Lecture 5

The Big Picture/Language Modeling

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Administrivia

- Feedback from last lecture.
  - More on the big picture?
  - Be careful what you wish for.
- Lab 1
  - Not graded yet; will be graded by next lecture.
  - Awards ceremony for evaluation next week.
- Lab 2
  - Handed out last Saturday; due nine days from now (Thursday, Oct. 15) at midnight.
  - Start early! Avail yourself of Courseworks.

Outline

- The Big Picture (review).
  - The story so far, plus situating Lab 1 and Lab 2.
- Language modeling.
  - Why?
  - How? (Grammars, *n*-gram models.)
  - Parameter estimation (smoothing).
  - Evaluation.

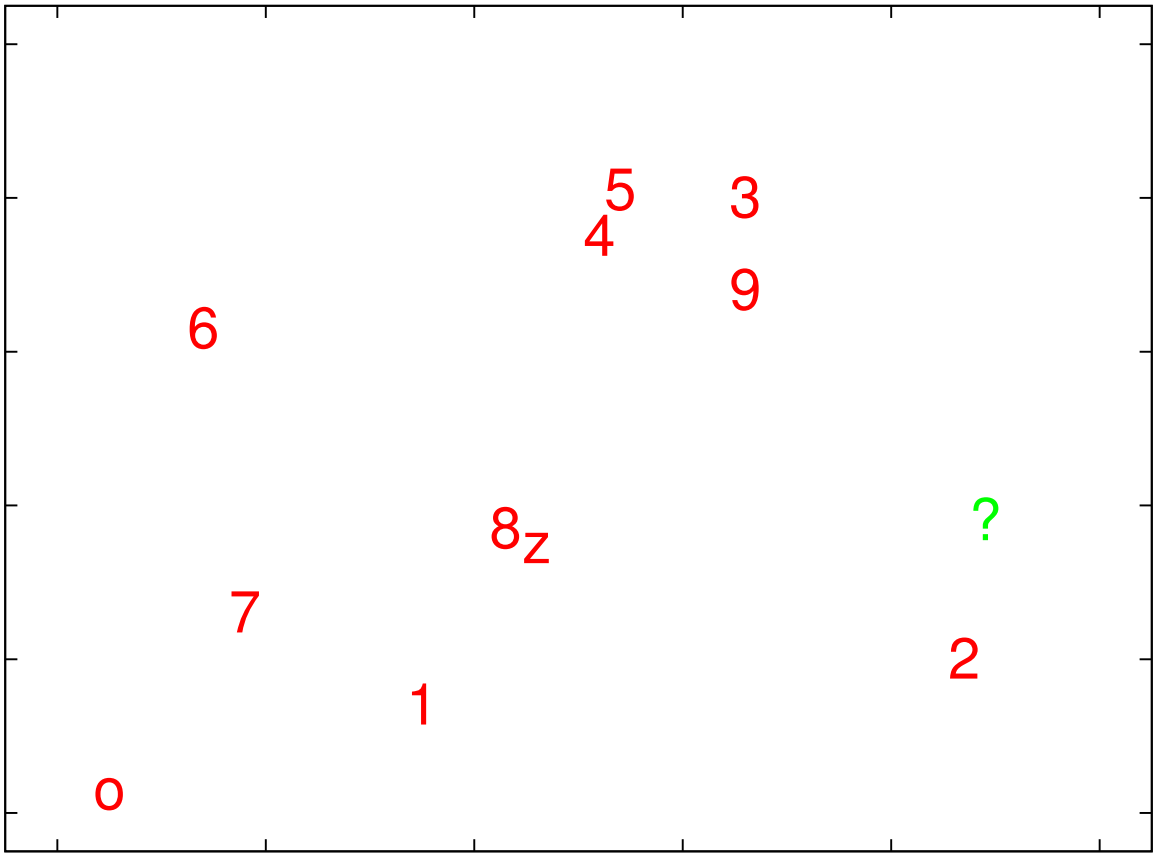
Part I

The Big Picture

Where Are We?

DTW as Nearest Neighbor Classification

- One template/exemplar per class.
- Find nearest class.

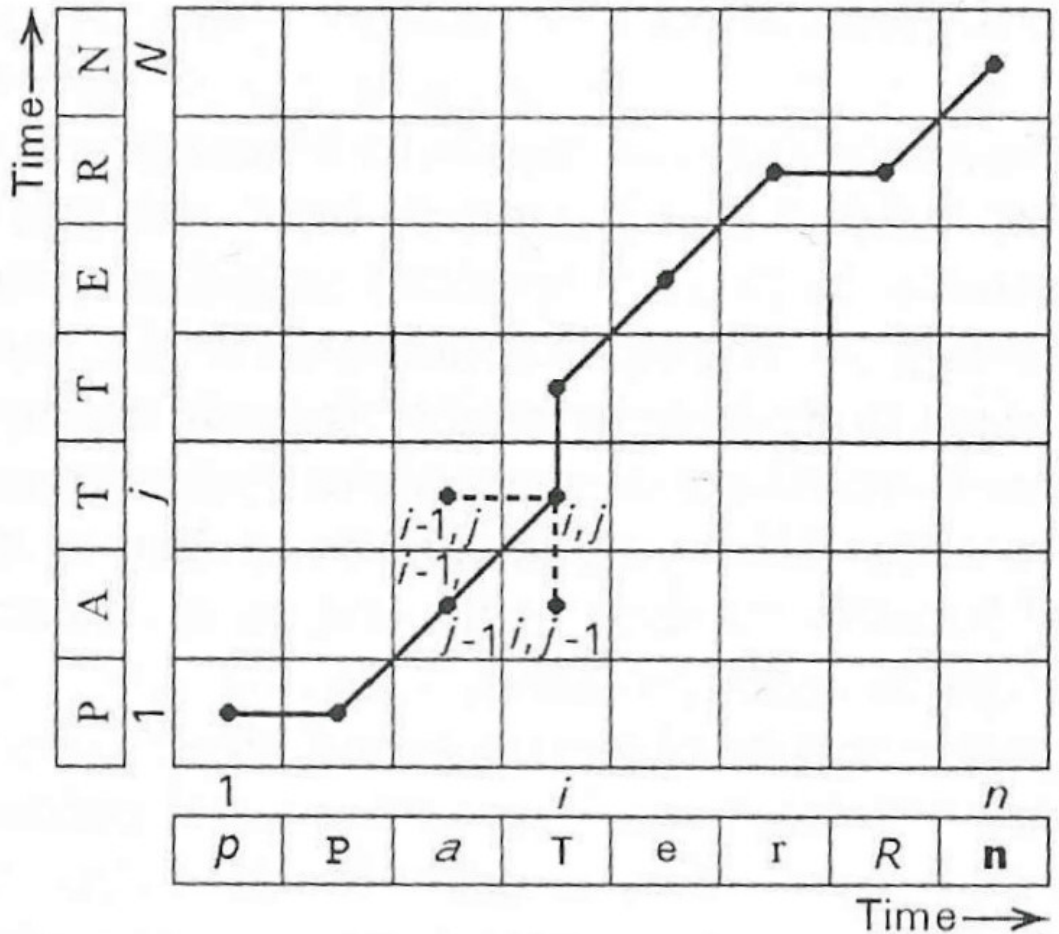


DTW as Nearest Neighbor Classification

- In reality . . .
  - Each example is an audio signal of arbitrary length/dimension.
- The key question . . .
  - What's a good distance measure/feature space?

Problem 1: Varying Utterance Lengths

- An *alignment* maps each test frame to a template frame.
- Sum distances between aligned frames.

