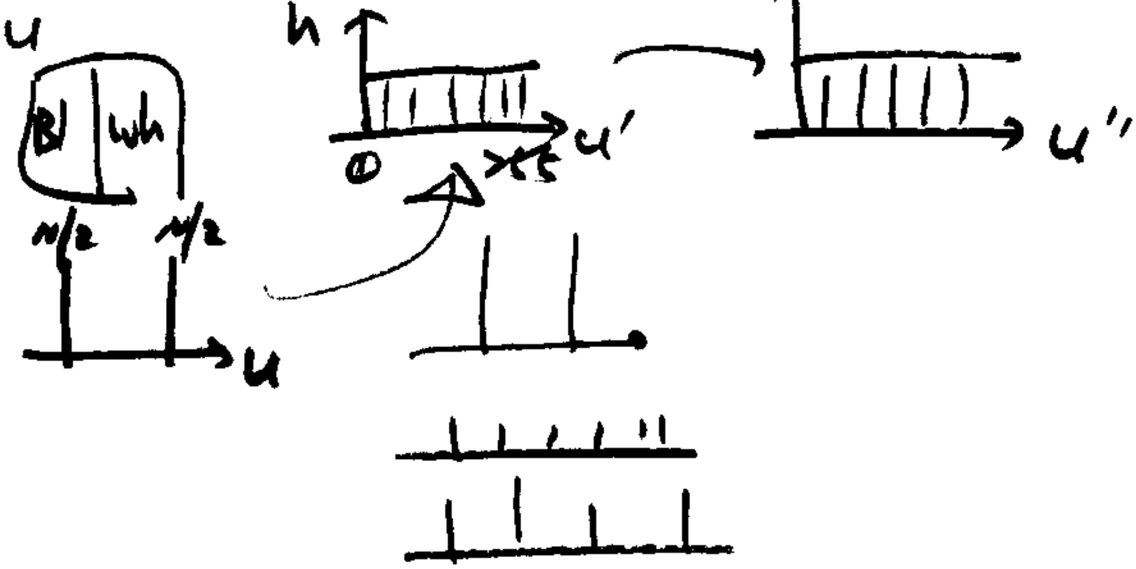
EE 4830 March 24th '05 Lecture 8

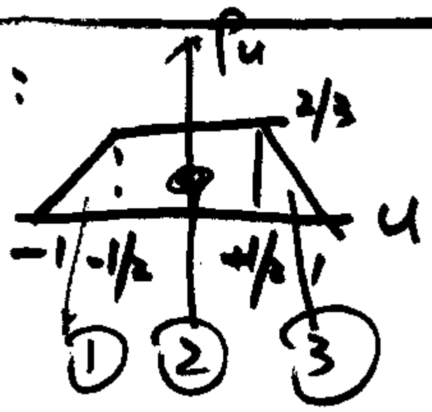
Midterm Solutions

P1: $u \xrightarrow{} \boxed{\text{hist EQ}} \xrightarrow{} u' \xrightarrow{} \boxed{\text{hist EQ}} \xrightarrow{} u''$

$u' = u''$?

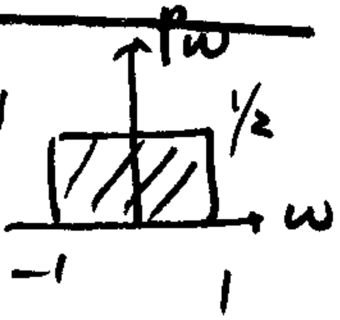


P2:



$u \longrightarrow \boxed{T} \longrightarrow w$

$F_u = F_w$



① $P_u = \begin{cases} \frac{4}{3}(u+1) & -1 \le u < \frac{-1}{2} \\ 2/3 & -1/2 \le u < 1/2 \\ \frac{-4}{3}(u-1) & 1/2 \le u \le 1 \end{cases}$

