DIP EE4830 Feb. 24 '05

DCT, DWT, KLT.

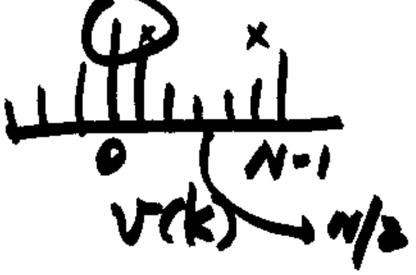
G&W Chap 4, 8.5, Ref Book 5.1

MidTerm March 10 '05

$$\xrightarrow[\text{2D, 1D}]{\text{Im}} \boxed{\text{Transform DFT}} \rightarrow \text{Processing} \rightarrow \boxed{\text{Trans}^{-1}} \rightarrow \text{Output}$$

LPF
HPF.

N-pt $\sum_{0}^{N-1} u^2$



$\sum_{0}^{N-1} v^2$

<u>Advantages:</u>

① More efficient Representation

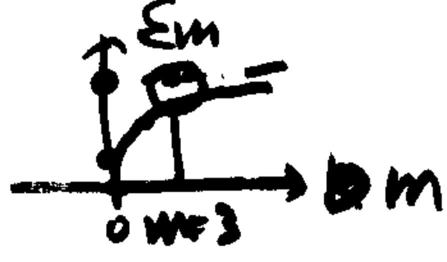
Pixel: N pt

$$\varepsilon_m \triangleq \sum_{n=0}^{m} |u(n)|^2 \Big/ \sum_{n=0}^{N-1} |u(n)|^2$$

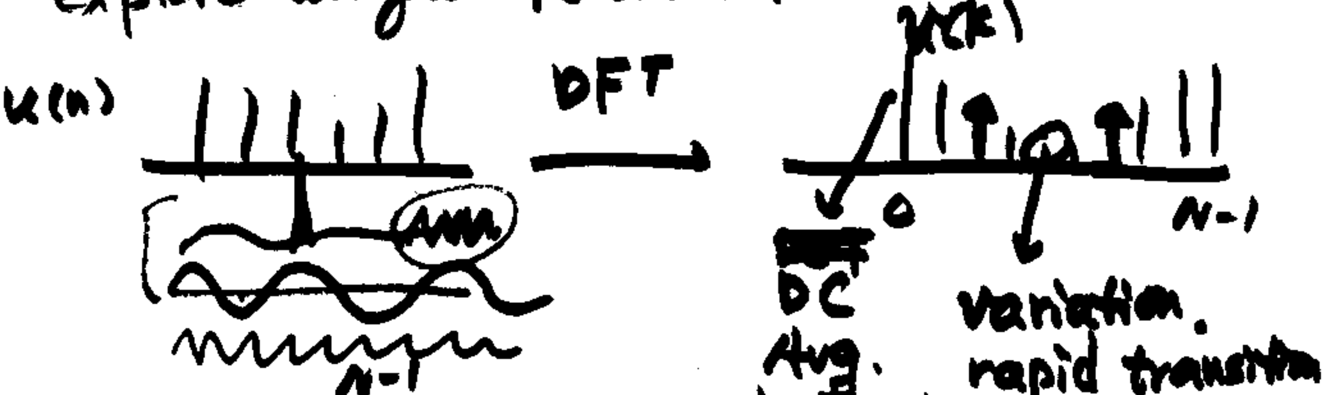
$$\hat{\varepsilon}_m \triangleq \sum_{k=0}^{m} |v(k)|^2 \Big/ \sum_{k=0}^{N-1} |v(k)|^2$$

DFT: Orthonormal Transform

$$\varepsilon_{N-1} = \hat{\varepsilon}_{N-1}$$



② Explore unique features.



$$v(k) = \sum_{n=0}^{N-1} u(n) \cdot \frac{1}{\sqrt{N}} e^{-j\frac{2\pi}{N} nk}$$

