DIP EE 4830

Feb 1st 2005

Color Space, HSI

~~Sa~~ Sampling, Interlaced Video

<u>Quantization</u>, JPEG Image Coding

Compression: Tradeoff between bit rate & distortion (quality)

$\boxed{\text{Im}}$ 512 × 512   RGB 8 bits/ch

⇒ Size $\frac{1}{4}$ Mega Pix × 3 B/pix

= 0.75 MB = 750 K Bytes

≈ 6 M bits

⇒ 50 K Bytes

⇒ Compression Ratio ≅ 15

Video 180 Mbps <u>Raw Data</u>

Color Perception is HVS

<u>RGB</u>, <u>HSI</u>, $L a^* b^*$, $Y C_R C_B$

Color Component

Chromanance.

YC_RC_B : Y sampled at rate 1

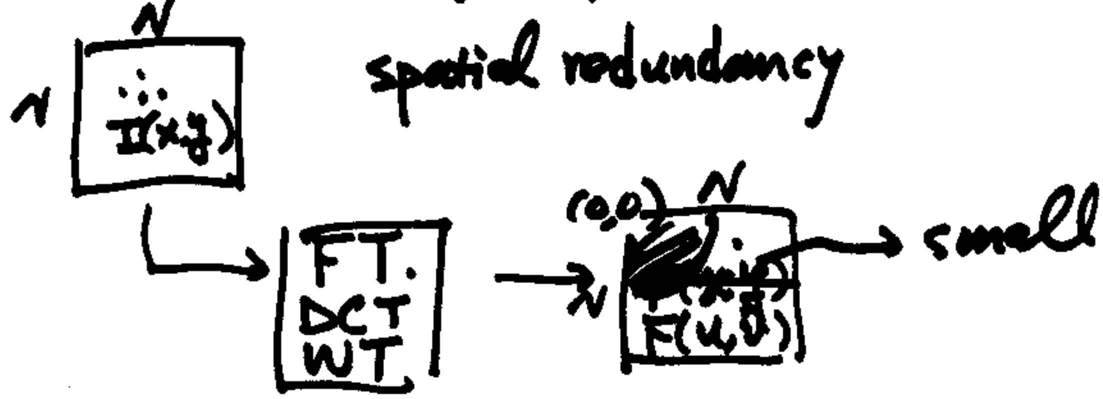
C_R at ½ × ½

N^2 $\frac{1}{4}N^2$ $\frac{1}{4}N^2$ Ratio = 2

N^2 N^2 N^2

Pixel → Frequency Domain

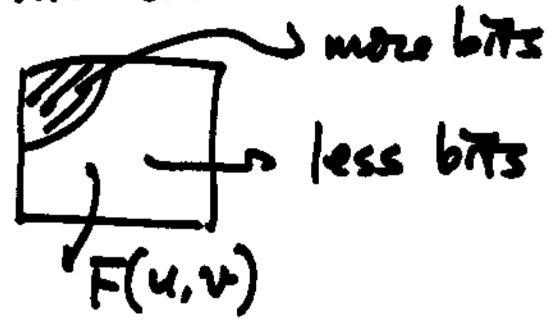
spatial redundancy

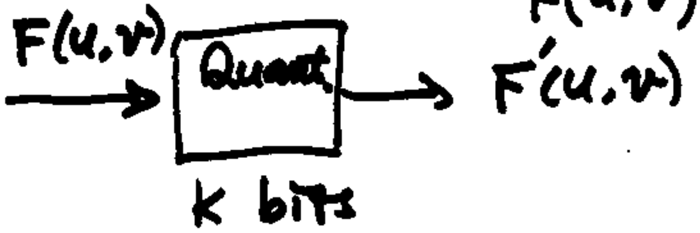


Non-Uniform Bit Allocation

Spatial

- pixel 24 bits
  8 bits/ch





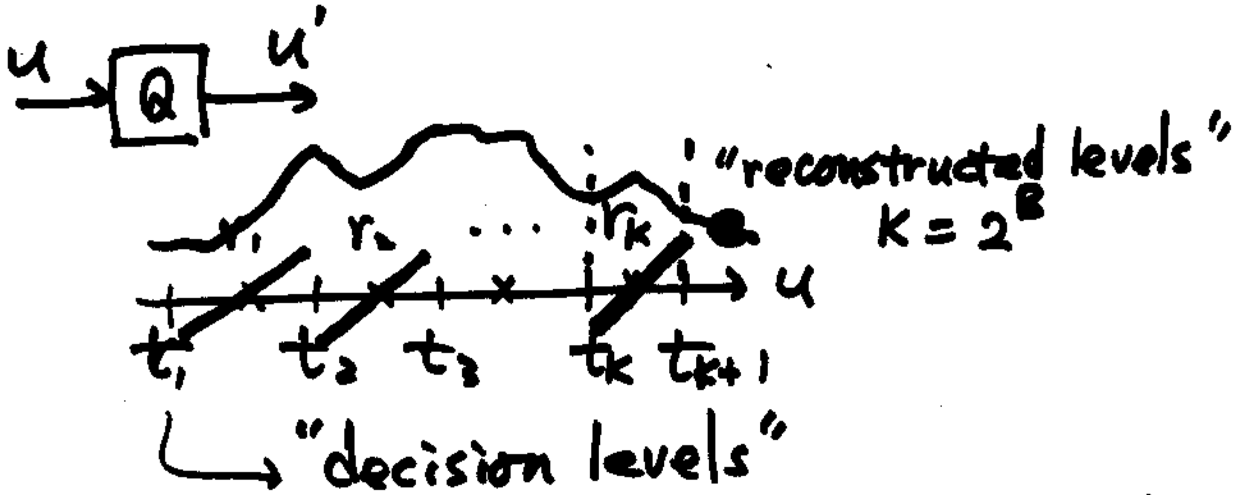
Questions :

① How many bits ?

