Lecture #5 DIP EE 4830
Feb 17 2005

Midterm Exam : March 10th
HW #2 Due Next wk.

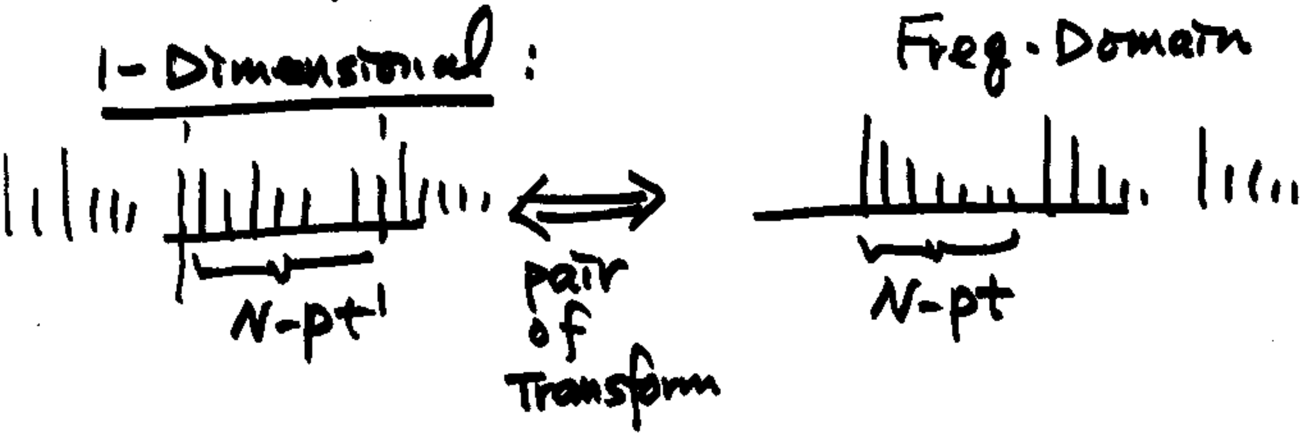
Image Transform :

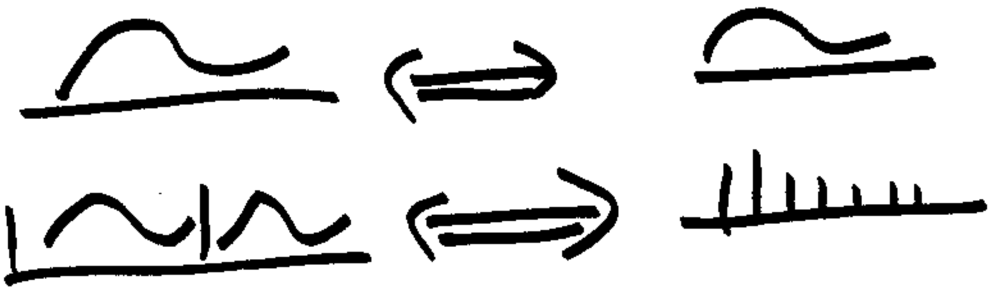
DFT, DCT, DWT

Chap 4 of G&W. Jain 5.1-5.6

Chap 8 : 1 section on DCT in G&W
8.5.2

1-Dimensional :





N-pt DFT

pixel domain $\xrightarrow{\text{Transform}}$

$$\vec{u} = \begin{bmatrix} u(0) \\ u(1) \\ \vdots \\ u(N-1) \end{bmatrix} \qquad \vec{v} = \begin{bmatrix} v(0) \\ v(1) \\ \vdots \\ v(N-1) \end{bmatrix}$$

Foward

Inverse

$$u(n) = \sum_{k=0}^{N-1} v(k) \left(\frac{1}{\sqrt{N}} e^{j\frac{2\pi kn}{N}} \right) \cdots\cdots (*)$$

$$v(k) = \sum_{n=0}^{N-1} u(n) \left(\frac{1}{\sqrt{N}} e^{-j\frac{2\pi kn}{N}} \right)$$