$\sum = v(0)$

$v\left(\frac{N}{2}\right)$

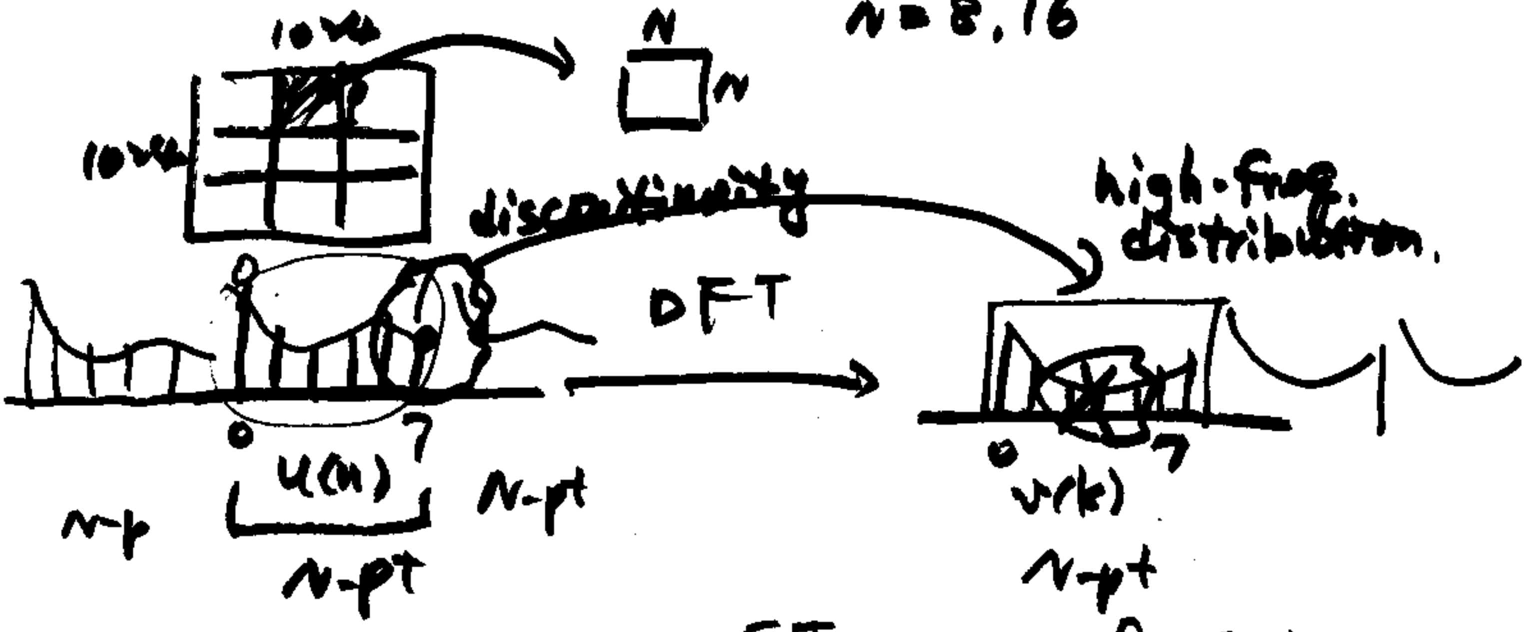
$k = N/2$

$$e^{-j\frac{2\pi}{N} n \cdot (N/2)}$$

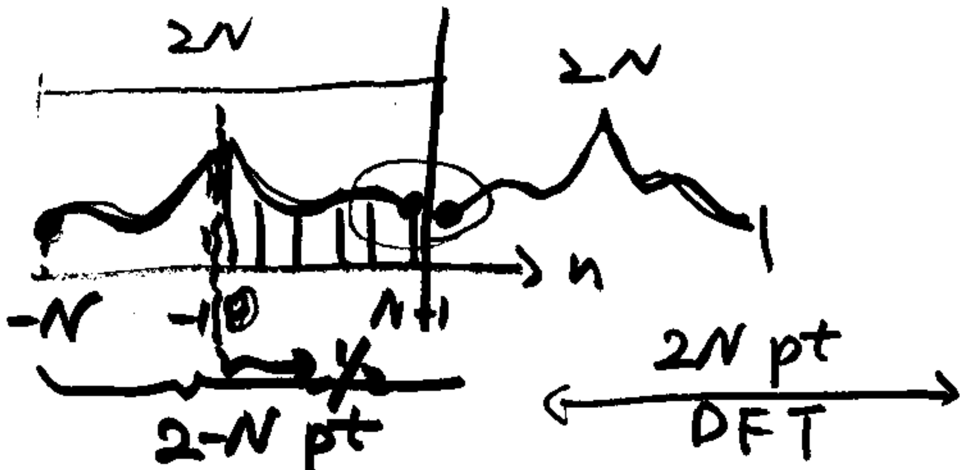
$$= e^{-j\pi n}$$

$$= \cos(\pi n) - \underbrace{j \sin(\pi n)}_{0}$$

Problem w/ block-based F.T.



Inifinite Series $\xrightarrow{\text{FT}}$ Inf. Series



$$v(k) = \frac{1}{\sqrt{2N}} \sum_{n=0}^{2N-1} \underbrace{u'(n)}\, e^{-j2\pi \underbrace{\frac{kn}{2N}}} \quad \begin{array}{c}2N\text{-pt}\\ \text{DFT}\end{array}$$

$$u'(n) = \begin{cases} u(n) & 0 \le n \le N-1 \\ u(2N-1-n) & N \le n \le 2N-1 \end{cases}$$

