Response to "Comment on 'Adaptive Q-S (lag, anticipated, and complete) time-varying synchronization and parameters identification of uncertain delayed neural networks'" [Chaos 17, 038101 (2007)]

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Parameter identification of dynamical systems from time series has received increasing interest due to its wide applications in secure communication, pattern recognition, neural networks, and so on. Given the driving system, parameters can be estimated from the time series by using an adaptive control algorithm. Recently, it has been reported that for some stable systems, in which parameters are difficult to be identified [Li *et al.*, Phys Lett. A **333**, 269–270 (2004); Remark 5 in Yu and Cao, Physica A **375**, 467–482 (2007); and Li *et al.*, Chaos **17**, 038101 (2007)], and in this paper, a brief discussion about whether parameters can be identified from time series is investigated. From some detailed analyses, the problem of why parameters of stable systems can be hardly estimated is discussed. Some interesting examples are drawn to verify the proposed analysis. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2749458]

Chaos synchronization has been intensively investigated since its wide applications in secure communication, automatic control, parameter identification, chemical reactor, physics, etc. Among these, one of the interesting applications is parameter identification from dynamical systems especially from chaotic systems, which can be applied to secure communication based on adaptive synchronization.[4,5] Many works[2,6–11] have studied chaos synchronization by using an adaptive control method,[12] and parameters can be estimated from time series of dynamical systems simultaneously. In Refs. 1 and 3, it has been reported that parameters cannot be estimated from some stable systems; thus someone may doubt if this adaptive control method is effective. Some analyses about why parameters of stable systems can hardly be estimated are studied in this paper, which may make some contribution to this field.

I. INTRODUCTION AND SUMMARY

Since the pioneering work proposed in Ref. 13, chaos synchronization has been widely investigated since its extensive applications in secure communication, automatic control, artificial neural networks, chemical reactor, physics, etc. In Ref. 13, the authors introduced a new method to synchronize two identical systems with different initial conditions, which are known as driving and response systems now. The idea of synchronization here is to use the output of the driving system to control the dynamics of the response system so that the output of the response system can synchronize with the output of the driving system.

In Ref. 14, the author first estimated model parameters from time series by autosynchronization. This idea is quite impressive and incited a new way to estimate model parameters from the output of the system (time series). Then, in Refs. 1 and 3, the authors found that the method used in Refs. 7, 14, and 15 can be ineffective for estimating parameters from the stable system, and some counterexamples are also proposed in Refs. 1 and 3. Later, in Ref. 16, the author declared that the statements in Ref. 1 are inaccurate and the detailed analysis is given by studying the largest invariance set based on LaSalle invariance principle, which provided a clear view about this topic. In Ref. 16, the author showed that for chaotic or periodic systems, parameters can be estimated by using the method in Refs. 14. Recently, the authors in Refs. 17 and 18 all studied parameters identification of dynamical systems from time series. In Ref. 17, the author stressed that "the chaotic behavior is necessary to realize such techniques of parameter estimation," and in Ref. 18, the authors also demonstrated that "both adaptive synchronization and parameter identification are more rapidly achieved" by using such control techniques. In addition, some explanations about whether parameters of chaotic systems can be estimated are also discussed.

From what has been introduced above, it is easy to obtain that parameters can be hardly estimated from some stable systems and may be clearly identified from chaotic systems. Therefore, in order to investigate clearly the parameter identification problem of dynamical systems from time series, the main question should be answered: Why cannot parameters of stable systems be estimated from time series? Is there any problem in the previous works?[2,6–11,15] Next, we will focus on this problem, and a detailed analysis will be addressed. Then, it is obvious to see why parameters cannot be estimated from time series of stable systems; however, parameters of some chaotic systems may be estimated effectively by using LaSalle invariance principle as in Refs. 17 and 18.

