Adaptive Q-S (lag, anticipated, and complete) time-varying synchronization and parameters identification of uncertain delayed neural networks

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In this paper, a new type of generalized Q-S (lag, anticipated, and complete) time-varying synchronization is defined. Adaptive Q-S (lag, anticipated, and complete) time-varying synchronization and parameters identification of uncertain delayed neural networks have been considered, where the delays are multiple time-varying delays. A novel control method is given by using the Lyapunov functional method. With this new and effective method, parameters identification and Q-S (lag, anticipated, and complete) time-varying synchronization can be achieved simultaneously. Simulation results are given to justify the theoretical analysis in this paper. © *2006 American Institute of Physics*. [DOI: 10.1063/1.2204747]

Chaos synchronization is a hot topic recently since its extensive applications in secure communication, automatic control, artificial neural networks, chemical reactor, physics, etc. In this paper, a new type of Q-S (lag, anticipated, and complete) time-varying synchronization is defined. Since most of the synchronization methods belong to master-slave (drive-response) type, the drive chaotic neural network system with unknown parameters and multiple time-varying delays is stated. We give a systematic, powerful, and concrete scheme to identify all unknown parameters. The unknown parameters can be precisely estimated dynamically through an adaptive control process and Q-S (lag, anticipated, and complete) time-varying synchronization between the drive system and response system can be achieved simultaneously based on Lyapunov functional method. Simulation results are given to show the effectiveness and feasibility of the theoretical analysis in this paper.

I. INTRODUCTION

Dynamical behaviors are interesting nonlinear phenomena and have been intensively investigated for many years due to its potential applications in secure communications, chemical reactions, biological systems and so on. Stability,[13,14] bifurcation,[24,25] and chaos synchronization[1–12,15–20,28] are also studied by many researchers.

Chaos synchronization, which has been a hot topic in nonlinear science, has attracted more attention in many fields such as physics, secure communication, automatic control, artificial neural networks, etc. Chaotic systems exhibit sensitive dependence on initial conditions. Because of this property, chaotic systems are difficult to be synchronized or controlled. In recent years, many important and fundamental results have been reported on the control and synchronization[15–21] of chaotic systems.

Many types of synchronization have been presented, such as complete synchronization, generalized synchronization, phase synchronization, lag synchronization, anticipated synchronization, antiphase synchronization, etc. Recently, Yan has considered Q-S (lag, anticipated, and complete) synchronization in Ref. 16. In this paper, we will define a new type of Q-S (lag, anticipated, and complete) time-varying synchronization that is based on Ref. 16.

An interesting application of chaos synchronization[1–3,7–12] is to estimate parameters of a chaotic system from time series when partial information about the system is available. In this paper, we address the problem of how chaos synchronization can be used for time series analysis. Suppose that you have obtained time series from a nonlinear (chaotic) system and that you have known the structure of the system, but the parameters are unknown. By obtaining the output of the system, another designed system identically synchronize with the original system. Although the parameters can be known in some cases, it would be desirable to have a feedback scheme to achieve synchronization in spite of the slave oscillator having least prior knowledge about the structure of the master system.

The organization of this paper is as follows: In Sec. II, we give the formulation and preliminaries for our main results. In Sec. III, an adaptive law is presented for the synchronization and parameters identification. Also, some corollaries and remarks are given to show the advantages of this paper. In Sec. IV, examples are given to show the effectiveness and feasibility of the proposed approach in this paper. In Sec. V, we give our conclusions.

II. MODEL FORMULATION AND PRELIMINARIES

In this section, we will give preliminary knowledge for our main results. Since most of the synchronization methods belong to the master-slave (drive-response) type, by one system driving another we mean that the two systems are coupled so that the behavior of the second is influenced by the behavior of the first one, and the behavior of the first is independent of the second. The first system will be called the master system or drive system, and the second one will be the slave system or response system.

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There has been increasing interest in investigating the dynamics of neural networks since Hopfield[22] constructed a simplified neural network model. To understand how neural networks compute, we will examine the electronic circuit of Hopfield neural network (HNN).[23] The elements of the hardware are a set of amplifiers with sigmoid I/O characteristics, input capacitance, output resistance, and a set of resistive connections connecting outputs of some amplifiers with inputs of others.

Now let us consider the following recurrent neural network model:

$$\dot{x}(t) = -Cx(t) + Af[x(t)] + Bk\{x[t-\tau(t)]\} + I, \tag{1}$$

or

$$\dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j[x(t)] + \sum_{j=1}^{n} b_{ij} k_j\{x[t-\tau_{ij}(t)]\} + I_i,$$
$$i = 1,2,\ldots,n, \tag{2}$$

where n denotes the number of units in a neural network, $x(t)=[x_1(t),x_2(t),\ldots,x_n(t)]^T \in R^n$ is the state vector associated with the neurons, $I=(I_1,I_2,\ldots,I_n)^T \in R^n$ is external input vector, $f[x(t)]=\{f_1[x(t)],f_2[x(t)],\ldots,f_n[x(t)]\}^T \in R^n$ and $k\{x[t-\tau(t)]\}=(k_1\{x[t-\tau(t)]\},k_2\{x[t-\tau(t)]\},\ldots,k_n\{x[t-\tau(t)]\})^T \in R^n$ corresponds to the activation functions and delayed activation functions of neurons, $\tau(t)=\tau_{ij}(t)$ $(i,j=1,2,\ldots,n)$ are the multiple time-varying delays, we suppose each $\tau_{ij}(t)$ $(i,j=1,2,\ldots,n)$ is bounded and the initial conditions of (1) are given by $x_i(t)=\phi_i(t) \in \mathcal{C}([-r,0],R)$ with $r=\max_{1\le i,j\le n, t\in R}\{\tau_{ij}(t)\}$, where $\mathcal{C}([-r,0],R)$ denotes the set of all continuous functions from $[-r,0]$ to R. $C=\mathrm{diag}(c_1,c_2,\ldots,c_n)$ is a diagonal matrix, and $A=(a_{ij})_{n\times n}$ and $B=(b_{ij})_{n\times n}$ are the connection weight matrix and the delayed connection weight matrix, respectively. Here matrices C, A, B are unknown.

Chaos dynamics has shown the interesting features that make it more attractive especially in secure communications. Among these strategies, those based on the identification of system parameters appear to be of great practical interest, especially when the system state is available to external measurements. The main reason is the possible modulation of a system parameter by the message to be transmitted. The difficulties of determining some system parameters in advance in practice make it more useful in secure communications. If the signal of the drive system is transmitted to the receiver of response system, then the encrypted ciphertext can be decrypted successfully.

The motivations for the study of the proposed paper in secure communication are given as follows:

First, many types of neural network, such as Hopfield neural network,[22] BAM neural network [a class of two-layer associative networks, called bidirectional associative memory (BAM) neural networks],[27] and cellular neural network[26] are special cases of the recurrent neural network model (1). These neural networks can be easy implemented by circuits systems, even implemented by VLSI when the networks are large-scale networks.

Second, the estimation of nonlinear function f and time-varying delays $\tau_{ij}(t)(i,j=1,2,\ldots,n)$ are still challenging problems nowadays. Also, Q-S time-varying synchronization (lag, anticipation, and completeness) $r(t)$ given in the definition below in this paper is novel and even more difficult to be estimated. Time-varying delays $\tau_{ij}(t)(i,j=1,2,\ldots,n)$, nonlinear function f, and Q-S time-varying synchronization (lag, anticipation, and completeness) $r(t)$ can be regarded as key parameters. These are known *a priori* by the receiver.