$$= \underbrace{\boxed{e^{j\frac{2\pi}{2N}\cdot\frac{k}{2}}}}_{\text{shift due to } -1/2} \underbrace{\frac{1}{\sqrt{2N}} \sum_{n=0}^{N-1} u(n) \boxed{\cos\left(\frac{\pi}{2N} k(2n+1)\right)}}_{\text{DCT basis}}$$

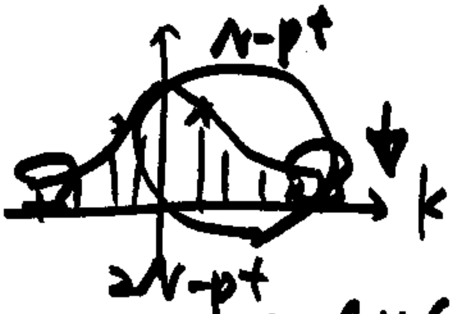
N-pt

N-pt DFT

$v_{N\text{-}DFT}(k)$

2N-pt DFT

$v_{2N\text{-}DFT}(k)$

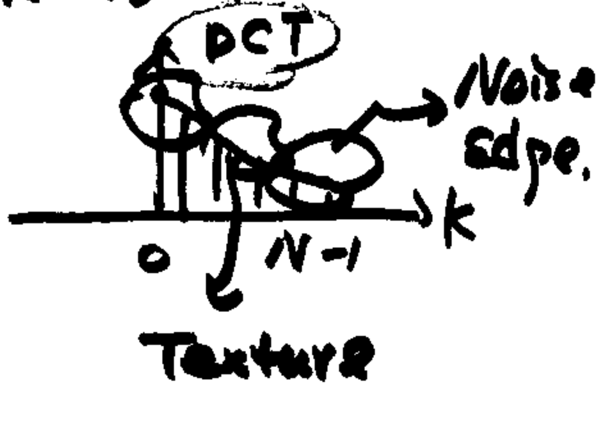


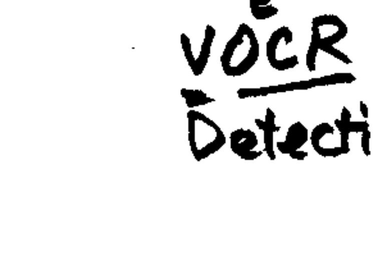
N-pt Real Values

N-pt DCT

$$v_{N\text{-}DCT}(k) = \alpha(k) \sum_{n=0}^{N-1} u(n) \cos\left(\frac{\pi}{2N} k (2n+1)\right)$$

$$\alpha(k) = \begin{cases} \sqrt{1/N} & k = 0 \\ \sqrt{2/N} & k = 1, \ldots N-1 \end{cases}$$