Matrix Form

$$\vec{u} = \underbrace{\left[\vec{b_0} \; \vdots \; \vec{b_1} \; \vdots \; \vec{b_2} \; \cdots \right]}_{\text{Matrix}} \begin{bmatrix} v(0) \\ v(1) \\ \vdots \\ v(N-1) \end{bmatrix}$$

$$\vec{b_0} = \begin{bmatrix} u(0) \\ u(1) \\ u(2) \\ \vdots \end{bmatrix}$$

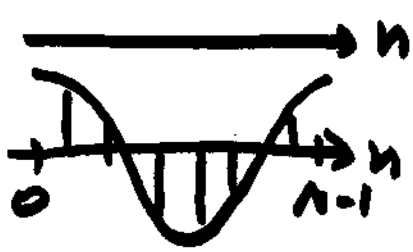
$$(*) \quad \vec{u} = \sum_{k=0}^{N-1} \vec{b_k} \cdot v(k)$$

$$\cos\left(\frac{2\pi}{N}kn\right) + j\sin\left(\frac{2\pi}{N}kn\right)$$

$$b_k(n) = \frac{1}{\sqrt{N}} e^{j\left(\frac{2\pi}{N}\right)k \cdot n}$$

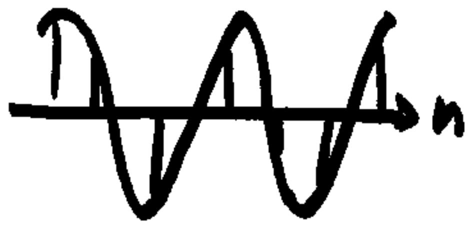
$k=0 \quad b_0(n)$

$k=1 \quad b_1(n)$



5-2

$k=2 \quad b_2(n)$



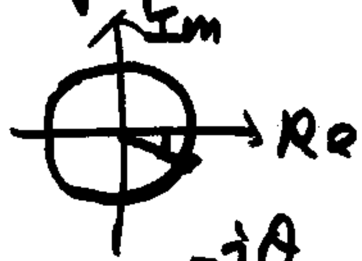
$k=3 \quad b_3$

$\vec{u}$ N-pt Vector $\xrightarrow{FT}$ $\vec{v} = A_{N\times N}\,\vec{u}_N$

$$A_{N\times N} = \frac{1}{\sqrt{N}}\begin{bmatrix} W_N^{0\cdot 0} & W_N^{0\cdot 1} & W_N^{0\cdot 2} & \cdots \\ W_N^{1\cdot 0} & W_N^{1\cdot 1} & W_N^{1\cdot 2} & \cdots \\ \vdots & & & \end{bmatrix}$$