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In order to derive the main results, some prior knowledge about adaptive synchronization and parameters identification in Ref. 15 are first introduced. For more details, please refer to Ref. 15. Consider the following driving system:

$$\dot{y}(t) = -Cy(t) + Ag(y(t)) + Bl(y(t-\tau(t))) \tag{1}$$

or

$$\dot{y}_i(t) = -c_i y_i(t) + \sum_{j=1}^{n} a_{ij} g_j(y(t)) + \sum_{j=1}^{n} b_{ij} l_j(y(t-\tau_{ij}(t))), \quad i=1,2,\ldots,n, \tag{2}$$

where n denotes the number of units in a neural network; $y(t)=(y_1(t),y_2(t),\ldots,y_n(t))^T \in \mathbb{R}^n$ is the state vector associated with the neurons; $g(y(t)) =(g_1(y(t)),g_2(y(t)),\ldots,g_n(y(t)))^T \in \mathbb{R}^n$ and $l(y(t-\tau(t))) =(l_1(y(t-\tau(t))),l_2(y(t-\tau(t))),\ldots,l_n(y(t-\tau(t))))^T \in \mathbb{R}^n$ correspond to the activation functions and delayed activation functions of neurons; and $\tau(t)=\tau_{ij}(t)$ $(i,j=1,2,\ldots,n)$ are the multiple time-varying delays. Suppose that each $\tau_{ij}(t)$ $(i,j=1,2,\ldots,n)$ is bounded and the initial conditions of (1) are given by $x_i(t)=\phi_i(t) \in \mathcal{C}([-r,0],\mathbb{R})$ with $r=\max_{1\le i,j\le n, t\in R}\{\tau_{ij}(t)\}$, where $\mathcal{C}([-r,0],\mathbb{R})$ denotes the set of all continuous functions from $[-r,0]$ to $\mathbb{R}$. $C=\text{diag}(c_1,c_2,\ldots,c_n)$ is a diagonal matrix, $A=(a_{ij})_{n\times n}$ and $B=(b_{ij})_{n\times n}$ are the connection weight matrix and the delayed connection weight matrix, respectively, which are unknown.

The response system is

$$\dot{z}(t) = -\bar{C}(t)z(t) + \bar{A}(t)g(z(t)) + \bar{B}(t)l(z(t-\tau(t))) + u, \tag{3}$$

namely,

$$\dot{z}_i(t) = -\bar{c}_i(t)z_i(t) + \sum_{j=1}^{n} \bar{a}_{ij}(t) g_j(z(t)) + \sum_{j=1}^{n} \bar{b}_{ij}(t) l_j(z(t-\tau_{ij}(t))) + u_i, \quad i=1,2,\ldots,n, \tag{4}$$

where $\bar{C}(t)=\text{diag}(\bar{c}_1(t),\bar{c}_2(t),\ldots,\bar{c}_n(t))$, $\bar{A}(t)=(\bar{a}_{ij}(t))_{n\times n}$ and $\bar{B}(t)=(\bar{b}_{ij}(t))_{n\times n}$ are matrix functions depending on the time t, $u(t)=(u_1(t),u_2(t),\ldots,u_n(t))$ is a controller. In practical situations, the output signals of the driving system (1) can be received by the response system (3), but the parameter matrices C, A and B of the driving system (1) are unknown, which are needed to be estimated.

Then, Theorem 1 in Ref. 15 is established.

Theorem 1: Assume that the Jacobian matrix of the vector function Q is invertible. Then, the driving system (1) Q-S time varying synchronizes with the response system (3) if one chooses

$$\dot{\bar{c}}_i(t) = \frac{1}{q_i}(1-\dot{r}(t))\left[\sum_{k=1}^{n} p_k e_k(t) \frac{\partial S_k(y(t-r(t)))}{\partial y_i(t-r(t))} y_i(t-r(t))\right], \tag{5}$$

$$\dot{\bar{a}}_{ij}(t) = -\frac{1}{r_{ij}}(1-\dot{r}(t)) \times \left[\sum_{k=1}^{n} p_k e_k(t) \frac{\partial S_k(y(t-r(t)))}{\partial y_i(t-r(t))} g_j(y(t-r(t)))\right], \tag{6}$$

$$\dot{\bar{b}}_{ij}(t) = -\frac{1}{s_{ij}}(1-\dot{r}(t)) \times \left[\sum_{k=1}^{n} p_k e_k(t) \frac{\partial S_k(y(t-r(t)))}{\partial y_i(t-r(t))} l_j(y(t-\tau_{kj}(t)-r(t)))\right], \tag{7}$$

$$\begin{aligned} u = &-[DQ(z(t))]^{-1}Me(t) + \bar{C}(t)z(t) - \bar{A}(t)g(z(t)) \\ &- \bar{B}(t)l(z(t-\tau(t))) + (1-\dot{r}(t))[DQ(z(t))]^{-1} \\ &\times [DS(y(t-r(t)))][-\bar{C}(t)y(t-r(t)) \\ &+ \bar{A}(t)g(y(t-r(t))) + \bar{B}(t)l(y(t-\tau(t)-r(t)))], \end{aligned} \tag{8}$$

where $p_i,q_i,r_{ij},s_{ij}(i,j=1,2,\ldots,n)$ are positive constants, $M=\text{diag}(m_1,m_2,\ldots,m_n)$ is a positive definite matrix. $\bar{C}(t)$, $\bar{A}(t)$, and $\bar{B}(t)$ are independent of C, A, and B.