VOCR

Detection

KLT, PCA Karhunen-Loeve Transform
Principle-Component Ana.

$u_i \quad u_j$

N-pt Signal
$u(0) \ldots u(N-1)$

: Covariance Matrix

$$\vec{u}_{N\times 1} = \begin{bmatrix} u(0) \\ u(1) \\ \vdots \\ u(N-1) \end{bmatrix}$$ Random Variable.

mean : $E[\vec{u}] = \vec{m}_u$

Covariance bet. $u_i \cdot u_j$

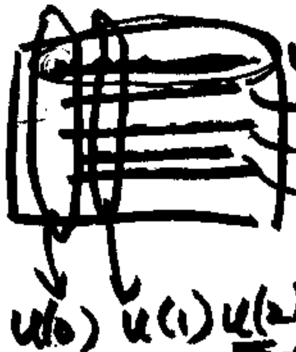
$$E\left((u_i - m_i)(u_j - m_j)^* \right)$$

Correlation Coeff.

$$= Cov(i,j) \Big/ Var(i)^{1/2} Var(j)^{1/2}$$

$C.C = +$
$= 0$
$= -$

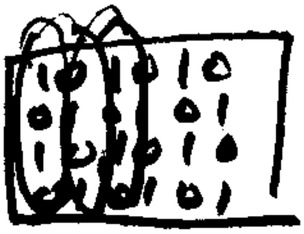
$cC = 1$



Row : $\vec{u}$
Column : Samples

$u(0)\ u(1)\ u(2)$

$$\underset{C.C.}{\text{}}(i,j) = \begin{array}{c} \\ N \end{array} \overset{N}{\begin{bmatrix} 1 & 1 & 1 & & \\ 1 & & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{bmatrix}}$$



$$C.C(i,j) = \begin{bmatrix} 1 & -1 & 1 & -1 & 1 \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{bmatrix}$$

$$\underline{u(0) \cdots\ u(N-1)}$$

N-pt

Cov Matrix

$$\underset{N\times N}{R_u} = E\left[(\vec{u}-\vec{m}_u)(\vec{u}-\vec{m}_u)^{*T}\right]$$

$N \times N$

DFT, DCT, KLT, DWT.

$\vec{v} = A\vec{u}$ $\quad$ A : unitary Transf.

$A^{-1} = A^{*t}$

$$R_v = A\, R_u\, A^{*T}$$

R_v = DFT of R_u

Example :

$u(0)\ u(1)$ $\quad R_u = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$ $\quad 0<\rho<1$

A : transform $= \frac{1}{2}\begin{pmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{3} \end{pmatrix}_{2\times 2}$

$$R_v = A\, R_u\, A^{*t}$$

$$= \begin{pmatrix} 1+\sqrt{3}\rho/2 & \rho/2 \\ \rho/2 & 1-\sqrt{3}\rho/2 \end{pmatrix}$$

corr. bet 1st. and

$\rho = 0.95$ $\quad = \begin{bmatrix} 91\% & 0.475 \\ 0.475 & 9\% \end{bmatrix}$

$$R_v = \Phi^{*t} R_u \Phi$$

$$= \Phi^{*t} R_u \left[\vec{\phi}_0 \,\middle|\, \vec{\phi}_1 \,\middle|\, \vec{\phi}_2 \,\middle|\, \cdots \vec{\phi}_{N-1} \right]$$

$$= \Phi^{*t} \left[\lambda_0 \vec{\phi}_0 \,\middle|\, \lambda_1 \vec{\phi}_1 \,\middle|\, \lambda_2 \vec{\phi}_2 \cdots \right]$$

