 directly from Theorem 1. The proof is completed.

Remark 1: In Corollary 1, it is easy to see that the designed controller is independent of the time delays, the bounds of the unknown parameters and system functions. In addition, the nonsmooth functions is considered in this brief. We can ensure the global asymptotical stability of the system even without knowing any condition on the unknown parts of the system.

IV. Simulation Examples

In this section, two simulation examples are given to show the effectiveness of the designed controllers.

Example: Consider the following modified Cohen–Grossberg neural network with time delays:

$$\begin{pmatrix}\frac{dy_1(t)}{dt}\\ \frac{dy_2(t)}{dt}\end{pmatrix}=-\begin{pmatrix}b_1(y_1(t))\\ b_2(y_2(t))\end{pmatrix}+A\begin{pmatrix}\tanh(y_1(t))\\ \tanh(y_2(t))\end{pmatrix}+B\begin{pmatrix}\tanh(y_1(t-1)\\ \tanh(y_2(t-1)\end{pmatrix}\tag{29}$$

where

$$b_i(u)=\begin{cases}u, & \text{if } u\geq 0\\ 2u, & \text{if } u<0\end{cases}$$

for $i=1,2$. It is easy to see that the assumptions A1)–A4) are satisfied. Moreover, $a_i(y_i(t))=1$ $(i=1,2)$ are bounded, positive continuous functions, $f_i(y_i(t))=\tanh(y_i(t))$ is globally

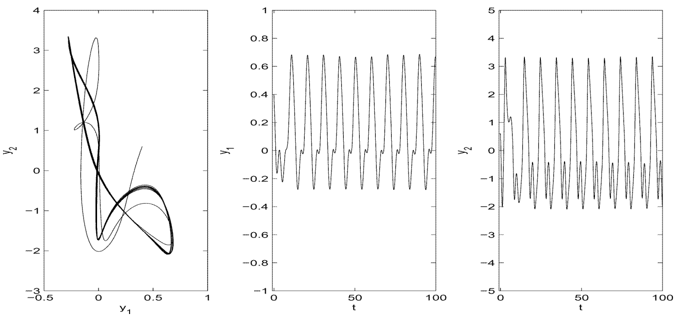

Fig. 1. Trajectories of state variables $y_1(t)$ and $y_2(t)$ without control.

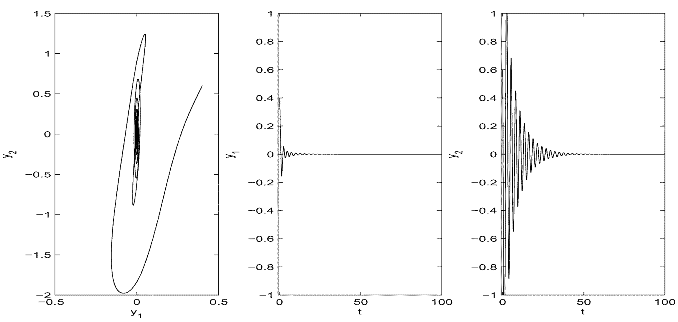

Fig. 2. Trajectories of state variables $y_1(t)$ and $y_2(t)$ with control.

Lipschitz and nondecreasing. Obviously, $K = E$, where E is the identity matrix, and

$$A = \begin{pmatrix} 2.0 & -0.1 \\ -5.0 & 3.2 \end{pmatrix} + \Delta A$$
$$B = \begin{pmatrix} -1.5 & -0.1 \\ -0.2 & -2.8 \end{pmatrix} + \Delta B.$$

Suppose all elements of the uncertainty matrices ΔA and ΔB are random in $[-0.02, 0.02]$ at all times.

The trajectories of system (29) without controller and with controller (27) and (28) are shown in Figs. 1 and 2, respectively. It is easy to see that without controller, system (29) is chaotic, but it is stable with controller (27) and (28) according to Corollary 1. The designed controller (27) and (28) is effective and can ensure the stability of system (29). Here, the behaved function is nonsmooth and the perturbations to the unknown parameters A and B are random.

V. Conclusion

In this brief, the robust adaptive control problem for unknown modified Cohen–Grossberg neural networks with time delays has been studied. Several effective controllers have been designed for ensuring the global asymptotical stability of the networks. One does not need to know the bounds of the unknown parameters and the time delays. Also, the functions in the networks can be nonsmooth. The effectiveness and feasibility of the designed controllers have been demonstrated through numerical simulations.

References

[1] R. T. Rockafellar and R. J. B. Wets, *Variational Analysis*. Berlin, Germany: Springer-Verlag, 1998.

[2] F. H. Clarke, *Optimization and Nonsmooth Analysis*. New York: Wiley, 1983.

[3] M. A. Cohen and S. Grossberg, "Absolute stability of global pattern formation and parallel memory storage by competitive neural networks," *IEEE Trans. Syst. Man Cybern.*, vol. SMC-13, no. 5, pp. 815–825, Sep./Oct. 1983.

[4] H. Ye, A. N. Michel, and K. Wang, "Qualitive analysis of Cohen–Grossberg neural networks with multiple delays," *Phys. Rev. E*, vol. 51, pp. 2611–2618, 1995.