Let $e(t)=Q(z(t))-S(y(t-r(t)))$, and subtracting (1) from (3) yields the synchronization error dynamical system as follows:

$$\begin{aligned} \dot{e}(t) = &-Me(t) + (1-\dot{r}(t))DS(y(t-r(t))) \\ &\times[-(\bar{C}(t)-C)y(t-r(t)) + (\bar{A}(t)-A)g(y(t-r(t))) \\ &+ (\bar{B}(t)-B)l(y(t-\tau(t)-r(t)))], \end{aligned} \tag{9}$$

namely,

$$\begin{aligned} \dot{e}_i(t) = &-m_i e_i(t) + (1-\dot{r}(t)) \\ &\times\Bigg[-\sum_{j=1}^{n}\frac{\partial S_i(y(t-r(t)))}{\partial y_j(t-r(t))}(\bar{c}_j(t)-c_j)y_j(t-r(t)) \\ &+\sum_{j=1}^{n}\sum_{k=1}^{n}\frac{\partial S_i(y(t-r(t)))}{\partial y_k(t-r(t))}(\bar{a}_{kj}(t)-a_{kj})g_j(y(t-r(t))) \\ &+\sum_{j=1}^{n}\sum_{k=1}^{n}\frac{\partial S_i(y(t-r(t)))}{\partial y_k(t-r(t))}(\bar{b}_{kj}(t)-b_{kj})l_j(y(t-\tau_{ij}(t) \\ &-r(t)))\Bigg], \quad i=1,2,\ldots,n. \end{aligned} \tag{10}$$

Consider the following Lyapunov functional candidate:

$$\begin{aligned} V(e(t),&\bar{C}(t)-C,\bar{A}(t)-A,\bar{B}(t)-B) \\ &= \tfrac{1}{2}\sum_{i=1}^{n}\Bigg\{p_i e_i^2(t) + q_i[\bar{c}_i(t)-c_i]^2 + \sum_{j=1}^{n} r_{ij}[\bar{a}_{ij}(t)-a_{ij}]^2 \\ &+ \sum_{j=1}^{n} s_{ij}[\bar{b}_{ij}(t)-b_{ij}]^2\Bigg\}; \end{aligned} \tag{11}$$

