EE E6887 Statistical Pattern Recognition

Solutions to Prob. 47 and 48 of Chap 3

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Note the unknown parameter $\theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$

In the E step, we need to derive the auxiliary function $Q(\theta;\theta^0)$.

$$Q(\theta;\theta^0) = E_{x_{32}}[\ln p(\mathbf{x}_g, \mathbf{x}_b; \theta) \,|\, \theta^0, D_g]$$

$$= \int_{-\infty}^{\infty} (\ln p(\mathbf{x}_1 \,|\, \theta) + \ln p(\mathbf{x}_2 \,|\, \theta) + \ln p(\mathbf{x}_3 \,|\, \theta))\, p(x_{32} \,|\, \theta^0, x_{31} = 2) dx_{32}$$

$$= \ln p(\mathbf{x}_1 \,|\, \theta) + \ln p(\mathbf{x}_2 \,|\, \theta) + \int_{-\infty}^{\infty} \ln p(\begin{bmatrix} 2 \\ x_{32} \end{bmatrix} | \theta) \frac{p(\begin{bmatrix} 2 \\ x_{32} \end{bmatrix} | \theta^0)}{\int p(\begin{bmatrix} 2 \\ x'_{32} \end{bmatrix} | \theta^0) dx'_{32}} dx_{32}$$

Now set $\theta^0 = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$. Plug this initial estimation to the above function to complete the E step.

Note by normalizing the pdf of x_1, we will be able to determine $\theta_1 = 1$

By analyzing the shape of Q and maximizing the Q function, we get $\theta_2 = 3$.

Therefore $\theta = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$.

Details of the Q functions and the maximization process is not included here.

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Similar to the above, except

$$Q(\theta;\theta^0) = E_{x_{32}}[\ln p(\mathbf{x}_g, \mathbf{x}_b; \theta) \,|\, \theta^0, D_g]$$

$$= \int_{-\infty}^{\infty} (\ln p(\mathbf{x}_1 \,|\, \theta) + \ln p(\mathbf{x}_2 \,|\, \theta) + \ln p(\mathbf{x}_3 \,|\, \theta))\, p(x_{31} \,|\, \theta^0, x_{32} = 2) dx_{31}$$

$$= \ln p(\mathbf{x}_1 \,|\, \theta) + \ln p(\mathbf{x}_2 \,|\, \theta) + \int_{-\infty}^{\infty} \ln p(\begin{bmatrix} x_{31} \\ 2 \end{bmatrix} | \theta) \frac{p(\begin{bmatrix} x_{31} \\ 2 \end{bmatrix} | \theta^0)}{\int p(\begin{bmatrix} x'_{31} \\ 2 \end{bmatrix} | \theta^0) dx'_{31}} dx_{31}$$

The final solution is $\theta = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.