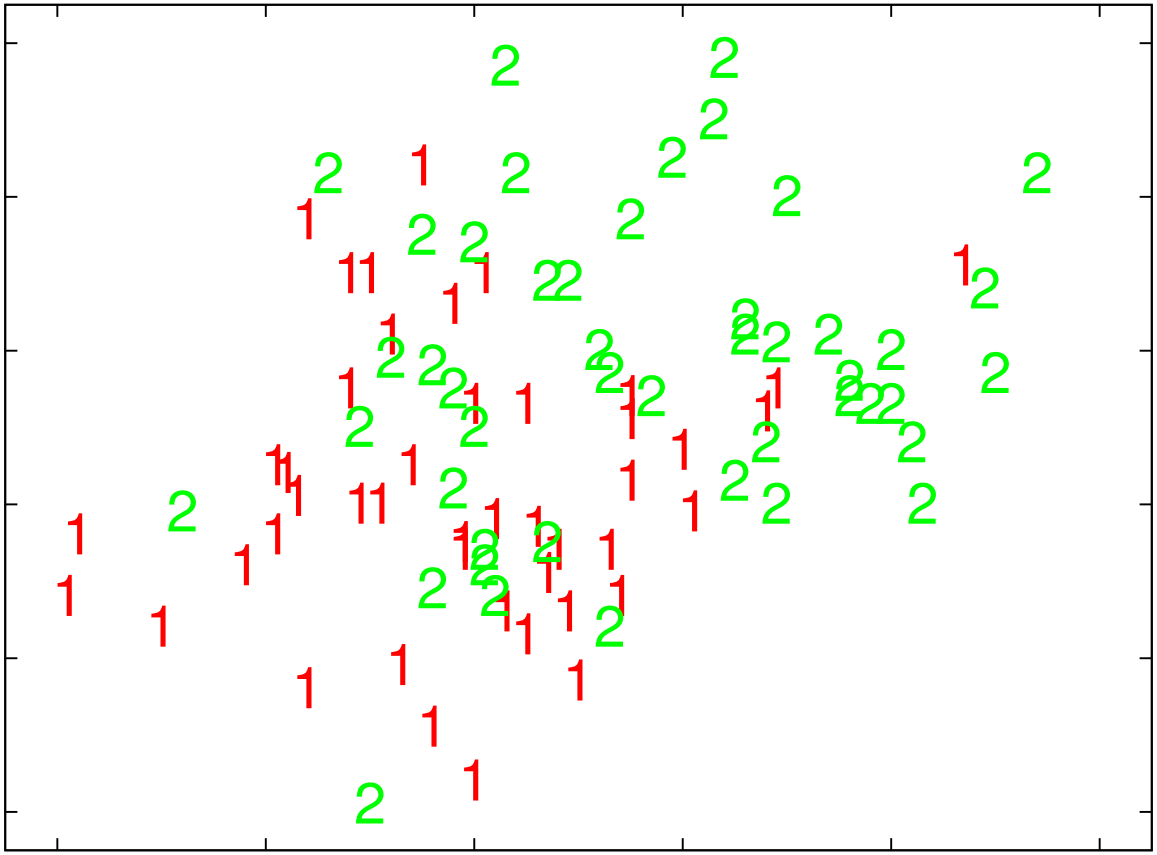


Dynamic Time Warping

- Which alignment to use?
  - Pick alignment that results in lowest overall distance.
- How to find this alignment efficiently?
  - *Dynamic programming* lets us search exponential number of alignments in polynomial time.
- Penalize “weird” alignments.
  - Add alignment cost into overall distance to penalize skips and the such.

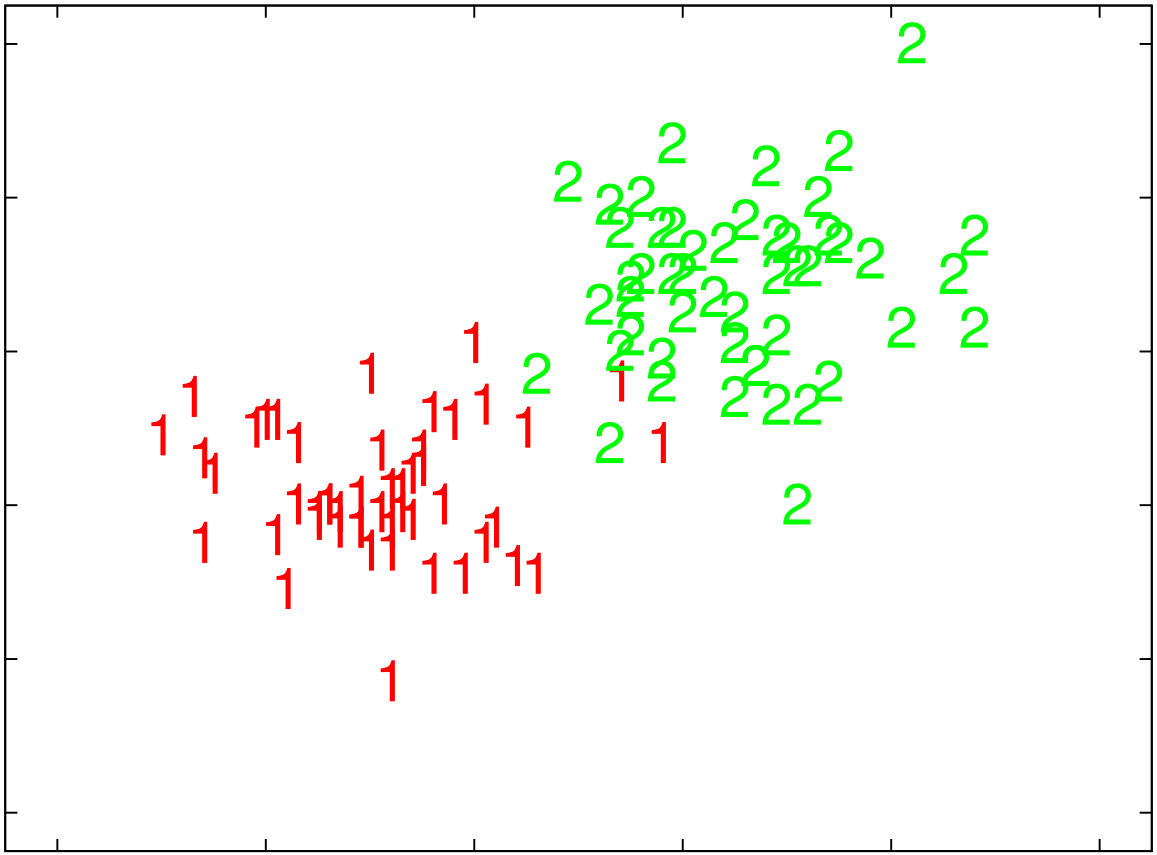
Problem 2: Computing Frame Distances

- With bad feature representation, classification is hard.



Problem 2: Computing Frame Distances

- With good feature representation, classification is easy.



What Did We Learn in Lab 1?

- Goal: maximize separability between examples from different classes.
- A smoothed frequency-domain representation works well.
  - FFT computes energy at each frequency (fine-grained).
  - Mel-binning does coarse-grained grouping (93.6% accuracy).
  - Log+DCT, Hamming window help.
  - Not completely unlike human auditory system.
- Data reduction makes later processing go faster.
  - Instead of processing 20000 values/sec, process 100×12 values/sec.

What Did We Learn in Lab 1?

- DTW is OK for speaker-dependent small-voc isolated word ASR.
  - $>$95% accuracy in clean conditions.
- Can we do better?

The Key: A Good Distance Measure

- Can we improve upon our *ad hoc* distance measure?
- There are lots of parameters that one can tune, *e.g.*,
  - dimension-dependent weights
  - class-dependent weights
  - alignment costs
  - relative weight of frame distances and alignment costs
  - etc.
- Is there a framework for choosing these parameters in a principled manner?

Where Are We?

The Idea

- Old way: choose class that minimizes

$$\text{distance}(\mathit{class}, \mathit{example})$$

- New way: choose class that maximizes

$$P(\mathit{example}|\mathit{class})$$

Probabilities as Frequencies

- What do we mean by $P(example|class)$?
  - The (relative) frequency with which the word *class* . . .
  - Is realized as the feature vector *example*.

What Does Probabilistic Modeling Give Us?

- If we can estimate $P(example|class)$ perfectly . . .
  - We can perform classification optimally!
- *e.g.*, two-class classification (w/ classes equally frequent)
  - Choose YES if $P(example|\text{YES}) > P(example|\text{NO})$.
  - Choose NO otherwise.
  - This is the best you can do!

Can We Find "Perfect" Models?