$$W_N = e^{-j2\pi/N}$$

$j = \sqrt{-1}$



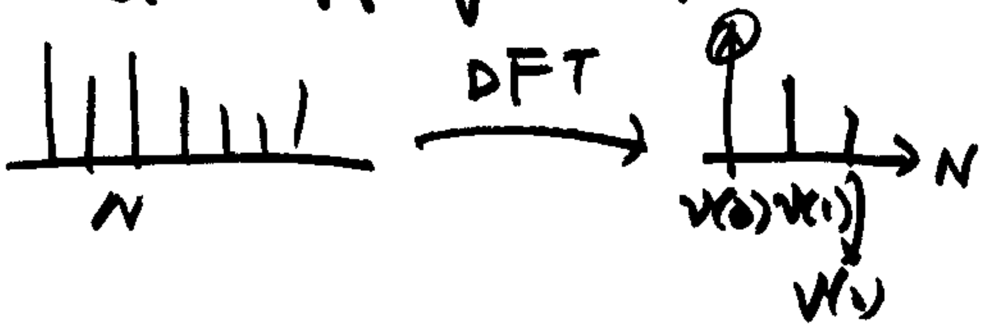
$e^{-j\theta}$

A : unitary matrix

$A^{-1} = A^{t*}$

$$\vec{v} = A\,\vec{u}_N$$

$$\vec{u} = A^{-1}\vec{v} = A^{*t}\vec{v}$$



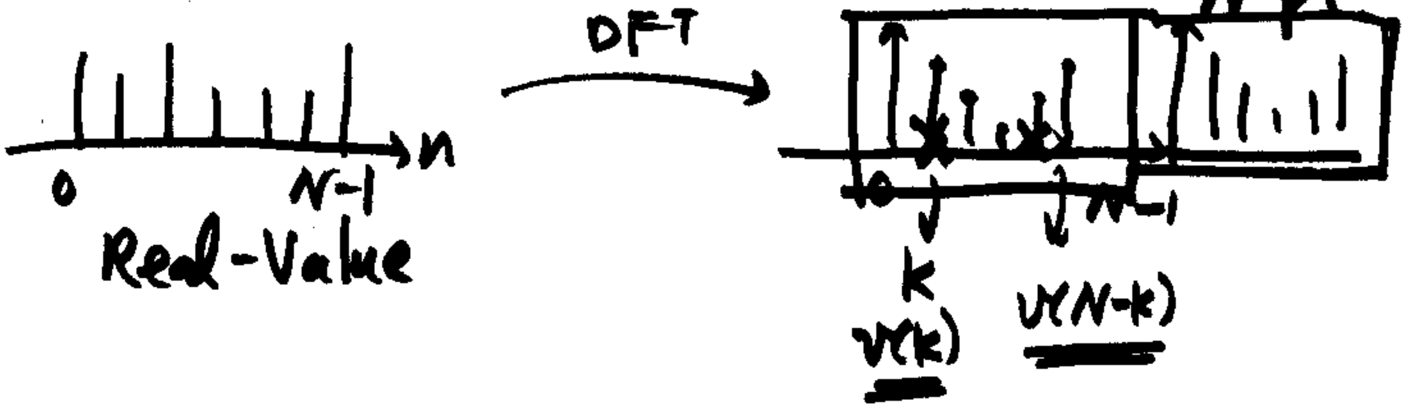
$u(n)$

$$\vec{u} \longrightarrow \vec{v}$$

$$\vec{v} = A\,\vec{u}$$

* Conjugate Symmetry

$$v(N-k) = v^*(k) \quad , \text{ if } u(n) \text{ is real}$$



Proof:

$$v(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} u(n)\, e^{-j\frac{2\pi}{N} k n}$$

$$v(N-k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} u(n)\, e^{-j\frac{2\pi}{N}(N-k)n}$$

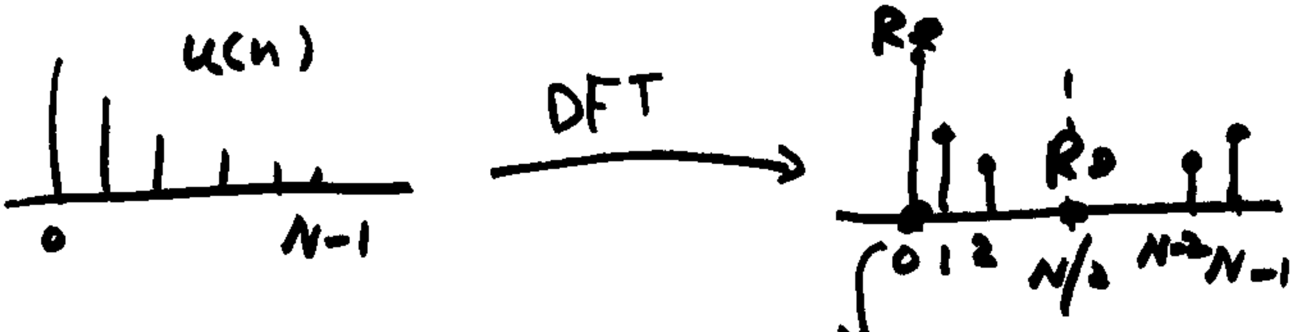
$$\underbrace{e^{-j\frac{2\pi}{N} N \cdot n}}_{e^{-j2\pi n}} \cdot\, e^{-j\frac{2\pi}{N}(-k)n}$$