[5] T. Chen and L. Rong, "Robust global exponential stability of Cohen–Grossberg neural networks with time delays," *IEEE Trans. Neural Networks*, vol. 15, no. 1, pp. 203–206, Jan. 2006.

[6] Z. Wang, Y. Liu, M. Li, and X. Liu, "Stability analysis for stochastic Cohen–Grossberg neural networks with mixed time delays," *IEEE Trans. Neural Netw.*, vol. 17, no. 3, pp. 814–820, May 2006.

[7] K. Yuan and J. Cao, "An analysis of global asymptotic stability of delayed Cohen–Grossberg neural networks via nonsmooth analysis," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 52, no. 9, pp. 1854–1861, Sep. 2005.

[8] J. Cao and J. Wang, "Global asymptotic and robust stability of recurrent neural networks with time delays," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 52, no. 2, pp. 417–426, Feb. 2005.

[9] S. Q. Hu and J. Wang, "Global exponential stability of continuous-time interval neural networks," *Phys. Rev. E*, vol. 65, pp. 1–9, 2002.

[10] H. Ye and A. N. Michel, "Robust stability of nonlinear time-delay systems with applications to neural networks," *IEEE Trans. Circuits Syst. I, Fundam. Theory Appl.*, vol. 43, no. 7, pp. 532–543, Jul. 1996.

[11] X. F. Liao, K. W. Wong, Z. Wu, and G. Chen, "Novel robust stability criteria for interval-delayed Hopfield neural networks," *IEEE Trans. Circuits Syst. I, Fundam. Theory Appl.*, vol. 48, no. 11, pp. 1355–1359, Nov. 2001.

[12] O. M. Kwon and J. H. Park, "On improved delay-dependent robust control for uncertain time-delay systems," *IEEE Trans. Autom. Contr.*, vol. 49, no. 11, pp. 1991–1995, Nov. 2004.

[13] W. Yu and J. Cao, "Synchronization control of stochastic delayed neural networks," *Phys. A*, vol. 373, pp. 252–260, 2007.

[14] F. Zheng, Q. Wang, and T. Lee, "Adaptive robust control of uncertain time delay systems," *Automatica*, vol. 41, pp. 1375–1383, 2005.

[15] W. Yu and J. Cao, "Adaptive Q-S (lag, anticipated, and complete) time-varying synchronization and parameters identification of uncertain delayed neural networks," *Chaos*, vol. 16, 2006, article 023119.

[16] S. S. Ge, F. Hong, and T. H. Lee, "Robust adaptive control of nonlinear systems with unknown time delays," *Automatica*, vol. 41, pp. 1181–1190, 2005.

[17] S. S. Ge, F. Hong, and T. H. Lee, "Adaptive neural network control of nonlinear systems with unknown time delays," *IEEE Trans. Autom. Contr.*, vol. 48, no. 12, pp. 2004–2010, Dec. 2003.

[18] F. J. Lin, R. J. Wai, W. D. Chou, and S. P. Hsu, "Adaptive backstepping control using recurrent neural network for linear induction motor drive," *IEEE Trans. Ind. Electron.*, vol. 49, no. 1, pp. 134–146, Feb. 2002.

[19] V. Singh, "Global robust stability of delayed neural networks: An LMI approach," *IEEE Trans. Circuits Syst. II*, vol. 52, no. 1, pp. 33–36, Jan. 2005.

[20] V. Singh, "Robust stability of cellular neural networks with delay: Linear matrix inequality approach," *Proc. IEE Control Theory Appl.*, vol. 151, no. 1, Jan. 2004.

[21] S. Arik, "Global robust stability of delayed neural networks," *IEEE Trans. Circuits Syst. I, Fundam. Theory Appl.*, vol. 50, no. 1, pp. 156–160, Jan. 2003.

[22] K. Ichikawa, "Adaptive control of delay systems," *Int. J. Contr.*, vol. 43, pp. 1653–1659, 1986.

[23] R. Ortega and R. Lozano, "Globally stable adaptive controller for systems delay," *Int. J. Contr.*, vol. 47, pp. 17–23, 1988.

[24] O. M. Kwon and J. H. Park, "On improved delay-dependent robust control for uncertain time-delay systems," *IEEE Trans. Autom. Contr.*, vol. 49, no. 11, pp. 1991–1995, Nov. 2004.

[25] W. Yu and J. Cao, "Adaptive synchronization and lag synchronization of uncertain dynamical system with time delay based on parameter identification," *Phys. A*, vol. 375, no. 2, pp. 467–482, 2007.

[26] J. Cao and Q. Song, "Stability in Cohen–Grossberg type BAM neural networks with time-varying delays," *Nonlinearity*, vol. 19, no. 7, pp. 1601–1617, 2006.

[27] J. Cao and X. Li, "Stability in delayed Cohen–Grossberg neural networks: LMI optimization approach," *Phys. D*, vol. 212, no. 1–2, pp. 54–65, 2005.

[28] J. Cao and J. Liang, "Boundedness and stability for Cohen–Grossberg neural networks with time-varying delays," *J. Math. Anal. Appl.*, vol. 296, no. 2, pp. 665–685, 2004.