- Models have *parameters*, *e.g.*, consider a Gaussian.

$$P(x|\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

  - Parameters: μ, σ.
- Idea: *Maximum likelihood estimation* (MLE).
  - Choose parameters that maximize the likelihood of the training data $\{x_i\}$:

$$(\mu_{\text{MLE}}, \sigma_{\text{MLE}}) = \arg\max_{(\mu,\sigma)} \prod_i P(x_i|\mu,\sigma)$$

Maximum Likelihood Estimation

- Assume we have the “correct” model form.
- Then, in the presence of infinite training samples . . .
  - Maximum likelihood estimation (MLE) approaches the “true” parameter estimates.
  - For most models, MLE is asymptotically consistent, unbiased, and efficient.
- Maximum likelihood estimation is easy for a wide class of models.
  - $\Rightarrow$ Count and normalize!

Long Live Probabilistic Modeling!

- If we choose the form of our model correctly and have lots of data . . .
  - We win!
- ASR moved to probabilistic modeling in mid-70's.
  - In ASR, no serious challenges to this framework since then.
- By and large, probabilistic modeling . . .
  - Does as well or better than all other techniques . . .
  - On all classification problems.
  - (Not everyone agrees.)

What's the Catch?

- Do we know form of probability distribution that data comes from?
- Can we actually find the MLE?
  - In training HMM's w/ FB, can only find local optimum.
  - MLE solution for GMM's is pathological.
- Do we have enough training data?
  - "There's no data like more data."
- These are the challenges!

Where Are We?

A Probabilistic Model of Speech

- Isolated word recognition w/ front end processing.

$$P(\textit{feature vectors} \mid \textit{word})$$

- Notation: $P(\mathbf{x}|\omega) \Rightarrow P_{\omega}(\mathbf{x})$

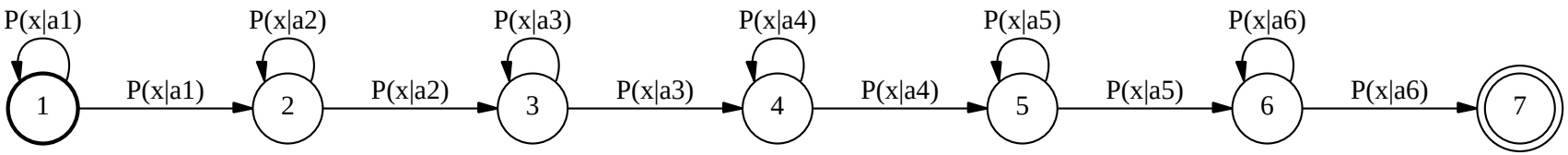
Problem 1: Varying Utterance Lengths

- An *alignment* $A = a_1 \cdots a_t$ maps each test frame t to a template frame a_t.
  - To compute probability of feature vectors $\mathbf{x}$ ...
  - Sum over all possible alignments A.

$$\begin{aligned} P_\omega(\mathbf{x}) &= \sum_A P_\omega(\mathbf{x}, A) \\ &= \sum_A P_\omega(A) \times P_\omega(\mathbf{x}|A) \end{aligned}$$

A Model of Alignments

- How to model $P_\omega(A)$?
  - HMM's!?
- Replace template for each word with an HMM.
  - Each frame in template corresponds to state in HMM.
  - Each alignment corresponds to path through HMM ...
  - Mapping each frame t to arc a_t in HMM.



- Path probability is product of transition probabilities along path.

$$P_\omega(A) = \prod_{t=1}^{T} P(a_t)$$

Problem 2: Computing Frame Distances

$$
\begin{aligned}
P_\omega(\mathbf{x}) &= \sum_A P_\omega(\mathbf{x}, A) \\
&= \sum_A P_\omega(A) \times P_\omega(\mathbf{x}|A)
\end{aligned}
$$

- How to model $P_\omega(\mathbf{x}|A)$?
- Compute overall probability by combining probability of each frame.

$$
P_\omega(\mathbf{x}|A) = \prod_{t=1}^{T} P(\vec{x}_t|a_t)
$$

Problem 2: Computing Frame Distances

$$P_\omega(\mathbf{x}|A) = \prod_{t=1}^{T} P(\vec{x}_t|a_t)$$

- How to model frame probabilities?
  - GMM!?
- Lots of stuff is Gaussian.
  - If you sum enough Gaussians . . .
  - You can approximate any distribution arbitrarily well.

Putting It All Together

$$
\begin{aligned}
P_\omega(\mathbf{x}) &= \sum_A P_\omega(\mathbf{x}, A) = \sum_A P_\omega(A) \times P_\omega(\mathbf{x}|A) \\
&\approx \max_A P_\omega(A) \times P_\omega(\mathbf{x}|A) \\
&= \max_A \prod_{t=1}^{T} P(a_t) \prod_{t=1}^{T} P(\vec{x}_t|a_t) \\
\log P_\omega(\mathbf{x}) &= \max_A \left[\sum_{t=1}^{T} \log P(a_t) + \sum_{t=1}^{T} \log P(\vec{x}_t|a_t) \right] \\
P(\vec{x}_t|a_t) &= \sum_{m=1}^{M} \lambda_{a_t,m} \prod_{\text{dim } d}^{D} \mathcal{N}(x_{t,d}; \mu_{a_t,m,d}, \sigma_{a_t,m,d})
\end{aligned}
$$

DTW and HMM's

- Can design HMM such that (see Holmes, Sec. 9.13, p. 155).

$$\text{distance}^{\text{DTW}}(\mathbf{x}_{\text{test}}, \mathbf{x}_{\omega}) \approx -\log P_{\omega}^{\text{HMM}}(\mathbf{x}_{\text{test}})$$

DTW	HMM
acoustic template	HMM
frame in template	state in HMM
DTW alignment	path through HMM
move cost	transition (log)prob
distance between frames	output (log)prob
DTW search	Forward/Viterbi algorithm

What Have We Gained?

- Automatically estimate parameters from data, scalably.
  - Maximum likelihood estimates improve with more data.
- Easy to encompass rich alignment and feature models.
  - *e.g.*, Gaussian variances $\Rightarrow$ class- and dimension-dependent distances.

Point to Ponder

- What independence assumptions have we made?

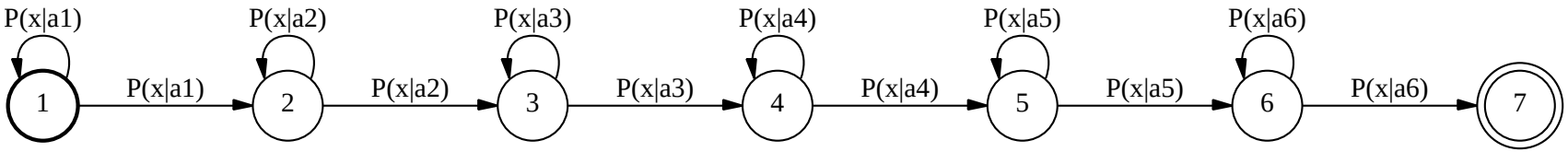
Where Are We?

Economizing on Parameters

- There is a cost to having too many parameters.
  - Too little data per parameter $\Rightarrow$ poor estimation.
  - Computational cost.
  - Local minima in EM training.
- Try to model the important stuff with as few parameters as possible.

How Big Should an HMM Be?

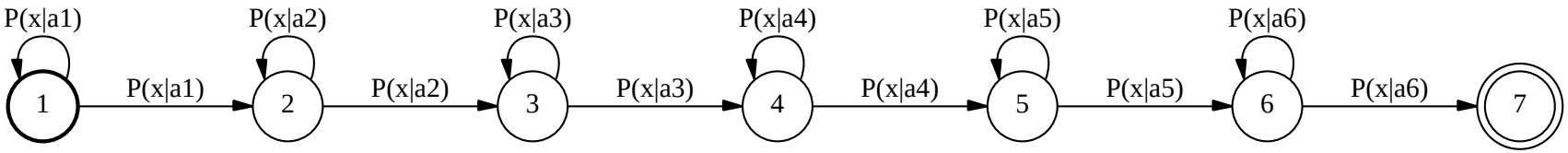
- Original view: each frame in DTW template for word ω corresponds to state in HMM.
  - But consecutive frames may be very similar to each other!



