EE E6887 Statistical Pattern Recognition

Homework #5

Due Date: Nov. 2nd 2005 Wed.

Please complete both problems.

P.1 (Vector Derivative)

In a general quadratic discriminant function, we have

$$g(\mathbf{x}) = w_0 + \sum_{i=1}^{d} w_i x_i + \sum_{i=1}^{d} \sum_{j=1}^{d} w_{ij} x_i x_j$$

Use the vector derivative rules we discussed in the class to find $\nabla_{\mathbf{x}}(g)$**,** namely gradient of g with respect to **x,** where $\mathbf{x} = \begin{bmatrix} x_1 & x_2 & \cdots & x_d \end{bmatrix}^t$ **.**

For notation simplicity, in your solutions use
$\mathbf{w} = \begin{bmatrix} w_1 & w_2 & \cdots & w_d \end{bmatrix}^t$ and $W = \begin{bmatrix} w_{ij} \end{bmatrix}$ (a $d \times d$ matrix)

P.2 (linearly separable)

The *convex hull* of a set of vectors $\mathbf{x}_i,\ i = 1, \cdots n$ is the set of all vectors of the form

$$\mathbf{x} = \sum_{i=1}^{n} \alpha_i \mathbf{x}_i$$

where the coefficients α_i are nonnegative and sum to 1. One way to check whether two sets of vectors are linearly separable is to see if their convex hulls overlap.

Use the above method to check whether the following data sets are linearly separable.

$\omega_1 : (1,2)^t \quad (2,-4)^t \quad (-3,-1)^t$
$\omega_2 : (2,4)^t \quad (-1,-5)^t \quad (5,0)^t$

P.3 (convergence of the gradient decent procedure)

Problem 15(a) in Chapter 5 of the textbook. (Part a only)

(Note in order to solve this problem, you will need to read the proof of Theorem 5.1, which we did not cover in the class.)