CDF $F_u = \begin{cases} \frac{2}{3}u^2 + \frac{4}{3}u + 2/3 & ① \\ 2/3\, u + 1/2 & ② \\ -2/3\, u^2 + 4/3\, u + \frac{1}{3} & ③ \end{cases}$

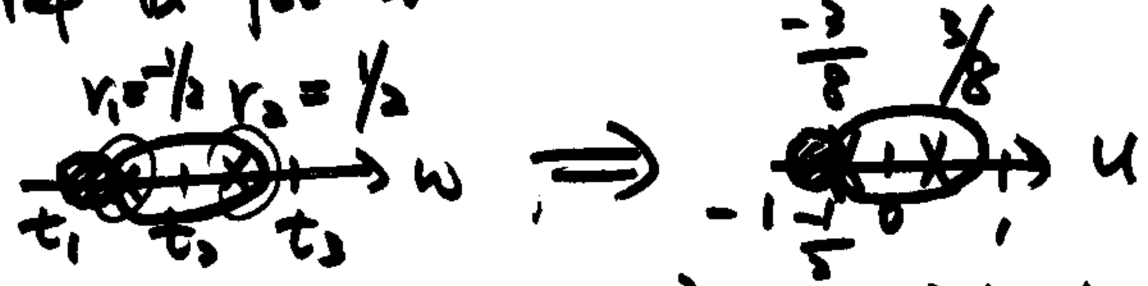
$$F_w(w) = \frac{w+1}{2}$$

$$F_u = F_w$$

$$w = \begin{cases} \frac{4}{3}u^2 + \frac{8}{3}u + \frac{1}{3} & ① \checkmark\ u \in [-1, \frac{-1}{5}) \\ \frac{4}{3}u & ② \ (W) \\ -\frac{4}{3}u^2 + \frac{8}{3}u - \frac{1}{3} & ③ \checkmark \end{cases}$$

2-Step Q for w

$r_1 = -1/2 \quad r_2 = 1/2$



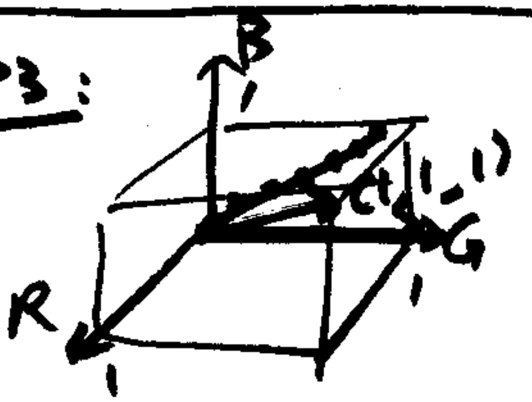
$w = 4/3 \cdot u \qquad u = \frac{3}{4} w = \frac{3}{4}(\pm 1/2)$

$= \pm \frac{3}{8}$

$$\frac{4}{3} \cdot \left(\frac{-1}{2}\right)^2 + \frac{8}{3}\left(\frac{-1}{2}\right) + 1/3$$

$$= \frac{1}{3} + \frac{-4}{3} + \frac{1}{3} = -2/3$$

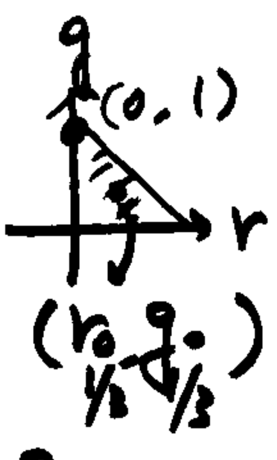
P3:



$r = R/(R+G+B)$

$g = G/(R+G+B)$

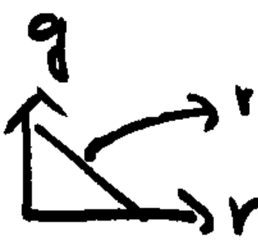
$b = B/(R+G+B)$



(a) $r = r_0 = R/(R+G+B)$

$\Rightarrow (1-r_0)R - r_0 \cdot G - r_0 \cdot B = 0$

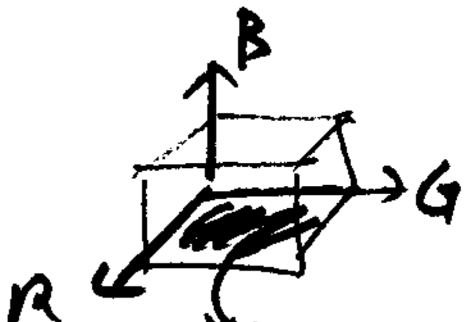
$g_0 = G/R+G+B \Rightarrow -g_0 R + (1-g_0)G - g_0 \cdot B = 0$

(b) 

$r+g=1$

$$\frac{R}{R+G+B}+\frac{G}{R+G+B}=1$$

$\Longrightarrow B=0$



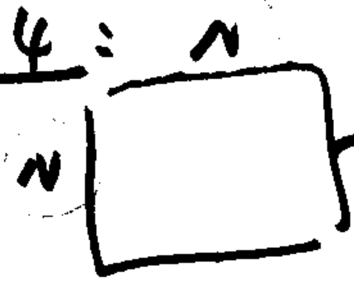
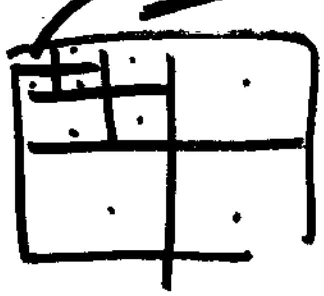
bottom surface

(c) a point in  $\Longrightarrow$ unique HSI ?

From part (a) $\longrightarrow$ a line in RGB space.

H S I

fixed × ×

P4 :  3-level Wavelet $\longrightarrow$ 

m, var. 10 subbands

$$\vec{u}=(m_1, v_{12}, \ldots\ldots, v_{20})^t \quad 20\times 1$$

$$\boxed{\vec{u}_1=(u(0), u(1) \ldots u(19))^t}$$

 } K Images

$$R_u(i,j)_{\boxed{20\times 20}}=E\big((u(i)-m(i))(u(j)-m(j))\big)$$
$$=\sum_{k=0}^{K-1}(u_k(i)-m)(u_k(j)-m)$$

8-3

$$\underset{20\times \underline{K}}{A} = \left[\vec{u}_{1\,20\times1},\ \vec{u}_{2\,20\times1},\ \cdots\ \vec{u}_{K\,20\times1}\right]$$

De-Mean $\vec{u}' = \vec{u} - \vec{m}$

$$\boxed{\underset{20\times1}{A'} = [\vec{u}_1',\ \vec{u}_2'\ \cdots\cdots \vec{u}_K']}$$

$$R_{u\,20\times20} = \underbrace{\underbrace{A'}_{20\times K}\ \underbrace{A'^{t}}_{K\times20}}_{20\times20}$$

Find eigenvector $\vec{\phi}_1 \cdots \vec{\phi}_{20}$

assuming $K > 20$

$$\underbrace{\underset{20\times20}{\Phi}} = [\vec{\phi}_1\ \vec{\phi}_2\ \cdots\ \vec{\phi}_{20}]$$