- How many states do we really need?
  - Rule of thumb: three states per *phoneme*.

Word HMM's

- Word TWO is composed of phonemes T UW.
- Two phonemes $\Rightarrow$ six HMM states.



- Convention: same output distribution for all arcs exiting a state . . .
  - Or place output distributions on states.
- Parameters: 12 transition probabilities, 6 output distributions.

How Big Should a GMM Be?

- For a Gaussian over $d \approx 40$ dimensions:
  - Diagonal covariance: $d + d$ parameters.
  - Full covariance: $d + \frac{d(d+1)}{2}$ parameters.
- How many components in each GMM?
  - Maybe 20–40, depending on how much data you have.

Where Are We?

Training for Isolated Word Recognition

- What do you need?
  - A set of audio utterances.
  - The reference transcript for each utterance.
  - An HMM topology for each word.

Training: The Basic Idea

- For each class (word) ω . . .
  - Collect all training samples **x** with that label.
  - Run FB on these samples to train parameters of corresponding HMM/GMM's.

The Way Things Actually Work

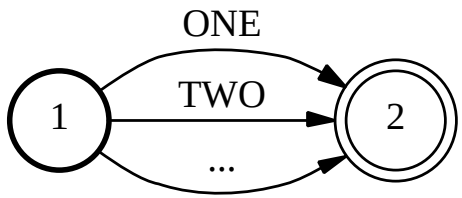
- Initialize training counts to 0.
- For each utterance $(\mathbf{x}_i, \omega_i)$ in training data . . .
  - Calculate acoustic feature vectors $\mathbf{x}_i$ using front end.
  - Construct HMM corresponding to word ω_i.
  - Run FB algorithm to update training counts for all transitions, GMM's in HMM.
- Reestimate HMM parameters using training counts.
- Repeat until training likelihood $\prod_i P_{\omega_i}(\mathbf{x}_i)$ converges.

Decoding: The Basic Idea

- For a test example $\mathbf{x}_{\text{test}}$.
  - For each word ω, compute (Viterbi or Forward) likelihood $P_\omega(\mathbf{x}_{\text{test}})$ using word HMM.
  - Pick word producing highest probability.
- Forward algorithm: $P_\omega(\mathbf{x}) = \sum_A P_\omega(\mathbf{x}, A)$
- Viterbi algorithm: $P_\omega(\mathbf{x}) \approx \max_A P_\omega(\mathbf{x}, A)$

The Way Things Actually Work

- Instead of separate HMM for each word ω . . .
  - Merge each word HMM into single big HMM.



- Use Viterbi algorithm (not Forward algorithm, why?)
  - In backtrace, collect all words along the way.
- Why one big graph?
  - Scalability to word sequences.
  - Speed: pruning during search; graph minimization.

From Isolated to Continuous ASR

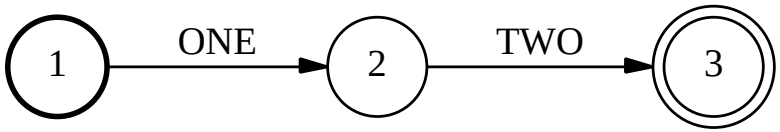
- *e.g.*, ASR for digit sequences of arbitrary length.
- The key insight:

$$P_{\omega_1\omega_2}(\mathbf{x}_1\mathbf{x}_2) = P_{\omega_1}(\mathbf{x}_1) \times P_{\omega_2}(\mathbf{x}_2)$$

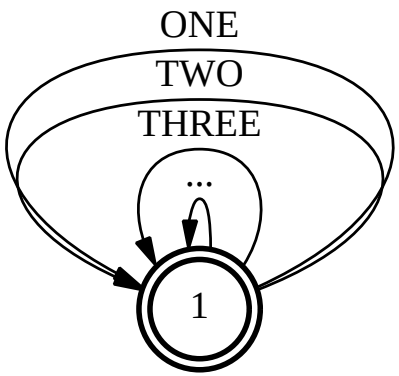
  - Can construct HMM for a word sequence . . .
  - By concatenating component word HMM's.
- Can use same exact word HMM's and HMM parameters as before!

Continuous Speech Recognition with HMM's

- Training: same as before, except . . .
  - HMM for utterance constructed by concatenating word HMM's for reference transcript.



- Decoding: same as before, except . . .
  - The single big HMM must include paths for all word sequences under consideration.

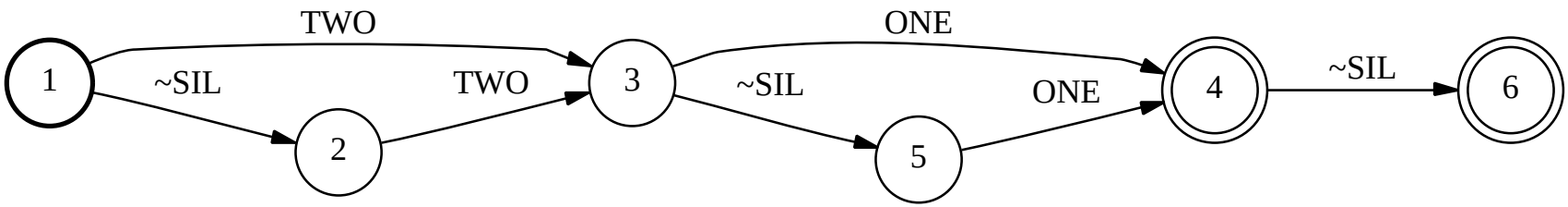


Silence is Golden

- What about all those pauses in speech?
  - At begin/end of utterance; in between words.
- Add silence word $\sim$SIL to vocabulary.
  - Three states? (Skip arcs?)
- Allow optional silence everywhere.

Silence is Golden

- *e.g.*, HMM for training with reference script: ONE TWO



- Lab 2: all these graphs are automatically constructed for you.
  - Silence is also used in isolated word training/decoding.

Part II

Language Modeling

Wreck a Nice Beach?

- The current story: *maximum likelihood classification*.
  - To decode a test sample $\mathbf{x}_{\text{test}}$ . . .
  - Find word sequence ω maximizing likelihood $P_\omega(\mathbf{x}_{\text{test}}|\omega)$.
- But:

  THIS IS OUR ROOM FOR A FOUR HOUR PERIOD .

  THIS IS HOUR ROOM FOUR A FOR OUR . PERIOD

The Fundamental Equation of ASR

$$\text{class}(\mathbf{x}_{\text{test}}) \stackrel{?}{=} \arg\max_{\omega} \; P(\mathbf{x}_{\text{test}}|\omega)$$

$$\text{class}(\mathbf{x}_{\text{test}}) \stackrel{!}{=} \arg\max_{\omega} \; P(\omega|\mathbf{x}_{\text{test}})$$

The Fundamental Equation of ASR

- Bayes' rule:

$$P(\mathbf{x}, \omega) = P(\omega)P(\mathbf{x}|\omega) = P(\mathbf{x})P(\omega|\mathbf{x})$$

$$P(\omega|\mathbf{x}) = \frac{P(\omega)P(\mathbf{x}|\omega)}{P(\mathbf{x})}$$

- The fundamental equation:

$$\begin{aligned} \text{class}(\mathbf{x}) &= \arg\max_{\omega} P(\omega|\mathbf{x}) \\ &= \arg\max_{\omega} \frac{P(\omega)P(\mathbf{x}|\omega)}{P(\mathbf{x})} \\ &= \arg\max_{\omega} P(\omega)P(\mathbf{x}|\omega) \end{aligned}$$

The Fundamental Equation of ASR

$$\text{class}(\mathbf{x}) = \arg\max_{\omega} P(\omega|\mathbf{x}) = \arg\max_{\omega} P(\omega)P(\mathbf{x}|\omega)$$

- $P(\omega)$ = *language model*.
  - Models frequency of each word sequence ω.
- $P(\mathbf{x}|\omega)$ = *acoustic model*.
  - Models frequency of acoustic feature vectors $\mathbf{x}$ given word sequence ω.

THIS IS OUR ROOM FOR A FOUR HOUR PERIOD .