then one has

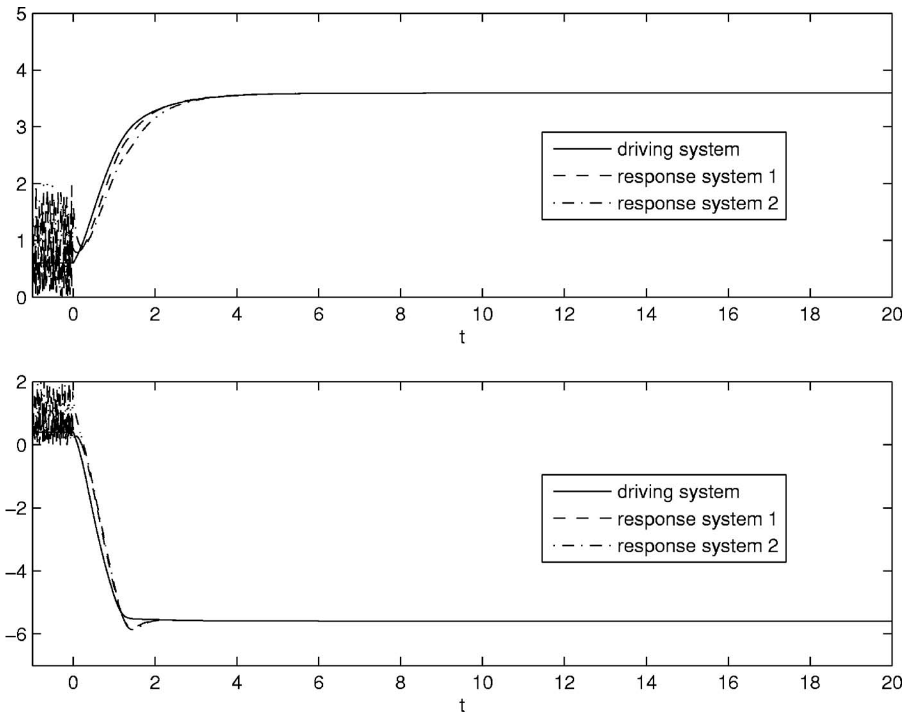

FIG. 1. Trajectories of driving system (solid line) and two response systems (dashed lines).

$$\frac{dV}{dt} \le -\sum_{i=1}^{n}\{p_i m_i e_i^2(t)\}. \tag{12}$$

From Lyapunov functional method, one can easily obtain the asymptotical stability of error system (9). Here, only stability results of the error system can be derived, and the parameters cannot be identified from (12) directly. However, a lot of existing results[2,6–11,15,17,18] investigated parameters identification of dynamical systems by using the similar method, which may not be accurate. In Refs. 17 and 18, the authors studied the parameters identification by using the LaSalle invariance principle and concluded that parameters of chaotic system can be estimated.

II. ANALYSIS AND EXAMPLES OF PARAMETERS IDENTIFICATION FOR STABLE SYSTEMS

Because of the lack of theoretical analyses of parameters identification, some counterexamples are proposed in Refs. 1 and 3, which show that parameters of stable systems can be hardly estimated. Next, we will investigate why the theoretical analysis cannot work for stable systems, and if there are some relationships between the original stable system and the identified stable system.

Consider a typical delayed Hopfield neural network as driving system,

$$\dot{y}(t) = -Cy(t) + Ag(y(t)) + Bg(y(t-\tau(t))), \tag{13}$$

where

$$C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad A = \begin{pmatrix} 5.0 & -0.1 \\ -5.0 & 2.8 \end{pmatrix}, \quad B = \begin{pmatrix} -1.6 & -0.1 \\ -0.3 & -2.5 \end{pmatrix},$$

$$\tau_{ij} = 1 \quad (i,j=1,2), \quad g(y) = \begin{pmatrix} \tanh y_1 \\ \tanh y_2 \end{pmatrix},$$

which is the same as in Ref. 3, where the authors changed the parameter $a_{11}=2$ in simulation example[15] to $a_{11}=5$. However, this changing model does not exhibit chaos phenomenon as illustrated in Fig. 1 shown above. In the following, a special case of complete synchronization is considered for simplicity, i.e., $z(t) \to y(t)$ as $t \to \infty$.

Here, two response systems (3)–(8) with different initial conditions are considered. From the above analysis, one obtains that the driving system (13) synchronizes with the two response systems as illustrated in Fig. 1. The solid line and two dashed lines are trajectories of the driving system and two response systems, respectively. The true parameter values in the driving system and trajectories of parameters estimation in two response systems are shown with solid and dashed lines as in Figs. 2–4, respectively. It is easy to see that parameters of driving systems cannot be estimated. Surprisingly, two response systems with different initial conditions converge to different values. Then, one can hardly decide which is the true parameter estimation of the original driving system. Why does this question occur? Next, some analyses concerning this problem are demonstrated.

Note that on the synchronization manifold, where $y(t) = z(t) = (\alpha_1 \ \alpha_2)^T$ (α_1 and α_2 are constants for stable systems), the error dynamical system (9) can be rewritten as