KLT $\vec{v} = \Phi^t \vec{u}$

IKLT $\vec{u} = \Phi \vec{v}$

corr. $\simeq 1$

(Bonus) $\vec{u} = [\underset{1}{0}\ 0\ \boxed{1}\ 0\ 0\ 0\ \underset{20}{0}]$

v_{31} corr $= -1$

KLT, IKLT
(subset of eigen. vector)

$$\vec{u}' = [0.2\ 0.4\ 0.9\ 0.3\ 0.5]$$

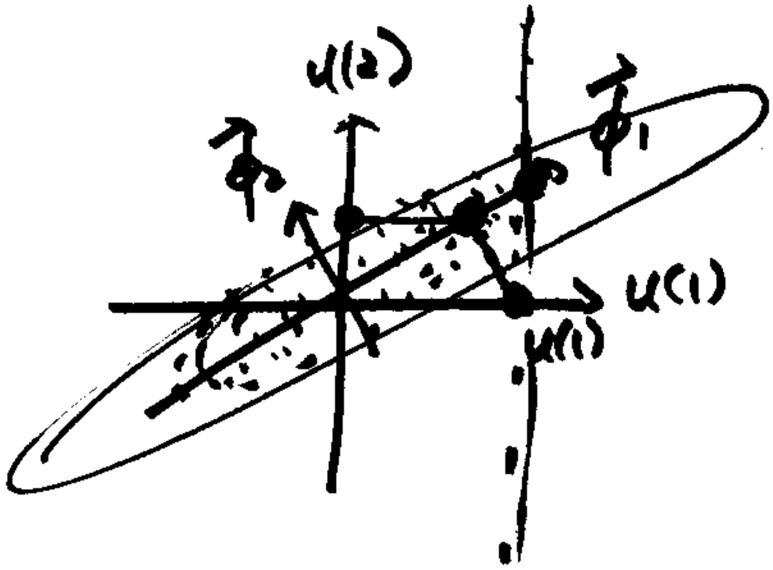
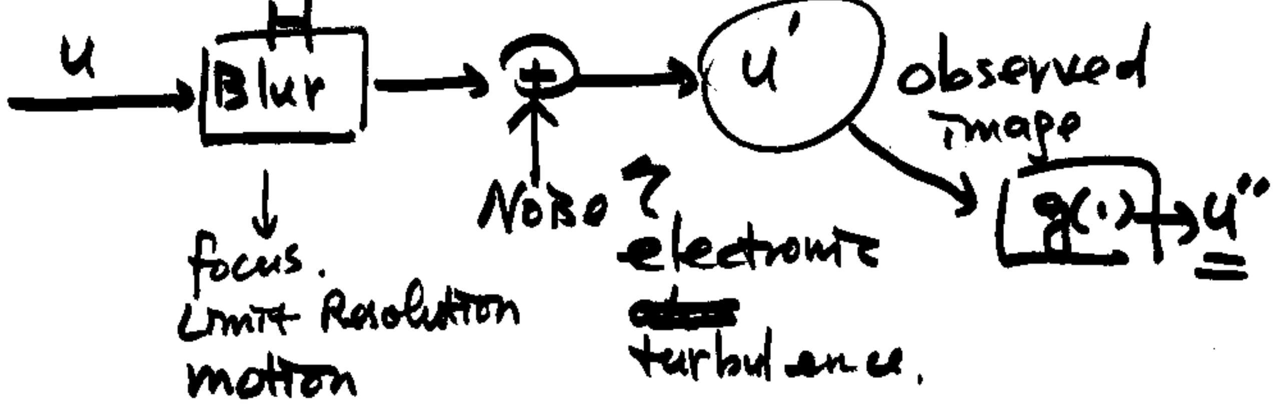


Image Restoration

Chap 5 G&W. Chap 8 Jain.