Third, the obtained results suggest that it would be easy for anyone to determine the unknown parameters from an intercepted signal using our algorithm if you know the key parameters time-varying delays $\tau_{ij}(t)(i,j=1,2,\ldots,n)$, nonlinear function f and Q-S time-varying synchronization (lag, anticipation, and completeness) $r(t)$. Here, the unknown parameters C, A, B are not key parameters. We may choose different parameters C, A, B to send different signals each time as long as the model can exhibit chaotic dynamics. It is difficult for the adversary to decrypt the plaintext without knowing key parameters time-varying delays $\tau_{ij}(t)(i,j=1,2,\ldots,n)$, nonlinear function f, and Q-S time-varying synchronization (lag, anticipation, and completeness) $r(t)$. Note that the unknown system parameters C, A, B need to be estimated. That is to say, all parameters of recurrent network (1) are unknown to the adversary. Recover the original recurrent network (1) is complex, even a small error may result in the wrong decryption.

Since the neural networks can be easy implemented by circuits systems and applied in secure communication, in this paper we consider the uncertain parameters of the neural network model as the master system. Also, chaos synchronization can be applied in other fields such as physics, chemical reactions, biological systems, automatical control, artificial neural networks, etc. Our objective is to find an adaptive way to let the slave system to Q-S time-varying synchronize with the master system, and also to identify the parameters simultaneously.

We assume that the model (1) has an equilibrium point $x^*=(x_1^*,x_2^*,\ldots,x_n^*)$ for a given I. To simplify the proofs, we will shift the equilibrium point x^* of (1) to the origin by using the following transformation:

$$y(t) = x(t) - x^*, \quad y[t-\tau(t)] = x[t-\tau(t)] - x^*.$$

The model (1) can be transformed into the following form:

$$\dot{y}(t) = -Cy(t) + Ag[y(t)] + Bl\{y[t-\tau(t)]\}, \tag{3}$$

namely,

$$\dot{y}_i(t) = -c_i y(t) + \sum_{j=1}^{n} a_{ij} g_j[y(t)] + \sum_{j=1}^{n} b_{ij} l_j\{y[t-\tau_{ij}(t)]\},$$
$$i = 1,2,\ldots,n, \tag{4}$$

where $g[y(t)]=\{g_1[y(t)],g_2[y(t)],\ldots,g_n[y(t)]\}^T \in R^n$ with $g_i[y(t)]=f_i[y(t)+x^*]-f_i(x^*)$ and $g(0)=0$, $l\{y[t-\tau(t)]\}=(l_1\{y[t-\tau(t)]\},l_2\{y[t-\tau(t)]\},\ldots,l_n\{y[t-\tau(t)]\})^T \in R^n$ with $l_i\{y[t-\tau(t)]\}=l_i\{y[t-\tau(t)]+x^*\}-l_i(x^*)$ and $l(0)=0$.

In this paper, we consider model (3) as the master system. The response system is

$$\dot{z}(t) = -\bar{C}(t)z(t) + \bar{A}(t)g[z(t)] + \bar{B}(t)l\{z[t-\tau(t)]\} + u, \tag{5}$$

namely,

$$\dot{z}_i(t) = -\bar{c}_i(t)z_i(t) + \sum_{j=1}^{n} \bar{a}_{ij}(t)g_j[z(t)] + \sum_{j=1}^{n} \bar{b}_{ij}(t)l_j\{z[t-\tau_{ij}(t)]\} + u_i, \quad i=1,2,\ldots,n, \quad (6)$$

where $\bar{C}(t)=\mathrm{diag}[\bar{c}_1(t),\bar{c}_2(t),\ldots,\bar{c}_n(t)]$, $\bar{A}(t)=[\bar{a}_{ij}(t)]_{n\times n}$ and $\bar{B}(t)=[\bar{b}_{ij}(t)]_{n\times n}$ are matrix functions depending on the time t, $u(t)=[u_1(t),u_2(t),\ldots,u_n(t)]$ is the controller. It has the same structure as the drive system but the matrix functions $\bar{C}(t)$, $\bar{A}(t)$, and $\bar{B}(t)$ are dependent on time t. In a practical situation, the output signals of the drive system (3) can be received by the response system (5), but the parameter matrices C, A, and B of the drive system (3) may not be known *a priori*, even waits for being identified.

In this paper, we will define a new type of Q-S (lag, anticipated, and complete) time-varying synchronization which is defined as the presence of certain relationship between the drive system and response system. Next, we give a definition about the main results talked in the proposed paper later.

Definition [Q-S (lag, anticipated, and complete) time-varying synchronization]: For the drive system (3) and the response system (5), let $Q[z(t)] =\{Q_1[z(t)],Q_2[z(t)],\ldots,Q_n[z(t)]\}^T\in R^n$ and $S[y(t)] =\{S_1[y(t)],S_2[y(t)],\ldots,S_n[y(t)]\}^T\in R^n$, where Q and S are continuous smooth vector functions, $Q:R^n\to R^n, S:R^n\to R^n$. Let $r(t)$ be the continuous smooth function of time t and the error states be

$$\begin{aligned} e(t) &= Q[z(t)] - S\{y[t-r(t)]\} \\ &= (Q_1[z(t)] - S_1\{y[t-r(t)]\}, Q_2[z(t)] \\ &\quad - S_2\{y[t-r(t)]\}, \ldots, Q_n[z(t)] - S_n\{y[t-r(t)]\}). \quad (7) \end{aligned}$$

It is said that the drive system (3) and the response system (5) are globally:

(i) Q-S lag time-varying synchronized if $r(t)>0, t\in R$, and $r(t)$ is called the Q-S time-varying synchronization lag;

(ii) Q-S complete time-varying synchronized if $r(t) =0, t\in R$, and $r(t)$ is called the Q-S time-varying synchronization completeness;

(iii) Q-S anticipated time-varying synchronized if $r(t) <0, t\in R$, and $r(t)$ is called the Q-S time-varying synchronization anticipation. If there exists a controller $u(y,z,t)$ such that all trajectories $\{y[t-r(t)],z(t)\}$ in (3) and (5) with any initial conditions approach the manifold $M=(\{y[t-r(t)]\},z(t)\,|\,Q_i[z(t)]=S_i\{y[t-r(t)]\},\ i=1,2,\ldots,n)$ as time t goes to infinity, that is to say

$$\lim_{t\to\infty}(Q_i[z(t)] - S_i\{y[t-r(t)]\}) \to 0 \quad (i=1,2,\ldots,n), \quad (8)$$

which implies that the error system is globally asymptotically stable.