THIS IS HOUR ROOM FOUR A FOR OUR . PERIOD

Source-Channel Modeling

$$\text{class}(\mathbf{x}) = \arg\max_{\omega} P(\omega|\mathbf{x}) = \arg\max_{\omega} P(\omega)P(\mathbf{x}|\omega)$$

- *Source-channel model*.
  - Source model $P(\omega)$ [language model].
  - (Noisy) channel model $P(\mathbf{x}|\omega)$ [acoustic model].
  - Recover source ω despite corruption from noisy channel.
- *Channel model* a.k.a. *direct model*.
  - Model $P(\omega|\mathbf{x})$ directly.
  - Why not?

Where Else Are Language Models Used?

- Handwriting recognition.
- Optical character recognition.
- Spelling correction.
- Machine translation.
- Natural language generation.
- Information retrieval.
- Any problem involving sequences?

Language Modeling and Domain

- Isolated digits: implicit language model.

$$P(\text{ONE}) = \frac{1}{11}, P(\text{TWO}) = \frac{1}{11}, \ldots, P(\text{ZERO}) = \frac{1}{11}, P(\text{OH}) = \frac{1}{11}$$

  - All other word sequences have probability zero.
  - Language models describe what word sequences the domain allow.
- The better you model acceptable/likely word sequences . . .
- The fewer acceptable/likely word sequences in a domain . . .
  - If not many choices, bad acoustic models look OK.
  - *e.g.*, isolated digit recognition, YES/NO recognition.

Real World Examples

- Isolated digits test set (*i.e.*, single digits).
- Language model 1.
  - Each digit sequence of length 1 equiprobable.
  - Probability zero for all other digit sequences.
- Language model 2.
  - Each digit sequence (of any length) equiprobable.
- LM 1: 1.8% error rate; LM 2: 11.5% error rate.
- Point: use all available domain knowledge in your LM!
  - *e.g.*, name dialer, phone number entry, UPS tracking number.

How to Construct a Language Model

For really simple domains:

- Enumerate all allowable word sequences.
  - *i.e.*, all word sequences ω with $P(\omega) > 0$.
  - *e.g.*, YES/NO, isolated digits.
- Use common sense to set $P(\omega)$.
  - Uniform distribution: $P(\omega) = \frac{1}{\text{size}(\{\omega\})}$.
  - Reduces to maximum likelihood classification:

$$\text{class}(\mathbf{x}) = \arg\max_{\omega} P(\omega)P(\mathbf{x}|\omega) = \arg\max_{\omega} P(\mathbf{x}|\omega)$$

How to Construct a Language Model

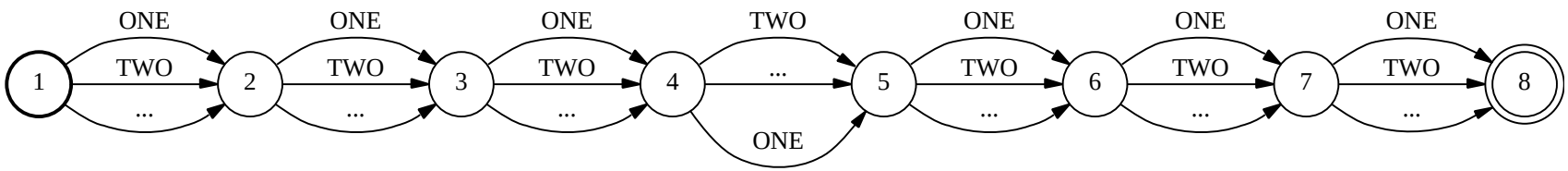
For pretty simple domains:

- *e.g.*, 7-digit phone numbers.
  - OH OH OH OH OH OH OH
  - OH OH OH OH OH OH ONE
  - OH OH OH OH OH OH TWO
  - etc.
- Is there a way we can compactly represent this list of strings?

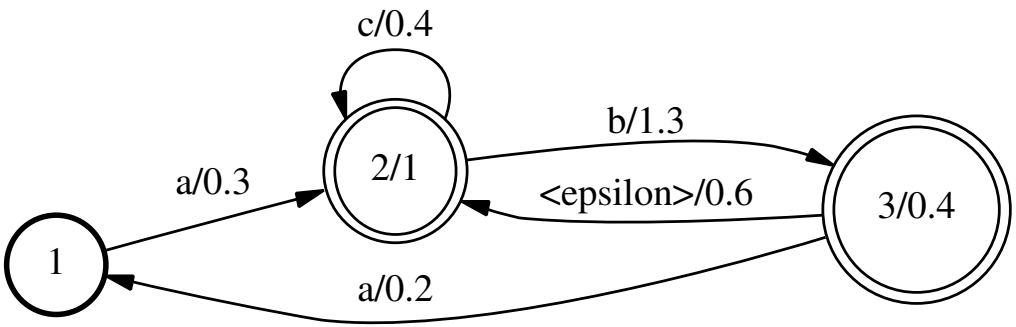
Where Are We?

Finite-State Automata

- What about (Hidden) Markov models?
- Let's use a different name.
  - Finite-state automaton/machine (FSA/FSM).
  - Or *weighted* finite-state automaton/machine (WFSA/WFSM).
  - Or *grammar*.



Weighted Finite-State Automata



- Conventions:
  - Single start state.
  - Arcs are labeled with a word and have a probability.
  - Can have multiple final states; final probability.

WFSA's as Language Models

- Probability of path (same as HMM):
  - Product of probabilities of each arc along path times ...
  - Final probability of state at end of path.

$$P(\text{path } A = a_1 \cdots a_T) = \prod_{t=1}^{T} P(a_t) \times P_{\text{final}}(\text{dest}(a_T))$$

- Probability of a word string ω (same as HMM):
  - Sum over all paths labeled with that word string.

$$P(\omega) = \sum_{A\,:\,\text{output}(A)=\omega} P(A)$$

Building a Grammar LM

- Step 1: Collect some in-domain training data.
  - Just need text, not audio.
- For some domains, easy:
  - News.
  - Data sources: the Web, Linguistic Data Consortium.
- For some domains, hard:
  - Telephone conversations; dialogue systems.
  - May need to pay people to collect/generate data.

Constructing WFSA's

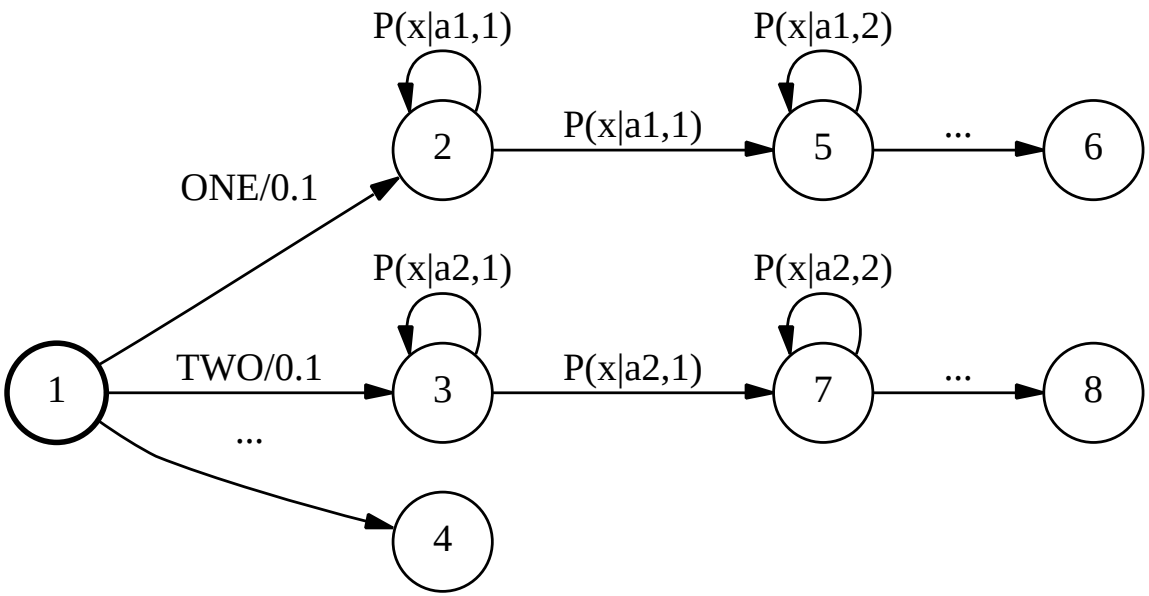
- The usual way.
  - Build WFSA topology manually . . .
  - So that "accepts" most of the training data.
  - Set probabilities manually.
- Another possible way.
  - Learn WFSA topology automatically from training data.
  - Train transition probabilities automatically. (How?)

