EE E6887 Statistical Pattern Recognition

Homework #1

Due Date: Sept. 21st 2005 Wed. 1pm

Complete any two of the following three questions.

P.1a

Suppose two equally probable 1-dimensional densities are of the form

$$p(x \mid \omega_i) \propto \exp(-\mid x - a_i \mid / b_i) \text{ for } i = 1, 2 \text{ and } b_i > 0$$

(a) Write an analytic expression for each density function. Namely, you have to normalize a_i and b_i for each function.

(b) Write the likelihood ratio $p(x \mid \omega_1) / p(x \mid \omega_2)$.

(c) Sketch a graph of the likelihood ratio for the case $a_1 = 1,\ b_1 = 2,\ a_2 = 0,\ b_2 = 1.$

P.1b

Under the Bayesian decision rule, the classification error is given by

$$P(error) = \int P(error \mid x)\, p(x) dx = \int \min[P(\omega_1 \mid x),\ P(\omega_2 \mid x)]\, p(x) dx$$

Show that for arbitrary density functions, an upper bound of the classification error can be found by replacing $\min[P(\omega_1 \mid x),\ P(\omega_2 \mid x)]$ with $2P(\omega_1 \mid x)P(\omega_2 \mid x)$. And a lower bound can be found by replacing $\min[P(\omega_1 \mid x),\ P(\omega_2 \mid x)]$ with $P(\omega_1 \mid x)P(\omega_2 \mid x)$.

P.3 (Matlab Exercise)

(a) Write a function to calculate the discriminant function of the following form

$$g_i(x) = -\frac{1}{2}(x - \mu_i)^t \Sigma_i^{-1}(x - \mu_i) - \frac{d}{2}\ln 2\pi - \frac{1}{2}\ln\left|\Sigma_i\right| + \ln P(\omega_i)$$

for a given mean vector and a covariance matrix.

(b) Write a function to calculate the Mahalanobis distance between an arbitrary point x and the mean, u **,** of a Gaussian distribution with covariance matrix Σ.

(c) Use the data set shown in the table at the beginning of the computer exercises of Chapter 2 (page 79 or 80). Assume each category has Gaussian distribution. Compute the mean and covariance matrix for each category. Assume the prior probabilities are $P(\omega_1) = 0.8,\ P(\omega_2) = P(\omega_3) = 0.1$. Then use your procedures developed in the previous parts (a) and (b) and the minimal error classifier to classify the following test data points: (1,2,1), (5,3,1), (0,0,0), and (1,0,0).