The problem is to design an adaptive synchronization algorithm

$$u = u(y,z,\bar{A},\bar{B},\bar{C},t), \quad \dot{\bar{A}} = \bar{A}(y,z,\bar{A},\bar{B},\bar{C},t),$$

$$\dot{\bar{B}} = \bar{B}(y,z,\bar{A},\bar{B},\bar{C},t), \quad \dot{\bar{C}} = \bar{C}(y,z,\bar{A},\bar{B},\bar{C},t),$$

here $\bar{A}(t)$, $\bar{B}(t)$,and $\bar{C}(t)$ are parameter estimates of the unknown parameters matrices A, B, and C. The object of this paper is to design u, $\bar{A}(t)$, $\bar{B}(t)$, and $\bar{C}(t)$ to force the state $z(t)$ of the response system (5) to Q-S time-varying synchronize with the state $y(t)$ of the drive system (3), i.e., to archive

$$Q[z(t)] - S\{y[t-r(t)]\} \to 0, \quad t\to\infty,$$

$$\bar{C}(t) - C \to 0, \quad t\to\infty,$$

$$\bar{A}(t) - A \to 0, \quad t\to\infty,$$

$$\bar{B}(t) - B \to 0, \quad t\to\infty.$$

Let

$$e(t) = Q[z(t)] - S\{y[t-r(t)]\},$$

and subtracting (3) from (5) yields the synchronization error dynamical system as follows:

$$\begin{aligned} \dot{e}(t) &= DQ[z(t)]\dot{z}(t) - DS\{y[t-r(t)]\}\dot{y}[t-r(t)][1-\dot{r}(t)] \\ &= DQ[z(t)](-\bar{C}(t)z(t) + \bar{A}(t)g[z(t)] \\ &\quad + \bar{B}(t)l\{z[t-\tau(t)]\} + u) - [1-\dot{r}(t)]DS\{y[t-r(t)]\} \\ &\quad \times(-Cy[t-r(t)] + Ag\{y[t-r(t)]\} \\ &\quad + Bl\{y[t-\tau(t)-r(t)]\}), \quad (9) \end{aligned}$$

where $DQ[z(t)]$ and $DS\{y[t-r(t)]\}$ are the Jacobian matrices of vector functions $Q[z(t)]$ and $S\{y[t-r(t)]\}$, respectively.

In order to establish our main results, it is necessary to make the following assumption:

(A_1): the determinant of matrices $DQ[z(t)]$ is nonzero, that is to say, $DQ[z(t)]$ is inverse.

III. MAIN RESULTS

In this section, we will give our main results by choosing an effective adaptive law.

Theorem 1: Under the Assumption (A_1), the drive system (3) will Q-S time-varying synchronize with the response system (5) if we choose

$$\begin{aligned} u &= -\{DQ[z(t)]\}^{-1}Me(t) + \bar{C}(t)z(t) - \bar{A}(t)g[z(t)] \\ &\quad - \bar{B}(t)l\{z[t-\tau(t)]\} + [1-\dot{r}(t)]\{DQ[z(t)]\}^{-1} \\ &\quad \times (DS\{y[t-r(t)]\})(-\bar{C}(t)y[t-r(t)] \\ &\quad + \bar{A}(t)g\{y[t-r(t)]\} + \bar{B}(t)l\{y[t-\tau(t)-r(t)]\}), \quad (10) \end{aligned}$$

$$\dot{\bar{c}}_i(t) = \frac{1}{q_i}[1-\dot{r}(t)]\left(\sum_{k=1}^{n} p_k e_k(t)\frac{\partial S_k\{y[t-r(t)]\}}{\partial y_i[t-r(t)]}y_i[t-r(t)]\right), \quad (11)$$

$$\dot{\bar{a}}_{ij}(t) = -\frac{1}{r_{ij}}[1-\dot{r}(t)]\left(\sum_{k=1}^{n} p_k e_k(t)\frac{\partial S_k\{y[t-r(t)]\}}{\partial y_i[t-r(t)]}\times g_j\{y[t-r(t)]\}\right), \quad (12)$$

$$\dot{\bar{b}}_{ij}(t) = -\frac{1}{s_{ij}}[1-\dot{r}(t)]\left(\sum_{k=1}^{n} p_k e_k(t)\frac{\partial S_k\{y[t-r(t)]\}}{\partial y_i[t-r(t)]}\times l_j\{y[t-\tau_{kj}(t)-r(t)]\}\right), \quad (13)$$

where $p_i, q_i, r_{ij}, s_{ij} (i,j=1,2,\ldots,n)$ are positive real values, $M=\mathrm{diag}(m_1,m_2,\ldots,m_n)$ is a positive definite matrix. $\bar{C}(t)$, $\bar{A}(t)$, and $\bar{B}(t)$ are independent of C, A, and B. Also, the unknown parameters can be identified simultaneously, i.e.,

$$e(t) = Q[z(t)] - S\{y[t-r(t)]\} \to 0, \quad t\to\infty, \quad (14)$$

$$\bar{c}_i(t) - c_i \to 0, \quad t\to\infty \quad (i=1,2,\ldots,n), \quad (15)$$

$$\bar{a}_{ij}(t) - a_{ij} \to 0, \quad t\to\infty \quad (i,j=1,2,\ldots,n), \quad (16)$$

$$\bar{b}_{ij}(t) - b_{ij} \to 0, \quad t\to\infty \quad (i,j=1,2,\ldots,n). \quad (17)$$

Proof: According to (10), we can rewrite the error system (9) as

$$\begin{aligned}\dot{e}(t) &= -Me(t) + [1-\dot{r}(t)]DS\{y[t-r(t)]\}\\ &\quad\times(-[\bar{C}(t)-C]y[t-r(t)] + [\bar{A}(t)-A]g\{y[t-r(t)]\}\\ &\quad + [\bar{B}(t)-B]l\{y[t-\tau(t)-r(t)]\}),\end{aligned} \quad (18)$$

namely,

$$\begin{aligned}\dot{e}_i(t) &= -m_i e_i(t) + [1-\dot{r}(t)]\Bigg(-\sum_{j=1}^{n}\frac{\partial S_i\{y[t-r(t)]\}}{\partial y_j[t-r(t)]}\\ &\quad\times[\bar{c}_j(t)-c_j]y_j[t-r(t)] + \sum_{j=1}^{n}\sum_{k=1}^{n}\frac{\partial S_i\{y[t-r(t)]\}}{\partial y_k[t-r(t)]}\\ &\quad\times[\bar{a}_{kj}(t)-a_{kj}]g_j\{y[t-r(t)]\}\\ &\quad+\sum_{j=1}^{n}\sum_{k=1}^{n}\frac{\partial S_i\{y[t-r(t)]\}}{\partial y_k[t-r(t)]}\\ &\quad\times[\bar{b}_{kj}(t)-b_{kj}]l_j\{y[t-\tau_{ij}(t)-r(t)]\},\\ &\quad i=1,2,\ldots,n.\end{aligned} \quad (19)$$

Choose the following Lyapunov functional:

$$\begin{aligned}&V[e(t),\bar{C}(t)-C,\bar{A}(t)-A,\bar{B}(t)-B]\\ &\quad=\tfrac{1}{2}\sum_{i=1}^{n}\Bigg(p_i e_i^2(t) + q_i[\bar{c}_i(t)-c_i]^2 + \sum_{j=1}^{n} r_{ij}[\bar{a}_{ij}(t)-a_{ij}]^2\\ &\quad+\sum_{j=1}^{n} s_{ij}[\bar{b}_{ij}(t)-b_{ij}]^2\Bigg).\end{aligned} \quad (20)$$