LM's and Training and Decoding

- How do LM's impact acoustic model training?
- How do LM's impact decoding?
  - Old way: build big HMM by taking FSA representing all allowable word sequences . . .
  - And replace each word with its HMM.

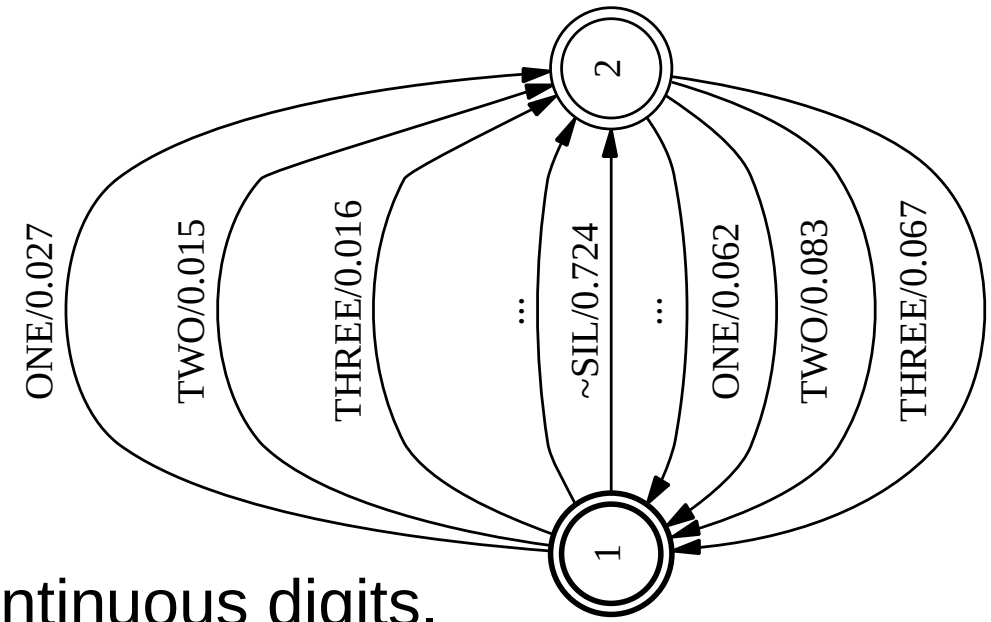
Using a Grammar LM in Decoding

- How do LM's impact decoding?
  - New way: build big HMM by taking *weighted* FSA representing all allowable word sequences . . .
  - And replace each word with its HMM.
- *e.g.*, digit grammar



Real World Example

- Decode continuous digit training data (4300 utts).
  - Why not just take reference transcript?
- Run Forward-Backward algorithm with following topology:



- Decoding continuous digits.
  - w/o LM: 13.1% WER (word error rate)
  - w/ LM: 12.0% WER
  - w/ LM, *LM weight*=10: 8.9% WER

Aside: What's Up With That LM Weight?

$$\text{class}(\mathbf{x}) = \arg\max_{\omega} P(\omega)^{\alpha} P(\mathbf{x}|\omega)$$

- Or is it the acoustic model weight?
- α often somewhere between 10 and 20
- One theory: modeling error.
  - If we could estimate $P(\omega)$ and $P(\mathbf{x}|\omega)$ perfectly ...
  - *e.g.*, at a given arc a_t, acoustic model assumes frames are independent

$$P(\vec{x}_t, \vec{x}_{t+1}|a_t = a_{t+1}) = P(\vec{x}_t|a_t)P(\vec{x}_{t+1}|a_t)$$

Aside: What's Up With That LM Weight?

- Another theory: higher variance in estimates of acoustic model probs.
  - Generally, $|\log P(\mathbf{x}|\omega)| \gg |\log P(\omega)|$
  - $\log P(\mathbf{x}|\omega)$ is computed by summing many more terms.
  - *e.g.*, continuous digits, $|\log P(\mathbf{x}|\omega)| \sim 1000$, $|\log P(\omega)| \sim 20$
  - Variance is cumulative.
  - Smaller differences in LM logprobs are more statistically significant.

Aside: What is This Word Error Rate Thing?

- Most popular evaluation measure for ASR systems

$$\text{wer} \equiv \frac{\sum_{\text{utts } u}(\#\text{ errors in } u)}{\sum_{\text{utts } u}(\#\text{ words in reference for } u)}$$

- # errors for hypothesized word sequence u_{hyp}; reference u_{ref}
  - Minimum number of word substitutions, word deletions, and word insertions needed to transform u_{ref} into u_{hyp}
- Example: WER = $\frac{3}{5}$ = 60%

u_{ref}:	THE		DOG	IS	HERE	NOW
u_{hyp}:	THE	UH	BOG	IS		NOW

  - Can WER be above 100%?

Aside: What is This Word Error Rate Thing?

- How can we compute the number of errors efficiently?
- Technical note: normalize word sequences beforehand.
  - *e.g.*, remove ∼SIL; change OH to ZERO
- *word accuracy*: percentage of reference words correct (ignores insertions).
- *sentence error rate*: percentage of utterances with at least one error.
- Not all errors are made the same.
  - *e.g.*, article *vs.* digit in voicemail.

Grammars Redux

- Is there a nicer syntax for specifying FSM's?
  - Awkward to type in FSM's.
  - *e.g.*, "arc from state 3 to state 6 with label SEVEN"
- Backus-Naur Form (BNF)

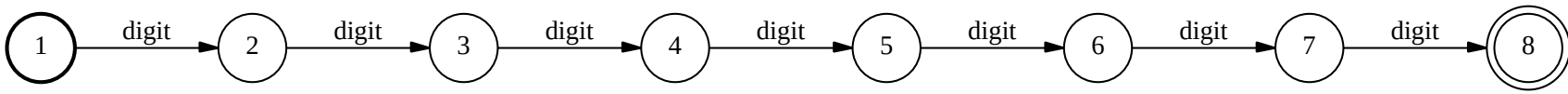
$$\begin{aligned} \text{[noun phrase]} &\rightarrow \text{[determiner] [noun]} \\ \text{[determiner]} &\rightarrow \text{A} \mid \text{THE} \\ \text{[noun]} &\rightarrow \text{CAT} \mid \text{DOG} \end{aligned}$$

- How to express 7-digit phone numbers in BNF?

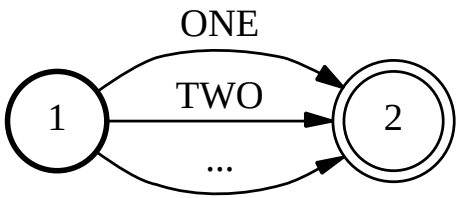
Compiling a BNF Grammar Into an FSM

- Express each individual rule as an FSM.

  phone-number



  digit



- Replace symbol with its FSM.
- Can we handle recursion/self-reference?

Aside: The Chomsky Hierarchy

- An FSA encodes a set of word sequences.
- A set of word sequences is called a *language*.
- Chomsky hierarchy.
  - Regular language: a language expressible by (finite) FSA.
  - Context-free language: a language expressible in BNF.
  - {regular languages} $\subset$ {context-free languages}
  - *e.g.*, the language $a^n b^n$ for $n = 1, 2, \ldots$

Aside: The Chomsky Hierarchy

- Is English regular? (*i.e.*, can it be expressed with an FSA?)
  - Probably not.
- Is English context-free? Are all natural languages context-free?
  - Swiss German, Bambara (Mali)
- Well, why don't we just write down a grammar for English?
  - We're too stupid.
  - Especially since people don't follow the rules.
  - Machine learning can't do it either.