pixel domain

N points

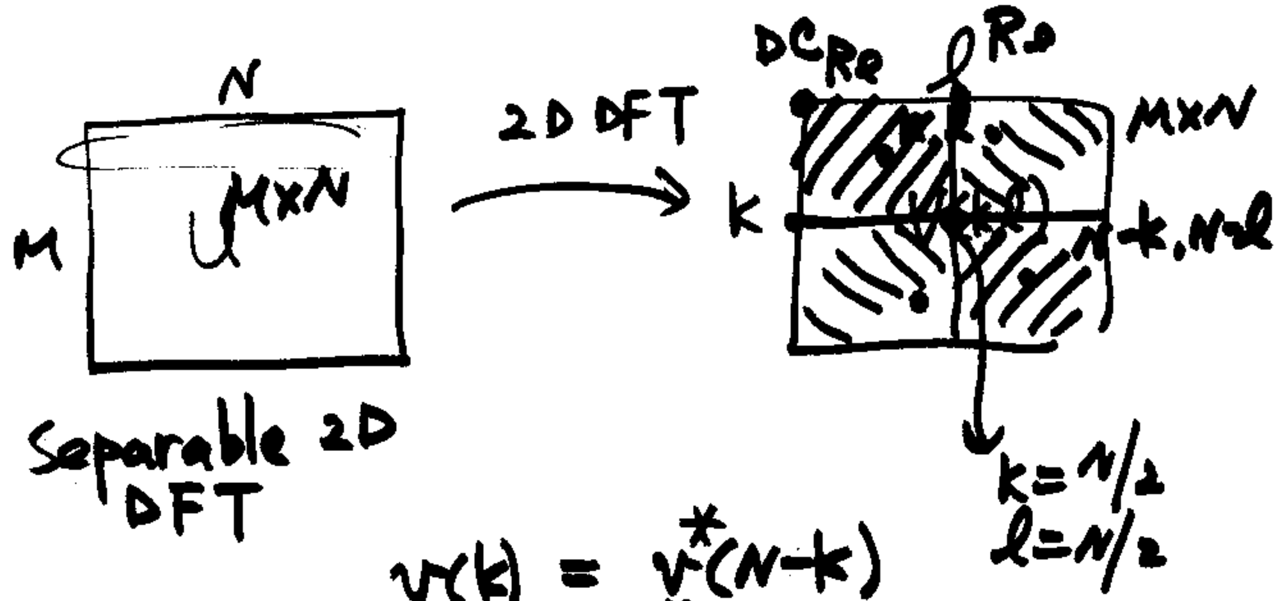
Real values

N coeff. × 2



$v(0)$: Real-Value

$$\left\{\begin{array}{l} v(0) = v^*(N) \\ v(0) = v(N) \end{array}\right. \Rightarrow v(0)\text{-Real}$$



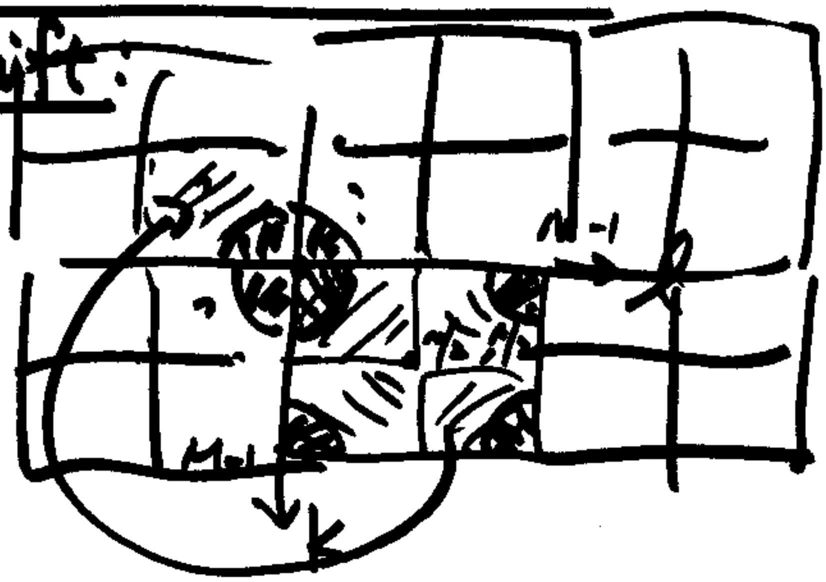
Separable 2D DFT

$$v(k) = v^*(N-k)$$

$$v(k,l) = v^*(N-k, N-l)$$

Center Shift:

DFT



2D DFT : fft2(img)