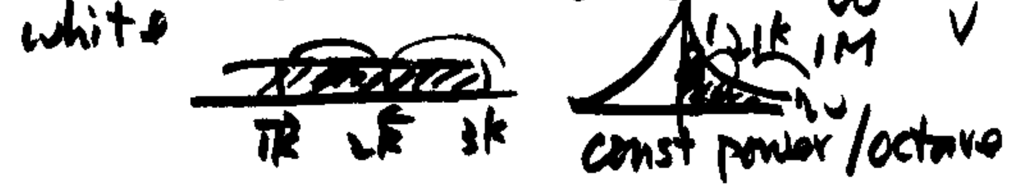
Quality Degration

Earlier : Hist. Equ.
Contrast Enhancement
Edge Sharpening
) Per Image

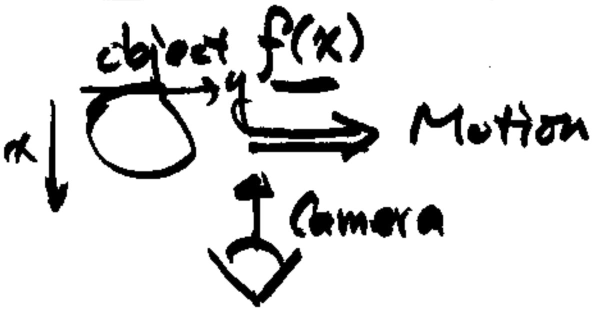
Here : model the degradation Sources

H , ?

$Power(\omega) = \frac{1}{\omega}$



Motion Blur



trajectory

$x(t) = \underline{a} \cdot t \quad t = 0 \cdots T$

speed

T: duration of imaging process
shutter open

$$f(x,t) = f(\underline{x} - \underline{at})$$

a: #pixels/sec ms.

$$\boxed{\underline{g(x)} = \int_{t=0}^{T} f(x-at)\,dt}$$

"Blur Effect" ... ⊛

observed Im

Instantaneous Input

Estimate the Effect by using Filter H

$$\xrightarrow{f(x)} \boxed{h(\;)} \rightarrow g(x)$$

$$F(\omega) \cdot \underline{H(\omega)} = G(\omega)$$

transfer Function

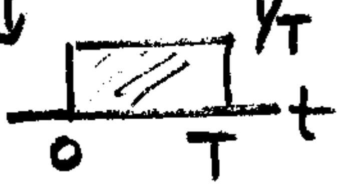
F.T on ⊛

$$G(\omega) = \int_{-\infty}^{\infty} \underbrace{\boxed{\int_{t=0}^{T} f(x-at)\,dt}}_{g(x)} e^{-j\omega x} \cdot dx$$

For your exercise.

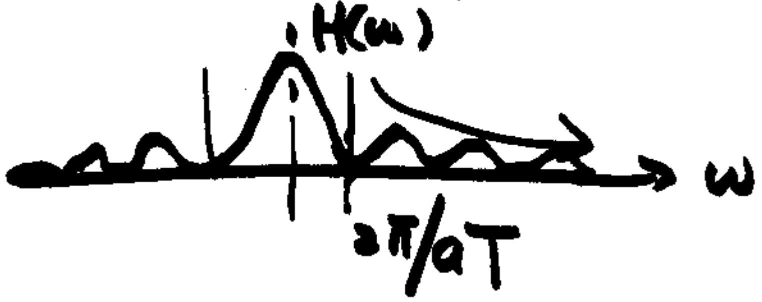
$$= \boxed{F(\omega)} \left(\int_0^T e^{-j\omega a t}\,dt \right)$$

$\Downarrow$

F.T. 

$$H(w) = \frac{2}{a \cdot w} \sin\left(\frac{w a T}{2}\right) e^{-j\frac{wa}{2}}$$

"Sinc"



$\underline{T} \uparrow \Rightarrow$ Narrow BW

$\Rightarrow$ More Blur

$\underline{a} \uparrow \Rightarrow$ faster motion

$\hookrightarrow$ H

H = fspecial('motion', Len, Ang)