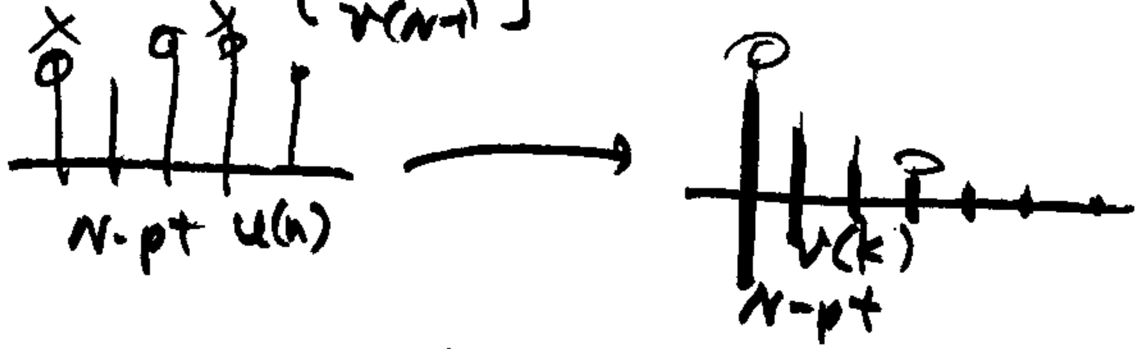
$$\boxed{R_v = \begin{bmatrix} \lambda_0 & & & 0 \\ & \lambda_1 & & \\ & & \lambda_2 & \\ 0 & & & \ddots \end{bmatrix}}$$

PCA , KLT

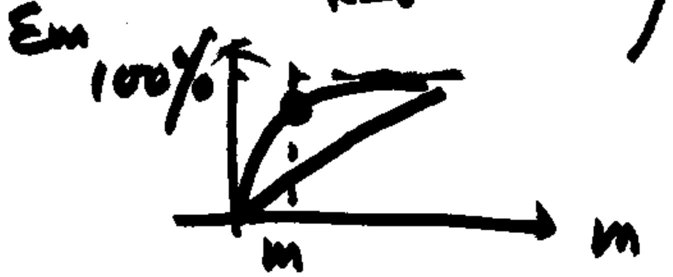
Non-Diag. c.c = 0

$$\vec{v} = \begin{bmatrix} v(0) \\ v(1) \\ \vdots \\ v(N-1) \end{bmatrix}$$

$\lambda_0 \geq \lambda_1 \geq \lambda_2 \cdots$



$$\hat{\varepsilon}_m = \sum_{k=0}^{m} |v(k)|^2 \Big/ \text{total}$$



PCA . KLT

$$\xrightarrow[\vec{u}]{N\times 1} \boxed{KLT} \longrightarrow \overset{N\times 1}{\vec{v}}$$

Goal: Best Transform A_{opt}
s.t. c.c bet variables min.
s.t. energy concentration in $\vec{v}$ is opt.

Procedure:

① Get $\underline{R_u}_{N\times N}$ <u>Covariance Matrix</u>

② Find Eigen Vectors $\vec{\phi}_k$ of R_u
$k = 0, \cdots N-1$

$$R\vec{\phi}_k = \lambda_k \vec{\phi}_k$$

λ_k : eigen value

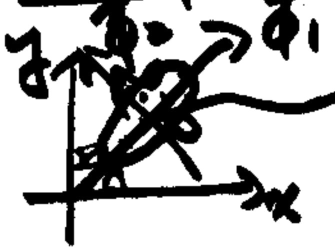
③ $\underline{\Phi} = \left[\vec{\phi}_0 \,\vdots\, \vec{\phi}_1 \,\vdots\, \vec{\phi}_2 \cdots \,\vdots\, \vec{\phi}_{N-1} \right]$