Differentiating V with respect to time along the solution of (19), we obtain

$$\begin{aligned}\frac{dV}{dt} &= \sum_{i=1}^{n}\left(p_i e_i(t)\dot{e}_i(t) + q_i[\bar{c}_i(t)-c_i]\dot{\bar{c}}_i(t) + \sum_{j=1}^{n} r_{ij}[\bar{a}_{ij}(t)-a_{ij}]\dot{\bar{a}}_{ij}(t) + \sum_{j=1}^{n} s_{ij}[\bar{b}_{ij}(t)-b_{ij}]\dot{\bar{b}}_{ij}(t)\right)\\
&= \sum_{i=1}^{n}\Bigg[-p_i m_i e_i^2(t) + [1-\dot{r}(t)]\Bigg(-\sum_{j=1}^{n} p_i e_i(t)\frac{\partial S_i\{y[t-r(t)]\}}{\partial y_j[t-r(t)]}[\bar{c}_j(t)-c_j]y_j[t-r(t)] + \sum_{j=1}^{n}\sum_{k=1}^{n} p_i e_i(t)\frac{\partial S_i\{y[t-r(t)]\}}{\partial y_k[t-r(t)]}[\bar{a}_{kj}(t)\\
&\quad - a_{kj}]g_j\{y[t-r(t)]\}\\
&\quad + \sum_{j=1}^{n}\sum_{k=1}^{n} p_i e_i(t)\frac{\partial S_i\{y[t-r(t)]\}}{\partial y_k[t-r(t)]}[\bar{b}_{kj}(t)-b_{kj}]l_j\{y[t-\tau_{ij}(t)-r(t)]\}\Bigg) + q_i[\bar{c}_i(t)-c_i]\dot{\bar{c}}_i(t)\\
&\quad + \sum_{j=1}^{n} r_{ij}[\bar{a}_{ij}(t)-a_{ij}]\dot{\bar{a}}_{ij}(t) + \sum_{j=1}^{n} s_{ij}[\bar{b}_{ij}(t)-b_{ij}]\dot{\bar{b}}_{ij}(t)\Bigg]\\
&= \sum_{i=1}^{n}\left[-p_i m_i e_i^2(t) + \left(-\sum_{k=1}^{n}[1-\dot{r}(t)]p_k e_k(t)\frac{\partial S_k\{y[t-r(t)]\}}{\partial y_i[t-r(t)]}y_i[t-r(t)] + q_i\dot{\bar{c}}_i(t)\right)[\bar{c}_i(t)-c_i(t)]\right]\\
&\quad + \sum_{j=1}^{n}\left[\sum_{k=1}^{n}[1-\dot{r}(t)]p_k e_k(t)\frac{\partial S_k\{y[t-r(t)]\}}{\partial y_i[t-r(t)]}g_j\{y[t-r(t)]\} + r_{ij}\dot{\bar{a}}_{ij}(t)\right][\bar{a}_{ij}(t)-a_{ij}(t)]\\
&\quad + \sum_{j=1}^{n}\left[\sum_{k=1}^{n}[1-\dot{r}(t)]p_k e_k(t)\frac{\partial S_k\{y[t-r(t)]\}}{\partial y_i[t-r(t)]}l_j\{y[t-\tau_{kj}(t)-r(t)]\} + s_{ij}\dot{\bar{b}}_{ij}(t)\right][\bar{b}_{ij}(t)-b_{ij}(t)] = -\sum_{i=1}^{n}[p_i m_i e_i^2(t)].\end{aligned} \quad (21)$$

We can therefore conclude that V is a Lyapunov functional of the error system (18) corresponding to the adaptive laws given in the condition of the theorem and the unknown parameters can be identified simultaneously. This completes the proof.

Corollary 1: Under the Assumption (A_1), the drive system (3) will Q-S time-varying synchronize with the response system (5) if we choose

$$\begin{aligned} u = &-\{DQ[z(t)]\}^{-1}Me(t) + \bar{C}(t)z(t) - \bar{A}(t)g[z(t)] \\ &- \bar{B}(t)l\{z[t-\tau(t)]\} + [1-\dot{r}(t)]\{DQ[z(t)]\}^{-1} \\ &\times (DS\{y[t-r(t)]\})(-\bar{C}(t)y[t-r(t)] \\ &+ \bar{A}(t)g\{y[t-r(t)]\} + \bar{B}(t)l\{y[t-\tau(t)-r(t)]\}), \end{aligned}$$

$$\dot{\bar{c}}_i(t) = [1-\dot{r}(t)]\left(\sum_{k=1}^{n} p_k e_k(t)\frac{\partial S_k\{y[t-r(t)]\}}{\partial y_i[t-r(t)]}y_i[t-r(t)]\right),$$

$$\begin{aligned} \dot{\bar{a}}_{ij}(t) = &-[1-\dot{r}(t)]\left(\sum_{k=1}^{n} p_k e_k(t)\frac{\partial S_k\{y[t-r(t)]\}}{\partial y_i[t-r(t)]}\right. \\ &\left.\times g_j\{y[t-r(t)]\}\right), \end{aligned}$$

$$\begin{aligned} \dot{\bar{b}}_{ij}(t) = &-[1-\dot{r}(t)]\left(\sum_{k=1}^{n} p_k e_k(t)\frac{\partial S_k\{y[t-r(t)]\}}{\partial y_i[t-r(t)]}\right. \\ &\left.\times l_j\{y[t-\tau_{kj}(t)-r(t)]\}\right), \end{aligned}$$

where $p_i (i=1,2,\ldots,n)$ are positive real values, $M=\mathrm{diag}(m_1,m_2,\ldots,m_n)$ is a positive definite matrix. $\bar{C}(t)$, $\bar{A}(t)$, and $\bar{B}(t)$ are independent of C, A, and B. Also, the unknown parameters can be identified simultaneously, i.e.,

$$e(t) = Q[z(t)] - S\{y[t-r(t)]\} \to 0, \quad t\to\infty,$$

$$\bar{c}_i(t) - c_i \to 0, \quad t\to\infty \quad (i=1,2,\ldots,n),$$

$$\bar{a}_{ij}(t) - a_{ij} \to 0, \quad t\to\infty \quad (i,j=1,2,\ldots,n),$$

$$\bar{b}_{ij}(t) - b_{ij} \to 0, \quad t\to\infty \quad (i,j=1,2,\ldots,n).$$

Proof: Let $q_i=r_{ij}=s_{ij}=1$ $(i=1,2,\ldots,n)$, we can get Corollary 1 directly from Theorem 1. Here we omit it.

Corollary 2: Under the Assumption (A_1), the drive system (3) will Q-S time-varying synchronize with the response system (5) if we choose

$$\begin{aligned} u = &-\{DQ[z(t)]\}^{-1}Me(t) + \bar{C}(t)z(t) - \bar{A}(t)g[z(t)] \\ &- \bar{B}(t)l\{z[t-\tau(t)]\} + [1-\dot{r}(t)]\{DQ[z(t)]\}^{-1} \\ &\times (DS\{y[t-r(t)]\})(-\bar{C}(t)y[t-r(t)] \\ &+ \bar{A}(t)g\{y[t-r(t)]\} + \bar{B}(t)l\{y[t-\tau(t)-r(t)]\}), \end{aligned}$$

$$\dot{\bar{c}}_i(t) = [1-\dot{r}(t)]\left(\sum_{k=1}^{n} e_k(t)\frac{\partial S_k\{y[t-r(t)]\}}{\partial y_i[t-r(t)]}y_i[t-r(t)]\right),$$

$$\begin{aligned} \dot{\bar{a}}_{ij}(t) = &-[1-\dot{r}(t)]\left(\sum_{k=1}^{n} e_k(t)\frac{\partial S_k\{y[t-r(t)]\}}{\partial y_i[t-r(t)]}\right. \\ &\left.\times g_j\{y[t-r(t)]\}\right), \end{aligned}$$

$$\begin{aligned} \dot{\bar{b}}_{ij}(t) = &-[1-\dot{r}(t)]\left(\sum_{k=1}^{n} e_k(t)\frac{\partial S_k\{y[t-r(t)]\}}{\partial y_i[t-r(t)]}\right. \\ &\left.\times l_j\{y[t-\tau_{kj}(t)-r(t)]\}\right), \end{aligned}$$