↳ vector

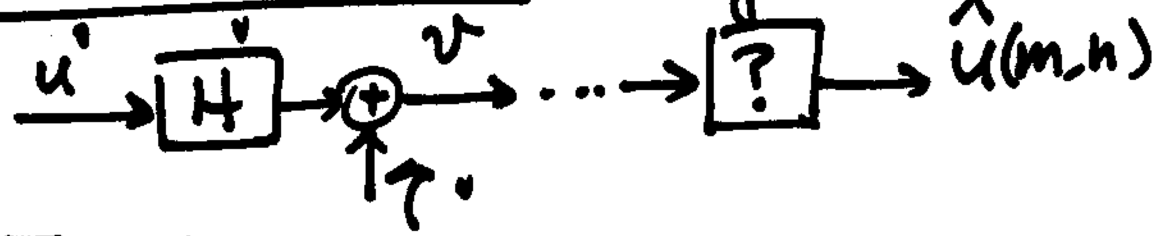
Len: 50 — 50 pixels, a·T

Ang: 45 → θ

I = Imread(. . .)

I' = imfilter (I, H)

Restoration Process



Freq. Domain

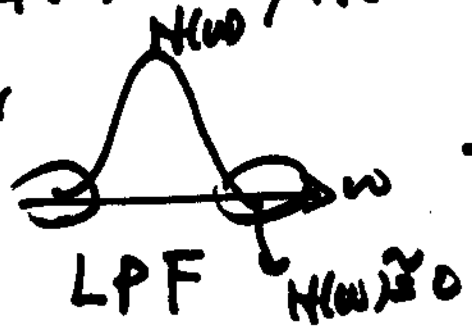
$$V(\omega) = U(\omega)\cdot H(\omega) + \underline{\underline{\eta(\omega)}}$$

Simple Case : $\underline{\eta = 0}$

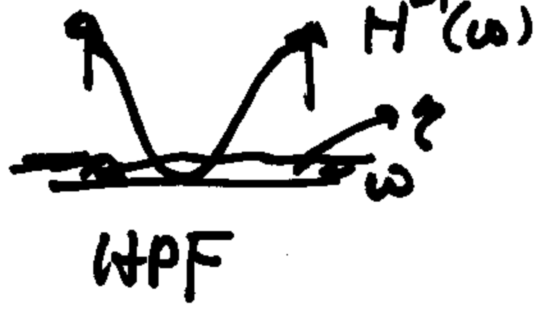
$$V(\omega) = U(\omega)\cdot H(\omega)$$

$$G(\omega) = 1/H(\omega) \qquad \text{Inverse Filter}$$

Blur



$\Longrightarrow$



In practice, $\eta \neq 0$

$H(\omega) \simeq 0 \Rightarrow 1/H$ large, amplify noise.

Pseudoinverse Filter

$$H^{-}(\omega) = \begin{cases} 1/H & H \neq 0 \\ 0 & H \simeq 0 \end{cases}$$

MSE (Min. Squared Error) Linear Estimator.

Find the optimal $\underline{g}$

s.t. $MSE = E[(u - \hat{u})^2]$ is minimized

given the knowledge of

H,

u, → Autocorrelation

η → Autocorrelation

<u>Wiener Filter</u>

$$G_{opt}(\omega) = \frac{H^*(\omega) S_{uu}(\omega)}{|H(\omega)|^2 S_{uu}(\omega) + S_{\eta\eta}(\omega)}$$

PSD of random sequence

Power spectral distribution

① if $\eta = 0$

$G_{opt}(\omega) = 1/H(\omega)$ Inverse Filter

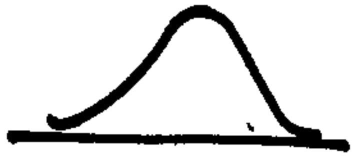
$|H(\omega)|^2 = H \cdot H^*$

② No Blur $H(\omega) = 1$: All Pass Filter

$$G_{opt}(\omega) = S_{uu}(\omega) / S_{uu} + S_{\eta\eta}$$

$$= 1 / 1 + (S_{\eta\eta}/S_{uu}) = \frac{1}{1 + SNR^{-1}}$$

③ General

$H \approx 1$  LPF

$\xi \approx 0$  white noise

~~S_{ff}~~ S_{uu} 

Q: what does $G_{opt}(\omega)$ look like?