When Grammars Just Won't Do

- Can't write or induce grammars for complex domains.
  - What to do?
- Can we come up with some sort of a grammar that:
  - Has a "regular" topology that we can just write down.
  - Has lots of parameters that can capture the most important dependencies in language.
  - But not too many, so we can estimate them using not too much data.

Where Are We?

N-Gram Models

$$
\begin{aligned}
& P(\omega = w_1 \cdots w_l) \\
&= P(w_1)P(w_2|w_1)P(w_3|w_1w_2)\cdots P(w_l|w_1 \cdots w_{l-1}) \\
&= \prod_{i=1}^{l} P(w_i|w_1 \cdots w_{i-1})
\end{aligned}
$$

- Markov assumption: identity of next word depends only on last $n-1$ words, say n=3

$$P(w_i|w_1 \cdots w_{i-1}) \approx P(w_i|w_{i-2}w_{i-1})$$

- Markov $\Rightarrow$ Markov Model $\Rightarrow$ grammar.

An LM Only an Engineer Could Love

- Markov assumption is clearly false.

$$P(w_i \mid \text{OF THE}) \Leftrightarrow P(w_i \mid \text{KING OF THE})$$

- Make n larger?

  FABIO, WHO WAS NEXT IN LINE, ASKED IF THE TELLER SPOKE ...

  - Too many parameters.

N-Gram Models

- Anyhoo, estimating $P(w_i|w_{i-2}w_{i-1})$ (MLE)

$$\begin{aligned} P_{\text{MLE}}(w_i|w_{i-2}w_{i-1}) &= \frac{\text{count}(w_{i-2}w_{i-1}w_i)}{\sum_{w_i}\text{count}(w_{i-2}w_{i-1}w_i)} \\ &= \frac{\text{count}(w_{i-2}w_{i-1}w_i)}{\text{count}(w_{i-2}w_{i-1})} \end{aligned}$$

  - With large training set, we see a reasonable fraction of all likely trigrams (or do we?)

N-Gram Models

- The workhorse of language modeling for ASR.
  - Around for 30 years; still primary technology for LVCSR.
  - Uses almost no linguistic knowledge.

    every time i fire a linguist the performance of the recognizer improves. Fred Jelinek (1988, IBM)
- Do we see a pattern here?

Technical Details: Sentence Begins

$$P(\omega = w_1 \cdots w_l) = \prod_{i=1}^{l} P(w_i | w_{i-2} w_{i-1})$$

- Pad with beginning-of-sentence token: $w_{-1} = w_0 = \triangleright$.

Technical Details: Sentence Ends

$$P(\omega = w_1 \cdots w_l) = \prod_{i=1}^{l} P(w_i | w_{i-2} w_{i-1})$$

- Want probabilities to normalize: $\sum_\omega P(\omega) = 1$
- Consider sum of probabilities of one-word sequences.

$$\sum_{w_1} P(\omega = w_1) = \sum_{w_1} P(w_1) = 1$$

  - Indeed, $\sum_{\omega:|\omega|=n} P(\omega) = 1$ for all $n \Rightarrow \sum_\omega P(\omega) = \infty$
- Fix: introduce an end-of-sentence token $w_{n+1} = \triangleleft$

$$P(\omega = w_1 \cdots w_l) = \prod_{i=1}^{\mathbf{l+1}} P(w_i | w_1 \cdots w_{i-1})$$

Bigram Model Example

JOHN READ MOBY DICK

MARY READ A DIFFERENT BOOK

SHE READ A BOOK BY CHER

$$P(\text{JOHN}|\triangleright) = \frac{\text{count}(\triangleright\ \text{JOHN})}{\sum_w \text{count}(\triangleright\ w)} = \frac{1}{3}$$

$$P(\text{READ}|\text{JOHN}) = \frac{\text{count}(\text{JOHN READ})}{\sum_w \text{count}(\text{JOHN}\ w)} = \frac{1}{1}$$

$$P(\text{A}|\text{READ}) = \frac{\text{count}(\text{READ A})}{\sum_w \text{count}(\text{READ}\ w)} = \frac{2}{3}$$

$$P(\text{BOOK}|\text{A}) = \frac{\text{count}(\text{A BOOK})}{\sum_w \text{count}(\text{A}\ w)} = \frac{1}{2}$$

$$P(\triangleleft|\text{BOOK}) = \frac{\text{count}(\text{BOOK}\ \triangleleft)}{\sum_w \text{count}(\text{BOOK}\ w)} = \frac{1}{2}$$

Bigram Model Example

$$
\begin{aligned}
& P(\text{JOHN READ A BOOK}) \\
&= P(\text{JOHN}|\triangleright)P(\text{READ}|\text{JOHN})P(\text{A}|\text{READ})P(\text{BOOK}|\text{A})P(\triangleleft|\text{BOOK}) \\
&= \frac{1}{3} \times 1 \times \frac{2}{3} \times \frac{1}{2} \times \frac{1}{2} \approx 0.06
\end{aligned}
$$

Data Sparsity

Will we see all the trigrams we need to see?

- (Brown *et al.*, 1992): 350M word training set
  - In test set, what percentage of trigrams unseen? $> 15\%$
- *i.e.*, In eight-word sentence, one unseen trigram on average

$$P(w_i|w_{i-2}w_{i-1}) = \frac{\text{count}(w_{i-2}w_{i-1}w_i)}{\sum_{w_i} \text{count}(w_{i-2}w_{i-1}w_i)}$$

  - One unseen trigram $\Rightarrow P(\omega = w_1 \cdots w_l) = 0$!
  - Decoders prefer word sequences with nonzero probability.

Life After Maximum Likelihood

Maybe MLE isn't such a great idea after all?

- Experiment: split 44M word data set into two halves (Church and Gale, 1992)
- For a bigram that occurs, say, 5 times in first half . . .
  - How many times does it occur in second half on average?
  - MLE predicts 5
  - In reality: $\sim$4.21 $\Rightarrow$ huh?
- What up?

Where Are We?

Bayesian Estimation

- Let's say I take a coin out of my pocket, flip it, and it's "heads".
  - What is $p = P(\text{heads})$?
  - MLE: $p_{\text{MLE}} = \arg\max_p P(\text{data } D \mid p) = \frac{1}{1} = 1$.
  - In reality, we believe $p \approx 0.5$.
- Instead of maximize probability of data given p ...
  - Maximize probability of p given data: $P(p \mid \text{data } D)$.
  - *Maximum a posteriori* (MAP) estimate.

Bayesian Estimation

$$
\begin{aligned}
p_{\text{MAP}} &= \arg\max_p P(p \mid D) \\
&= \arg\max_p \frac{P(p)P(D \mid p)}{P(D)} \\
&= \arg\max_p P(p)P(D \mid p)
\end{aligned}
$$

- $P(p)$ — *prior* probability of coin having a certain p
- $P(D \mid p)$ — *likelihood* of data given p
- $P(p \mid D)$ — *posterior* probability of a p value given the data

Bayesian Estimation

- prior distribution: $P(p = 0.5) = 0.99$
  $P(p = 0.000) = P(p = 0.001) = \cdots = P(1.000) = 0.00001$
- data D: 1 flip, 1 heads
  - $P(p = 0.5|D) \propto P(p = 0.5)P(D|p = 0.5) = 0.99 \times 0.5 = 0.495$
  - $P(p = 1.0|D) \propto P(p = 1)P(D|p = 1) = 0.00001 \times 1 = 0.00001$
  - MAP estimate: $p = 0.5$
- data D: 17 flips, 17 heads
  - $P(p = 0.5|D) \propto 0.99 \times 0.5^{17} \approx 0.000008$
  - $P(p = 1.0|D) \propto 0.00001 \times 1 = 0.00001$
  - MAP estimate: $p = 1$

Bayesian Estimation

- Little data, prior has a big effect.
- Lots of data, prior has little effect.
  - MAP estimate converges to ML estimate.

Language Model Smoothing

- How can we adjust maximum likelihood estimates . . .
  - To account for effects of prior distribution . . .
  - Especially in the presence of sparse data.
- Usually, don't actually come up with explicit prior.
  - Just use as justification for *ad hoc* methods.