where $M=\mathrm{diag}(m_1,m_2,\ldots,m_n)$ is a positive definite matrix. $\bar{C}(t)$, $\bar{A}(t)$, and $\bar{B}(t)$ are independent of C, A, and B. Also, the unknown parameters can be identified simultaneously, i.e.,

$$e(t) = Q[z(t)] - S\{y[t-r(t)]\} \to 0, \quad t\to\infty,$$

$$\bar{c}_i(t) - c_i \to 0, \quad t\to\infty \quad (i=1,2,\ldots,n),$$

$$\bar{a}_{ij}(t) - a_{ij} \to 0, \quad t\to\infty \quad (i,j=1,2,\ldots,n),$$

$$\bar{b}_{ij}(t) - b_{ij} \to 0, \quad t\to\infty \quad (i,j=1,2,\ldots,n).$$

Proof: Let $p_i=q_i=r_{ij}=s_{ij}=1 (i=1,2,\ldots,n)$, we can get Corollary 2 directly from Theorem 1. Here we omit it.

Corollary 3: Under the Assumption (A_1), the drive system (3) will Q-S time-varying synchronize with the response system (5) if we choose

$$\begin{aligned} u = &-\{DQ[z(t)]\}^{-1}Me(t) + \bar{C}(t)z(t) - \bar{A}(t)g[z(t)] \\ &- \bar{B}(t)l\{z[t-\tau(t)]\} + [1-\dot{r}(t)]\{DQ[z(t)]\}^{-1} \\ &\times (DS\{y[t-r(t)]\})(-\bar{C}(t)y[t-r(t)] \\ &+ \bar{A}(t)g\{y[t-r(t)]\} + \bar{B}(t)l\{y[t-\tau(t)-r(t)]\}), \end{aligned}$$

$$\dot{\bar{c}}_i(t) = q[1-\dot{r}(t)]\left(\sum_{k=1}^{n} e_k(t)\frac{\partial S_k\{y[t-r(t)]\}}{\partial y_i[t-r(t)]}y_i[t-r(t)]\right),$$

$$\begin{aligned} \dot{\bar{a}}_{ij}(t) = &-r[1-\dot{r}(t)]\left(\sum_{k=1}^{n} e_k(t)\frac{\partial S_k\{y[t-r(t)]\}}{\partial y_i[t-r(t)]}\right. \\ &\left.\times g_j\{y[t-r(t)]\}\right), \end{aligned}$$

$$\dot{\bar{b}}_{ij}(t) = -s[1-\dot{r}(t)]\left(\sum_{k=1}^{n} e_k(t)\frac{\partial S_k\{y[t-r(t)]\}}{\partial y_i[t-r(t)]}\right.$$
$$\left.\times l_j\{y[t-\tau_{kj}(t)-r(t)]\}\right),$$

where q, r, s are positive and real values, $M=\mathrm{diag}(m_1,m_2,\ldots,m_n)$ is a positive definite matrix. $\bar{C}(t)$, $\bar{A}(t)$, and $\bar{B}(t)$ are independent of C, A and B. Also, the unknown parameters can be identified simultaneously, i.e.,

$$e(t) = Q[z(t)] - S\{y[t-r(t)]\} \to 0, \quad t \to \infty,$$

$$\bar{c}_i(t) - c_i \to 0, \quad t \to \infty \quad (i=1,2,\ldots,n),$$

$$\bar{a}_{ij}(t) - a_{ij} \to 0, \quad t \to \infty \quad (i,j=1,2,\ldots,n),$$

$$\bar{b}_{ij}(t) - b_{ij} \to 0, \quad t \to \infty \quad (i,j=1,2,\ldots,n).$$

Proof: Let $q=p_i/q_i$, $r=p_i/r_{ij}$, $s=p_i/s_{ij}$ $(i=1,2,\ldots,n)$, we can get Corollary 3 directly from Theorem 1. Here we omit it.

In order to simplify the corollaries, the following assumption is established:

(A_2): $S\{y[t-r(t)]\}=(S_1\{y_1[t-r(t)]\},S_2\{y_2[t-r(t)]\},\ldots,S_n\{y_n[t-r(t)]\})\in R^n$, then it is easy to see that

$$\partial S_k\{y[t-r(t)]\}/\partial y_i[t-r(t)] = 0 \quad (k \neq i).$$

Next, we give a simplified corollary:

Corollary 4: Under the Assumptions (A_1) and (A_2), the drive system (3) will Q-S time-varying synchronize with the response system (5) if we choose

$$u = -\{DQ[z(t)]\}^{-1}Me(t) + \bar{C}(t)z(t) - \bar{A}(t)g[z(t)]$$
$$-\bar{B}(t)l\{z[t-\tau(t)]\} + [1-\dot{r}(t)]\{DQ[z(t)]\}^{-1}$$
$$\times (DS\{y[t-r(t)]\})(-\bar{C}(t)y[t-r(t)]$$
$$+\bar{A}(t)g\{y[t-r(t)]\} + \bar{B}(t)l\{y[t-\tau(t)-r(t)]\}),$$

$$\dot{\bar{c}}_i(t) = q[1-\dot{r}(t)]e_i(t)\frac{\partial S_i\{y[t-r(t)]\}}{\partial y_i[t-r(t)]}y_i[t-r(t)],$$

$$\dot{\bar{a}}_{ij}(t) = -r[1-\dot{r}(t)]e_i(t)\frac{\partial S_i\{y[t-r(t)]\}}{\partial y_i[t-r(t)]}g_j\{y[t-r(t)]\},$$

$$\dot{\bar{b}}_{ij}(t) = -s[1-\dot{r}(t)]e_i(t)\frac{\partial S_i\{y[t-r(t)]\}}{\partial y_i[t-r(t)]}$$
$$l_j\{y[t-\tau_{ij}(t)-r(t)]\},$$

where q,r,s are positive and real values, $M=\mathrm{diag}(m_1,m_2,\ldots,m_n)$ is positive definite matrix. $\bar{C}(t)$, $\bar{A}(t)$, and $\bar{B}(t)$ are independent of C, A, and B. Also, the unknown parameters can be identified simultaneously, i.e.,

$$e(t) = Q[z(t)] - S\{y[t-r(t)]\} \to 0, \quad t \to \infty,$$

$$\bar{c}_i(t) - c_i \to 0, \quad t \to \infty \quad (i=1,2,\ldots,n),$$

$$\bar{a}_{ij}(t) - a_{ij} \to 0, \quad t \to \infty \quad (i,j=1,2,\ldots,n),$$

$$\bar{b}_{ij}(t) - b_{ij} \to 0, \quad t \to \infty \quad (i,j=1,2,\ldots,n).$$

Proof: It is easy to obtain from Corollary 3. Here we omit it.

Remark 1: In Refs. 8, 10, and 12, the authors have studied a special type of chaotic system without time delays. However, in this paper, we have considered a general model.

Remark 2: In Refs. 3, 7, and 9, the authors have studied the adaptive synchronization of uncertain chaotic systems based on parameter identification. However, in this paper we have considered the adaptive Q-S (lag, anticipated, and complete) time-varying synchronization of uncertain neural networks with multiple time-varying delays. This delayed model is more general, and we also study adaptive Q-S (lag, anticipated, and complete) time-varying synchronization, which is a more generalized synchronization.