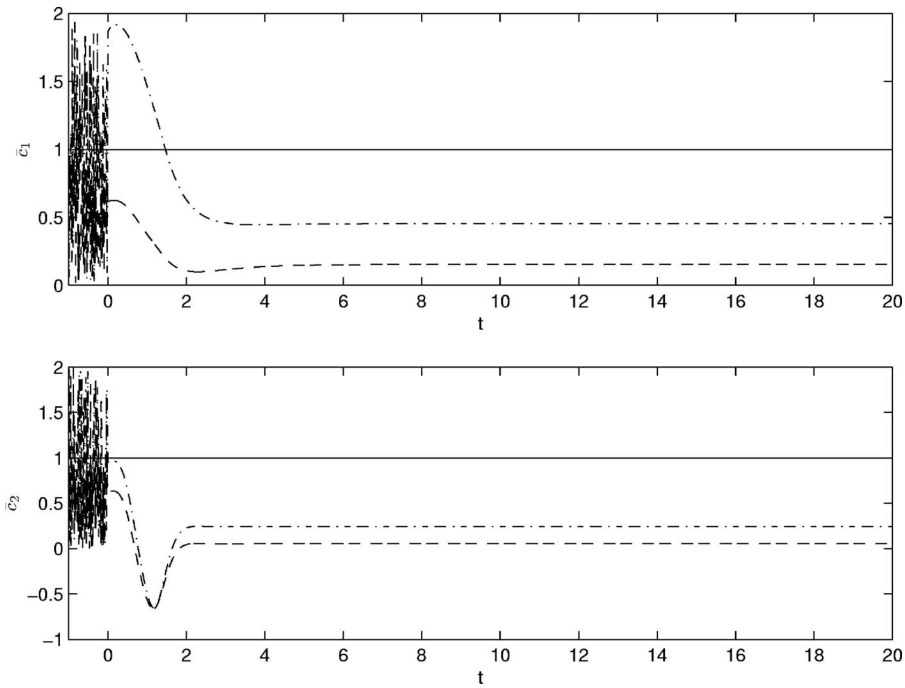

FIG. 2. True parameter value (solid line) in driving system and parameters estimation of $\bar{C}$ (dashed lines) in two response systems.

$$0=(\bar{c}_j(t)-c_j)\alpha_j+\sum_{j=1}^{n}(\bar{a}_{ij}(t)-a_{ij})\tanh(\alpha_j)+\sum_{j=1}^{n}(\bar{b}_{ij}(t)-b_{ij})\tanh(\alpha_j),\quad i=1,2. \tag{14}$$

Given this equation, where $\bar{c}_j$, $\bar{a}_{ij}$, and $\bar{b}_{ij}$ are unknown variables, there are no unique solutions for these variables. It is obvious that two completely different systems that have the same stable equilibrium can achieve synchronization. This means that the states of two response systems both converge to the same equilibrium as shown in Fig. 1. However, parameters in two systems are not the same.

Actually, the driving system (1) and the response system (3) and (5)–(8) can be considered as a whole system with dimensions $3n+2n^2$. The number of variables in the whole system are $3n+2n^2$, which include y_i, z_i, c_i, a_{ij}, and b_{ij} for $i,j=1,2,\ldots,n$. Note that in (14), there are n equations with $n+2n^2$ variables when y_i and z_i are known equilibrium points on the synchronization manifold. Therefore, there exist many different solutions c_i, a_{ij}, b_{ij} satisfying (14). From (12), one knows that y_i and z_i are the same on the synchronization

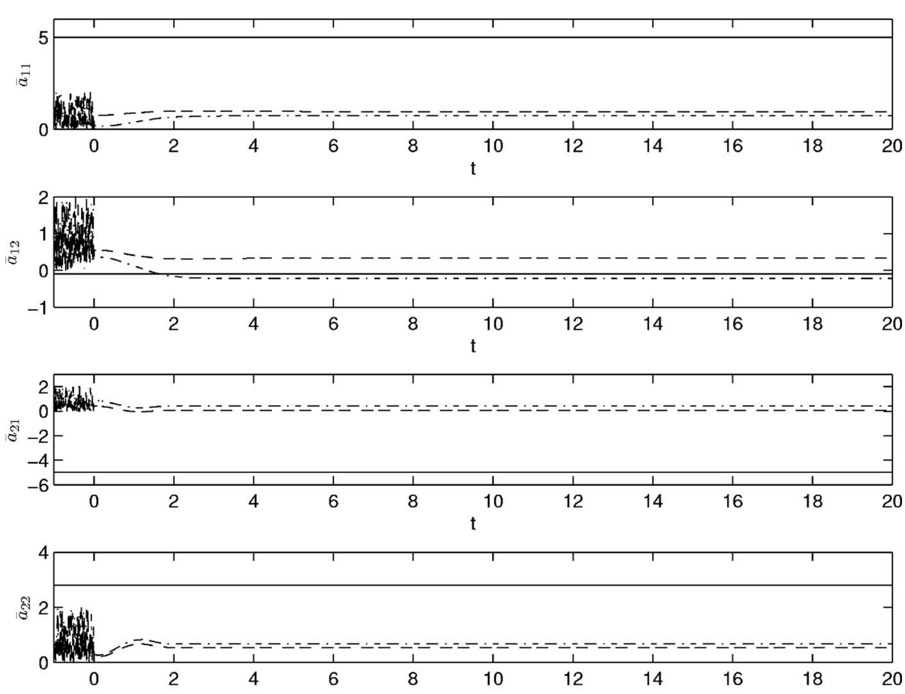

FIG. 3. True parameter value (solid line) in driving system and parameters estimation of $\bar{A}$ (dashed lines) in two response systems.