$\underline{\underline{u}}(m,n) \longleftrightarrow \underline{\underline{v}}(k,l)$

$$\sum_{k}\sum_{l} |v(k,l)|^2 = \sum_{m,n}\sum |u(m,n)|^2$$

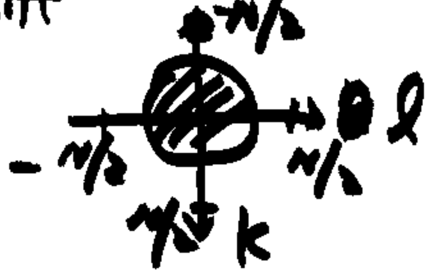
Reason for Center Shift :

① Visualization

$$LPF = \begin{cases} 1 & \text{freq low} \\ 0 & \text{freq high} \end{cases}$$

$$\begin{matrix}\text{Im} \\ u\end{matrix} \longrightarrow \underset{\text{fftshift}}{\boxed{DFT}} \longrightarrow \boxed{LPF} \longrightarrow \underset{\text{invfftshift}}{\boxed{DFT^{-1}}} \longrightarrow$$

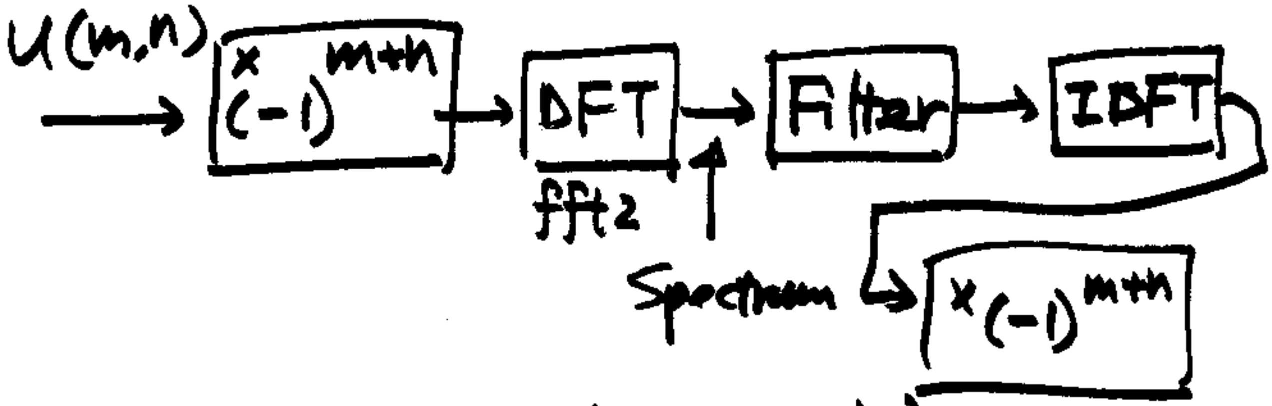
?



How to implement Center Shift ?

$$u(n)\cdot(-1)^n \xleftrightarrow{\text{DFT}} \underline{v}(k - N/2)$$

$$u(m,n)\cdot(-1)^{m+n} \Longleftrightarrow v(k - M/2,\ l - N/2)$$



<u>Fourier Spectrum</u> (Magnitude)

$$|v(k,l)| = \left(Re^2(v) + Im^2(v)\right)^{1/2}$$

−1 1

1 −1

<u>Phase</u>

$$\phi(k,l) = \tan^{-1}(Re,\ Im)$$

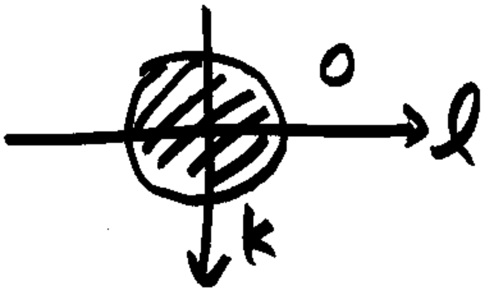
$$\tan^{-1}\left(\frac{Im}{Re}\right) \quad \text{Range}: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

<u>Power Spectrum</u>

$$P(k,l) = |v(k,l)|^2$$

Filtering :

LPF : $H(k,l) = \begin{cases} 1 & D(k,l) \le D_0 \\ 0 & D(k,l) > D_0 \end{cases}$



Ideal

$$V(k,l) \longrightarrow \boxed{H(k,l)} \longrightarrow H(k,l) \cdot V(k,l)$$

Transfer Function of Filter.