Remark 3: In Ref. 15, a generalized (lag, anticipated, and complete) synchronization of a class of continuous-time systems is defined by Yan. Also, a Q-S (lag or anticipated) synchronization of continuous-time dynamical systems is studied in Ref. 16 by Yan using a backstepping scheme. However, in this paper, the parameters of the drive system are unknown; the estimation of these parameters is investigated by the Lyaponov functional method with a proposed adaptive law. It is a simple approach by using the Lyapunov method. Also, the synchronization considered in this paper is a time-varying synchronization.

Remark 4: Huang and Guo have considered parameter identification by identical synchronization between two different systems in Ref. 17. A more general model with multiple time-varying delays is considered in this paper; Q-S (lag, anticipated, and complete) time-varying synchronization is also investigated. This is a more generalized form of synchronization.

Remark 5: In this paper, we define a new type of Q-S (lag, anticipated, and complete) time-varying synchronization. The parameters of the drive system are unknown, and the Q-S (lag, anticipated, and complete) time-varying synchronization and the unknown parameters are identified simultaneously.

IV. NUMERICAL EXAMPLES

In this section, we will give some simulation examples to justify the theoretical analysis in this paper.

Example: Consider a typical delayed Hopfield neural network as a drive system:

$$\dot{y}(t) = -Cy(t) + Ag[y(t)] + Bg\{y[t-\tau(t)]\},$$

where

$$C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad A = \begin{pmatrix} 2.0 & -0.1 \\ -5.0 & 2.8 \end{pmatrix}, \quad B = \begin{pmatrix} -1.6 & -0.1 \\ -0.3 & -2.5 \end{pmatrix}$$

$$\tau_{ij} = 1 \ (i,j=1,2), \quad g(y) = \begin{pmatrix} \tanh y_1 \\ \tanh y_2 \end{pmatrix}.$$

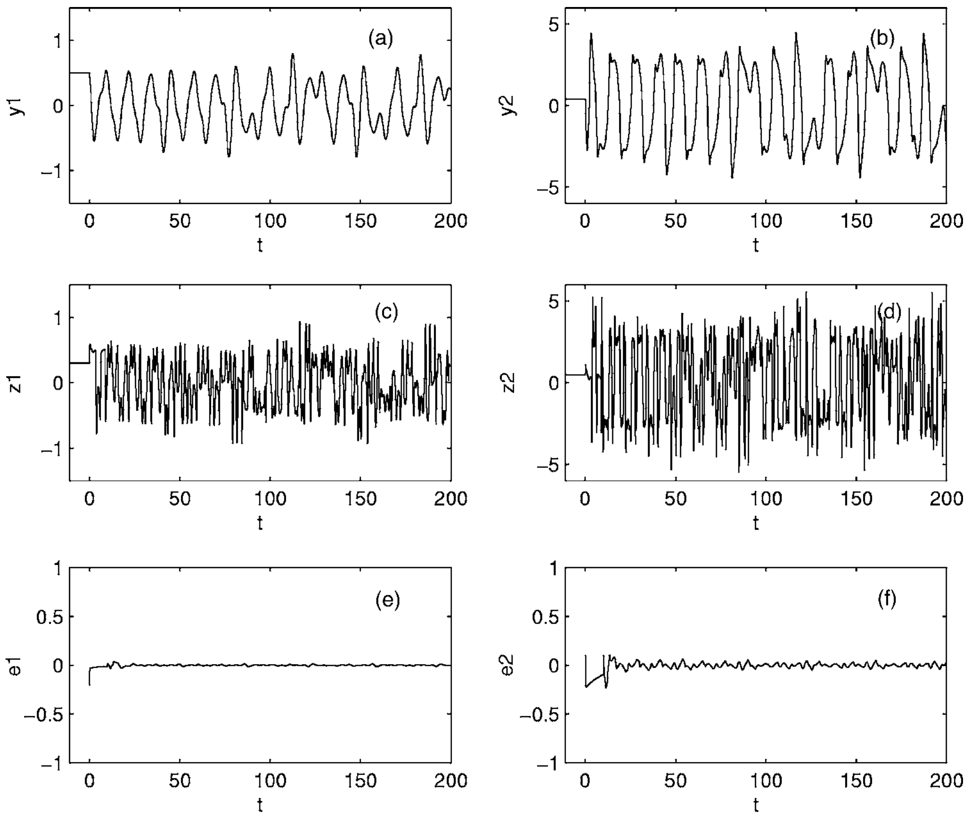

FIG. 1. State trajectories of drive, response, and error system: (a) $y_1(t)$; (b) $y_2(t)$; (c) $z_1(t)$; (d) $z_2(t)$; (e) $e_1(t)$; (f) $e_2(t)$.

Here we only consider the error state $e(t)=z(t)-y[t-r(t)]$, i.e., $Q[z(t)]=z(t), S\{y[t-r(t)]\}=y[t-r(t)]$. Thus $DQ[z(t)]=DS\{y[t-r(t)]\}=J$, where J is the identity matrix. Assumption $(A1)$ and $(A2)$ are satisfied. By Corollary 4, we obtain

$$\dot{z}(t) = -Me(t) + [1-\dot{r}(t)](-\bar{C}(t)y[t-r(t)] + \bar{A}(t)g\{y[t-r(t)]\} + \bar{B}(t)l\{y[t-\tau(t)-r(t)]\}),$$

$$\dot{\bar{c}}_i(t) = q[1-\dot{r}(t)]e_i(t)y_i[t-r(t)],$$

$$\dot{\bar{a}}_{ij}(t) = -r[1-\dot{r}(t)]e_i(t)g_j\{y[t-r(t)]\},$$

$$\dot{\bar{b}}_{ij}(t) = -s[1-\dot{r}(t)]e_i(t)l_j\{y[t-\tau_{ij}(t)-r(t)]\}.$$

Here, we choose

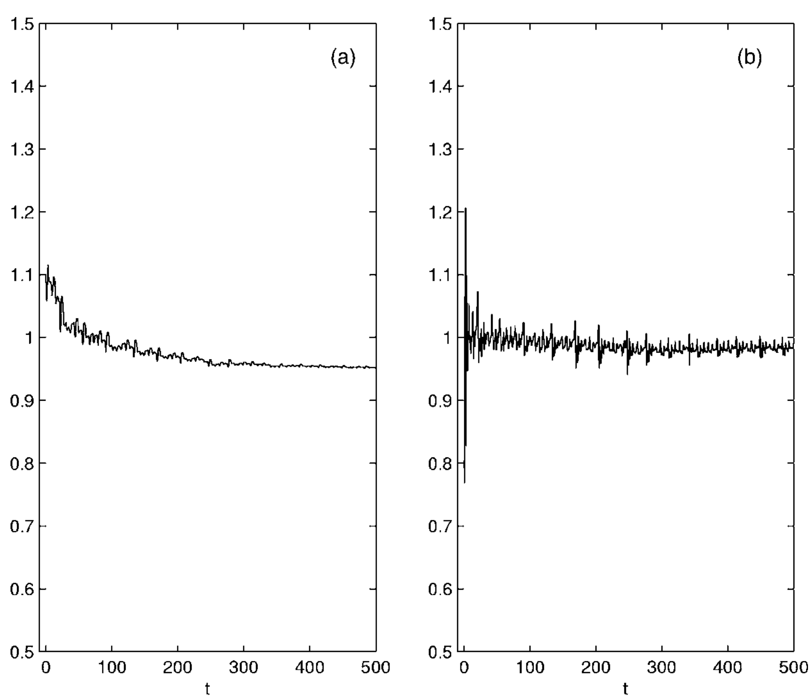

FIG. 2. Estimation parameters of C in response system: (a) $\bar{c}_1(t)$; (b) $\bar{c}_2(t)$.