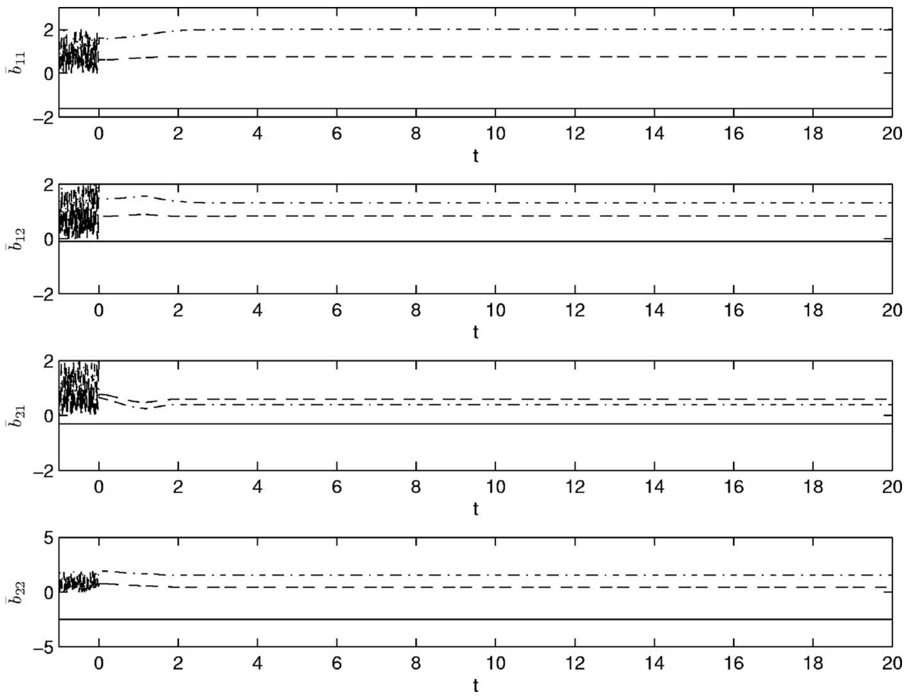

FIG. 4. True parameter value (solid line) in driving system and parameters estimation of $\bar{B}$ (dashed lines) in two response systems.

manifold for stable systems, however, c_i, a_{ij}, and b_{ij} can vary differently for different response systems.

The learning equations (5)–(7) can just achieve some parameters, which enables the response system to have the same stable equilibrium as the driving system. However, if the driving system is chaotic, the learning equation may work effectively. For some chaotic driving systems, the parameters may be estimated by adaptive control method and the LaSalle invariant principle.[17,18] It might be impossible for another response system with identical structure, but with different parameters, to track the same chaotic driving system. Once it is not in the synchronization state, the parameters' update law will be activated to make the estimated parameters approach the true value.

III. CONCLUSIONS

Since some counter examples are proposed in Refs. 1 and 3, which show that parameters cannot be estimated from stable systems, and one may doubt whether parameters can be precisely estimated from time series of dynamical systems. Some detailed explanations and analyses are derived in this paper. From previous analyses,[16–18] it has been reported that parameters of chaotic systems can be estimated from time series. Some detailed analyses and examples are given in this paper to investigate why parameters of stable systems cannot be estimated.

As for the stable systems, the dynamical asymptotic behavior is only an equilibrium point, which provides inadequate information for estimating their parameters. Two completely different systems with the same stable equilibrium point can synchronize with each other. However, for chaotic systems, chaotic attractor provides more information for identifying the parameters. If there is a mismatch in the parameters of two identical systems, it may be hard for them to achieve synchronization. However, this is not to say that parameters of chaotic systems can be surely estimated for time series. In Ref. 19, synchronization based parameter identification of dynamical systems from time series is carefully revisited. A linear independence condition is pointed out, which is sufficient for such parameter identification of general dynamical systems.

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