Example of Language Model Smoothing

$+\delta$ smoothing (Dirichlet prior)

- add δ to each trigram count, for some δ (*e.g.*, 0.01)
- count(ATE TWO DUCKS) $= 1$, count(ATE TWO) $= 1$

$$P_{+\delta}(\text{DUCKS} \mid \text{ATE TWO}) = \frac{1 + 0.01}{1 + 0.01 \times |V|}$$

- vocabulary V — set of all words under consideration
  - say $|V| = 10000$

$$P_{+\delta}(\text{DUCKS} \mid \text{ATE TWO}) = \frac{1 + 0.01}{1 + 0.01 \times 10000} = 0.01$$

Vocabulary Selection

- Tradeoff
  - The more words, the more things you can confuse the correct words with.
  - The fewer words, the more out-of-vocabulary (OOV) words.
  - You can't get a word correct if it's not in the vocabulary.
- In practice:
  - Just choose k most frequent words in training data.
  - $k \approx 50000$ for unconstrained English speech.
  - $k < 10000$ for constrained tasks.

Schemes for Language Model Smoothing

- Basic idea: if not enough counts in trigram (bigram) model:
  - Use information from bigram (unigram) model
  - $P(w|\text{ATE TWO})$, $P(w|\text{TWO})$, $P(w)$
- Backoff (Katz, 1987)
  - Use MLE if enough counts, otherwise *backoff* to lower-order model

$$P_{\text{Katz}}(w_i|w_{i-1}) = \begin{cases} P_{\text{MLE}}(w_i|w_{i-1}) & \text{if count}(w_{i-1}w_i) \geq 5 \\ P_{\text{GT}}(w_i|w_{i-1}) & \text{if } 0 < \text{count}(w_{i-1}w_i) < \\ \alpha_{w_{i-1}} P_{\text{Katz}}(w_i) & \text{otherwise} \end{cases}$$

  - Choose $\alpha_{w_{i-1}}$ so that $\sum_{w_i} P_{\text{Katz}}(w_i|w_{i-1}) = 1$

Schemes for Language Model Smoothing

- Interpolation (Jelinek, 1980)
  - Interpolate (weighted sum) with lower-order model.

$$P_{\text{interp}}(w_i|w_{i-1}) = \lambda_{w_{i-1}} P_{\text{MLE}}(w_i|w_{i-1}) + (1-\lambda_{w_{i-1}}) P_{\text{interp}}(w_i)$$

  - $\lambda_{w_{i-1}}$ depends on count of w_{i-1}
    - *e.g.*, if count$(w_{i-1}) = 0$, $\lambda_{w_{i-1}} = 0$
    - *e.g.*, if count$(w_{i-1}) \gg 0$, $\lambda_{w_{i-1}} \sim 1$
  - Can estimate $\lambda_{w_{i-1}}$ to maximize likelihood of *held-out* data.
- Empirically, interpolation is superior to backoff.
- State-of-the-art: modified Kneser-Ney smoothing (Chen and Goodman, 1999).

Optimizing Smoothing Parameters

- Smoothing: modeling difference between training data and "true" distribution (test data).
  - *e.g.*, in training, see $P_{\text{MLE}}(\text{DUCKS} \mid \text{ATE TWO}) = \frac{1}{1} = 1$
  - In test, estimate $P_{\text{smooth}}(\text{DUCKS} \mid \text{ATE TWO}) \approx 0.2$
- Optimize smoothing parameters on *held-out* data.

$$P_{\text{smooth}}(w_i|w_{i-1}) = \lambda_{w_{i-1}} P_{\text{MLE}}(w_i|w_{i-1}) + (1 - \lambda_{w_{i-1}}) P_{\text{smooth}}(w_i)$$

  - What if we chose $\lambda_{w_{i-1}}$ to optimize likelihood of *training* data?
  - Hint: what does "MLE" mean?

Deleted Estimation

- *Held-out estimation*
  - Reserve part of data as held-out.
  - Can we not “waste” data?
- *Deleted estimation*
  - Reserve part of data as held-out.
  - Rotate!
  - Collect counts from all different partitionings to estimate parameters.

Good-Turing Discounting

- Estimate how often word with k counts in training data . . .
  - Occurs in test set of equal size.
- Deleted estimation.
  - Delete one word at a time.
  - If "test" word occurs $k + 1$ times in whole data set . . .
    - Occurs k times in "training" set.
    - Give count to bucket for words with k counts.
  - Total count placed in bucket for k-count words

$$(\#\text{ words w/ } k + 1 \text{ counts}) \times (k + 1)$$

Good-Turing Discounting

$$(\text{avg. count, } k\text{-count word}) = \frac{(\#\text{ words w/ } k+1 \text{ counts}) \times (k+1)}{(\#\text{ words w/ } k \text{ counts})}$$

- How accurate is this?

k	GT estimate	actual
1	0.45	0.45
2	1.26	1.25
3	2.24	2.24
4	3.24	3.23
5	4.22	4.21

  - Less accurate for higher k; why?
- Corollary: ("true" count of unseen events) = (# 1-counts)
  - 53M trigrams w/ 1-count in 365MW of data $\Rightarrow$
    - $P(\text{test trigram is unseen}) \approx \frac{53}{365} \approx 14.5\%$

Whither Smoothing?

- Does smoothing matter?
  - No smoothing (MLE estimate): performance will be very poor (zero probabilities will kill you).
  - Difference between OK (bucketed linear interpolation) and the best (modified Kneser-Ney).
    - $\sim$1% absolute in WER for unconstrained speech.
  - No downside to better smoothing (except implementation effort).

Whither Smoothing?

- Big or small models? *e.g.*, 3-gram or 5-gram?
  - With smaller models, less sparse data issues — better probability estimates?
  - Empirically: bigger is better.
    - With best smoothing, little or no performance degradation if make model too large.
    - With lots of data (100MW+), significant gain from 3-gram to 5-gram.
    - Limiting resource: disk/memory.
  - *Count cutoffs* or *entropy-based pruning* can be used to reduce size of LM.

Where Are We?

Evaluating Language Models

- Best way: plug into ASR system, see how affects WER.
  - Expensive to compute.
- Is there something cheaper that predicts WER well?
  - *Perplexity* (PP) of test data (only needs text).
  - Doesn't really predict WER super well, but has theoretical significance.

Perplexity

- Compute (geometric) average probability p_{avg} ...
  - Assigned to each word in test data $w_1 \cdots w_l$ by model $P(\cdot)$.
  - $p_{\text{avg}} = \left[\prod_{i=1}^{l} P(w_i | w_1 \cdots w_{i-1})\right]^{\frac{1}{l}}$
- Invert it: $\text{pp} = \frac{1}{p_{\text{avg}}}$
  - Can be interpreted as average branching factor.
- Theoretical significance:
  - $\log_2 \text{pp}$ = average number of bits per word needed to encode test data using $P(\cdot)$.
  - Related to sampled estimate of Kullback-Leibler (KL) distance between $P(\cdot)$ and ideal language model.

Perplexity

- Estimate of human performance (Shannon, 1951)
  - Shannon game — humans guess next letter in text.
  - PP=142 (1.3 bits/letter), uncased, unpunctuated, open vocab.
- Estimate of trigram language model (Brown *et al.*1992).
  - PP=790 (1.75 bits/letter), cased, punctuated, open vocab.
- ASR systems (uncased, unpunctuated, closed vocab).
  - $\sim$100 for complex domains (Switchboard, Broadcast News).
  - Can be much lower for constrained domains.
- PP depends a lot on vocabulary size.
  - Map all OOV words to single token, the *unknown word*.
  - Larger the vocabulary, larger PP will be.

Using *N*-Gram Models in ASR

- How to build the HMM used in decoding?
- Actually, an *n*-gram model can be expressed as an FSA.
  - Just pretend it's a big grammar; do as before.
  - Need lots of memory, though.
- Whole subfield of ASR devoted to this task: *search*.
- To be continued . . .

The Road Ahead

- The road behind: small vocabulary ASR.
- Lecture 6-8: large vocabulary ASR.

Course Feedback

1. Was this lecture mostly clear or unclear? What was the muddiest topic?
2. Comments on difficulty of Lab 1?
3. Other feedback (pace, content, atmosphere)?