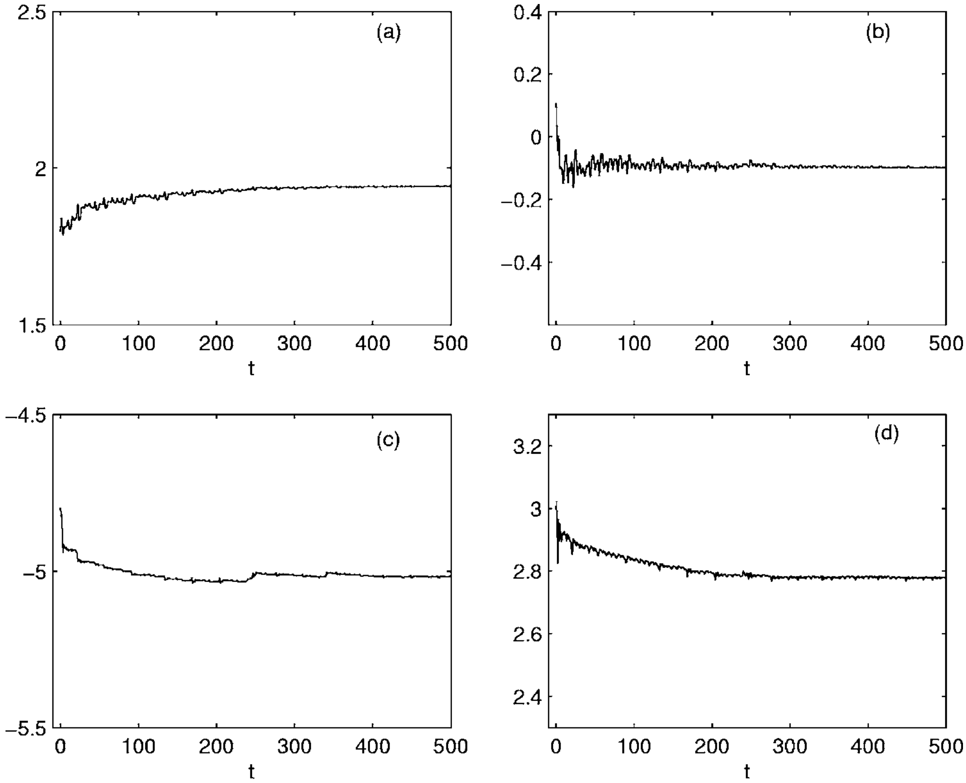

FIG. 3. Estimation parameters of A in response system: (a) $\bar{a}_{11}(t)$; (b) $\bar{a}_{12}(t)$; (c) $\bar{a}_{21}(t)$; (d) $\bar{a}_{22}(t)$.

$$M = \begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix}, \quad q = r = s = 1.$$

It is obvious to see that the above equations are independent of the unknown parameters C, A, and B. We do not need to know the parameters of the drive system; just following the above adaptive equation and the output of the drive system, we find that the drive system will synchronize with the response system according to the adaptive law stated above.

First, we choose $r(t)=5(1+\sin t)$, $\dot{r}(t)=5\cos t$, and it is the synchronization lag. The trajectories of the drive, re-

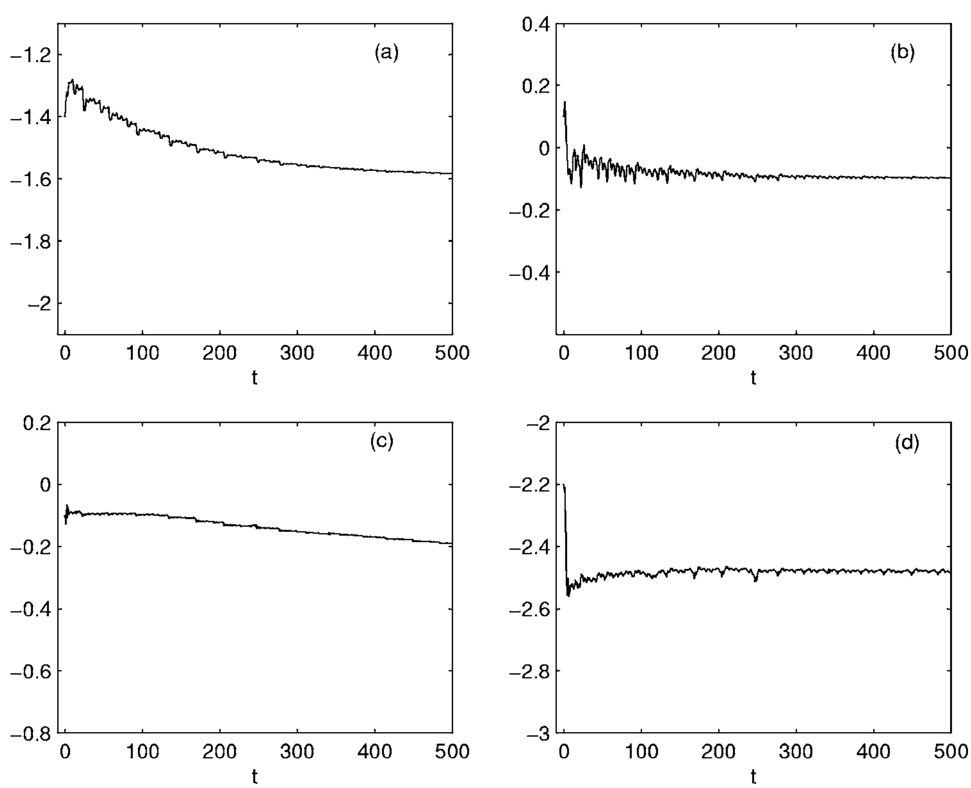

FIG. 4. Estimation parameters of B in response system: (a) $\bar{b}_{11}(t)$; (b) $\bar{b}_{12}(t)$; (c) $\bar{b}_{21}(t)$; (d) $\bar{b}_{22}(t)$.