Butterworth LPF

$$H(k,l) = \frac{1}{1 + \left[\frac{D(k,l)}{D_0}\right]^{2n}}$$

n : order $n = 0$

$n = 1$

$n = 2$

High Pass Filter

$1 - LPF$

Ideal $H = \begin{cases} 0 & \text{if } D \le D_0 \\ 1 & \text{if } D > D_0 \end{cases}$

Why Transform ?

Filter : $A_{M\times M}$

$A_{M\times M} \cdot U_{M\times N}$

$M \times N \times M = M^2 \cdot N$ operation

DFT(A)   DFT(U)

$\boxed{H(k,l) \cdot U(k,l)}$

$M \times N \implies M \times N$ ✓

$U_{M\times N}$: DFT

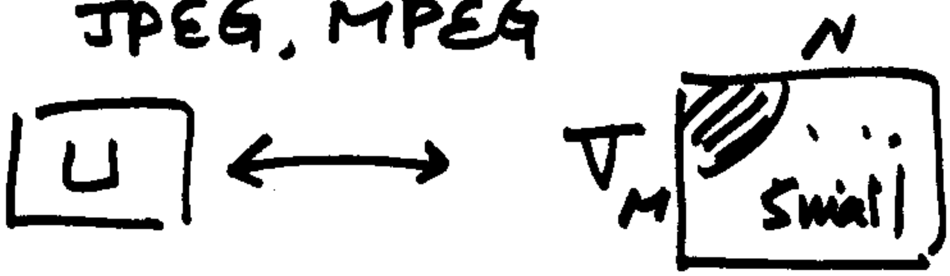
$u(n)$: N-pt $N \cdot \log N$

$U_{M\times N}$: $M \cdot N \cdot \log M + M \cdot N \cdot \log N$

$= MN(\log M + \log N)$ ✓

Truncation of Freq. Coeff

JPEG, MPEG

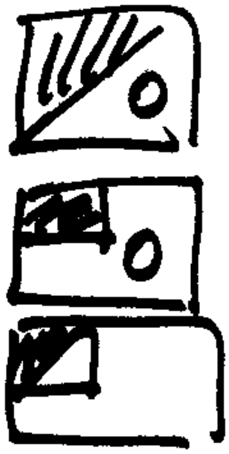


Zonal Filtering

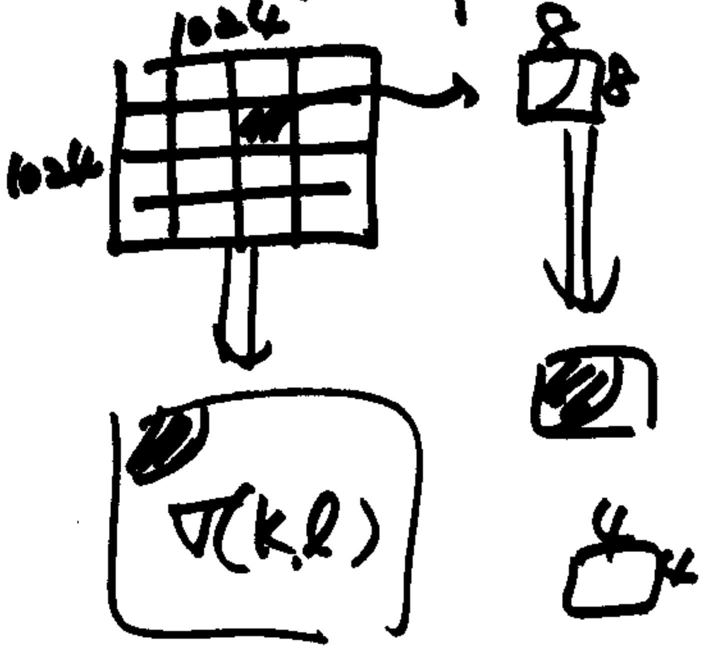
1/2

1/4

1/8



Why in Image Coding, Transform Size (Block) is 8 pixels?



per pixel complexity

$\log_8 N + \log_8 M$

$= \underline{6 \text{ "x"}}$

cf. $\underline{20 \text{ "x"}}$