KLT transform

④ $\underline{\underline{\vec{v}}} = \underline{\Phi}^{*t} \,\underline{\underline{\vec{u}}}$

$\underline{\underline{\vec{u}}} = \underline{\Phi}\, \underline{\vec{v}}$

Example :



$\vec{u} = \begin{bmatrix} x \\ y \end{bmatrix}$ of points ~~in~~ the object

M pts in the object : # of samples

R_u 2x2

$= [\ \mathrm{cov}(u(0), u(1))$

$\vec{\phi}_0, \vec{\phi}_1$

$\underbrace{\lambda_0}_{\text{Var.}} \quad \lambda_1$

Var.

Example :

Multi-Spectral Images (R.S, Medical)

M bands

$\vec{u} = (u(0), u(1), u(2) \cdots u(N^2-1))$

$\vec{u} = [u(0), u(1) \cdots u(M-1)]^t_{M\times 1}$

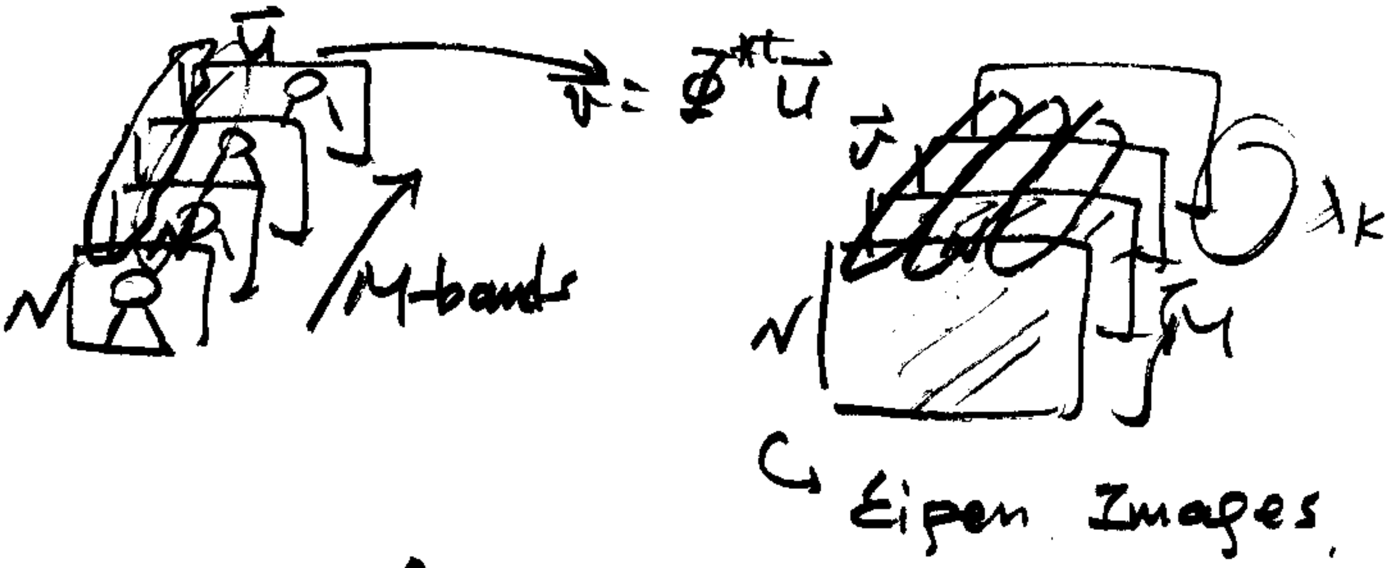
Samples : N^2

$\underbrace{R_u}_{}$ MxM

$\vec{\phi}_0, \vec{\phi}_1 \cdots \vec{\phi}_{M-1}$

KLT Φ

$\vec{v} = \Phi^{*t} \vec{u}$



Eigen-face