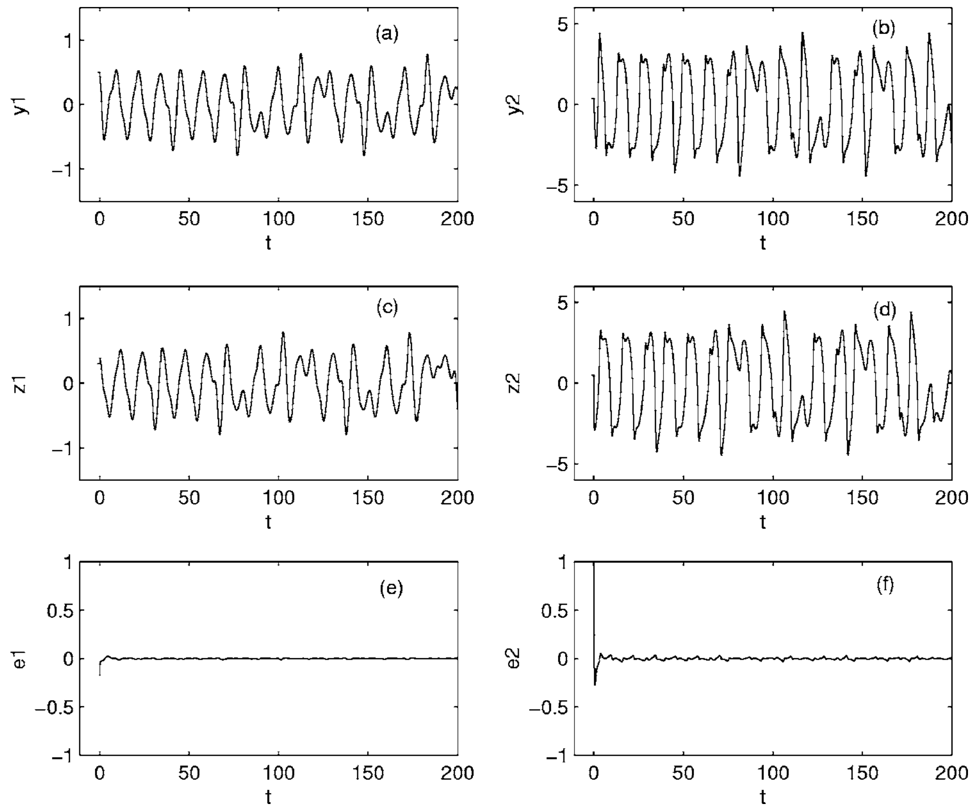

FIG. 5. State trajectories of drive, response, and error system: (a) $y_1(t)$; (b) $y_2(t)$; (c) $z_1(t)$; (d) $z_2(t)$; (e) $e_1(t)$; (f) $e_2(t)$.

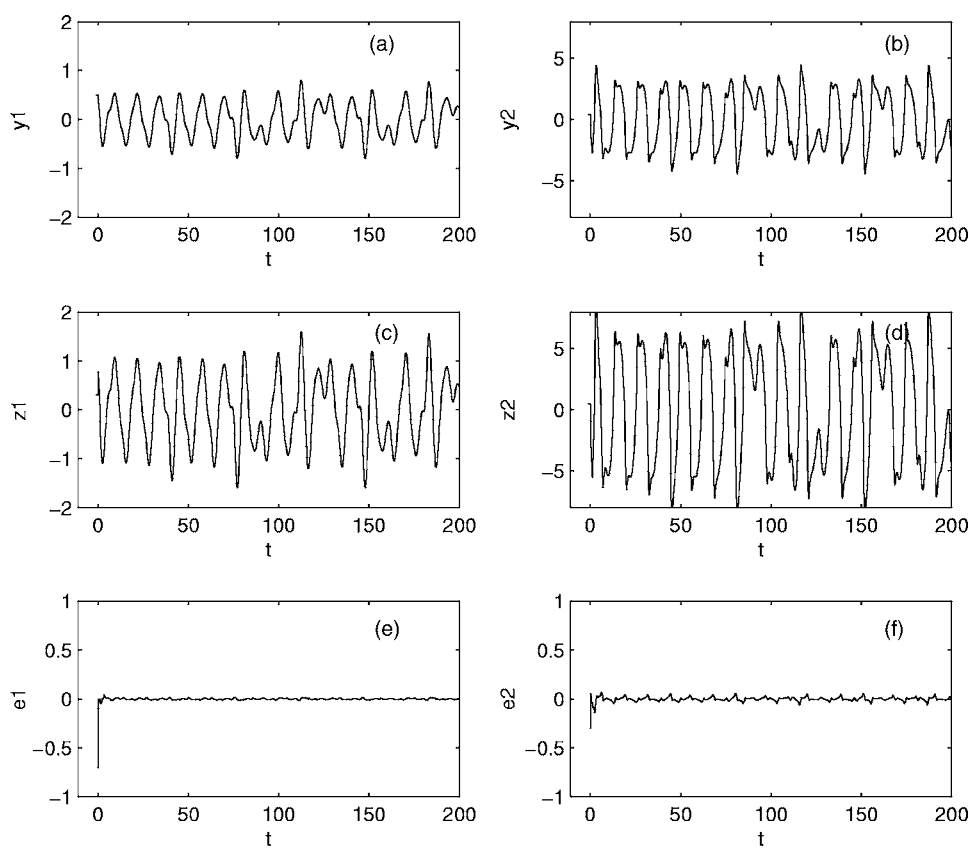

FIG. 6. State trajectories of drive, response, and error system: (a) $y_1(t)$; (b) $y_2(t)$; (c) $z_1(t)$; (d) $z_2(t)$; (e) $e_1(t)$; (f) $e_2(t)$.

sponse, and error systems are shown in Fig. 1. The parameter estimation trajectories of response system are shown in Figs. 2–4. From Figs. 1–4, we see that the drive system will Q-S lag time-varying synchronize with the response system and the parameter estimation in the response system tends to the object we want.

Next, we choose $r(t)=-10$, $\dot{r}(t)=0$, and it is the synchronization anticipation. The trajectories of the drive, response, and error systems are shown in Fig. 5. From Fig. 5, we see that the drive system will Q-S anticipated synchronize with the response system.

Finally, we choose the error state $e(t)=z(t)-2y(t)$, i.e., $Q[z(t)]=z(t), S\{y[t-r(t)]\}=2y(t)$ to show Q-S complete synchronization. Thus $DQ[z(t)]=J$, $DS[y(t)]=2J$, where J is the identity matrix. Assumptions $(A1)$ and $(A2)$ are satisfied. By Corollary 4, we obtain

$$\begin{aligned}\dot{z}(t) = &- Me(t) + [1-\dot{r}(t)](DS\{y[t-r(t)]\})\\ &\times(-\bar{C}(t)y[t-r(t)] + \bar{A}(t)g\{(y[t-r(t)]\}\\ &+\bar{B}(t)l\{y(t-\tau(t)-r(t)]\}),\end{aligned}$$

$$\dot{\bar{c}}_i(t) = q[1-\dot{r}(t)]e_i(t)\frac{\partial S_i\{y[t-r(t)]\}}{\partial y_i[t-r(t)]}y_i[t-r(t)],$$

$$\dot{\bar{a}}_{ij}(t) = -r[1-\dot{r}(t)]e_i(t)\frac{\partial S_i\{y[t-r(t)]\}}{\partial y_i[t-r(t)]}g_j\{y[t-r(t)]\},$$

$$\begin{aligned}\dot{\bar{b}}_{ij}(t) = &- s[1-\dot{r}(t)]e_i(t)\frac{\partial S_i\{y[t-r(t)]\}}{\partial y_i[t-r(t)]}\\ &\times l_j\{y[t-\tau_{ij}(t)-r(t)]\}.\end{aligned}$$

Here we choose

$$M = \begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix}, \quad q=r=s=1, r(t)=0.$$

Also it is obvious to see that the above equations are independent of the unknown parameters C, A, and B. The trajectories of the drive, response, and error systems are shown in Fig. 6. From Fig. 6, we see that the response system Q-S completely synchronize with the drive system.

V. CONCLUSION

In this paper, adaptive Q-S (lag, anticipated and complete) time-varying synchronization and parameter identification of uncertain neural networks with multiple time-varying delays have been considered. We do not need to know the unknown parameters of the system, but the slave system can Q-S time-varying synchronize with the master system and the parameter identification can be achieved by the output of the master system. It is easy and convenient to use this method by adopting an adaptive law. Numerical simulations are given to show the effectiveness and feasibility of the developed method.

Chaos synchronization can be applied especially in secure communications. Using the approach established in this paper, we can design some neural networks to achieve Q-S time-varying synchronization and apply this method in transmitting messages in the networks. These will be our present and future work.

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