② How to quantize ?

③ Quality measurement ?
metrics

2005
3-2



$u' = Q(u) = r_k$, if $t_k \le u < t_{k+1}$

Uniform quantizer

$t_k = k \cdot q$ $\quad q$: quan. step size

Mean Square Error

$e(u) = u - u' = u - r_k$, if $u \in [t_k, t_{k+1}]$

$$\text{(MSE)}\ \varepsilon^2 = \sum_{k=1}^{L} \int_{t_k}^{t_{k+1}} (u - r_k)^2 p(u)\, du$$

$p(u)$: prob. den. func. of random variable U.

$\int_{-\infty}^{\infty} p(u)\, du = 1$

MSE depends on L, t_k, r_k

Goal : minimize MSE, given L

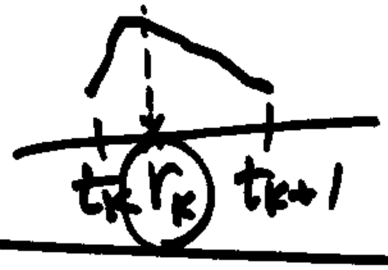
Solution : Opt. MSE Quantizer

Lloyd - Max Quantizer

$$\frac{\partial \varepsilon^2}{\partial t_k} = 0 \implies t_k = (r_k + r_{k-1})/2$$

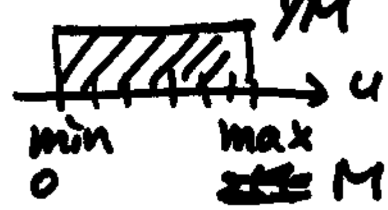
$$\frac{\partial \varepsilon^2}{\partial r_k} = 0 \implies r_k = \int_{t_k}^{t_{k+1}} u \cdot p(u)\,du \Big/ \int_{t_k}^{t_{k+1}} p(u)\,du$$

"Center of Mass"



Specific Case:

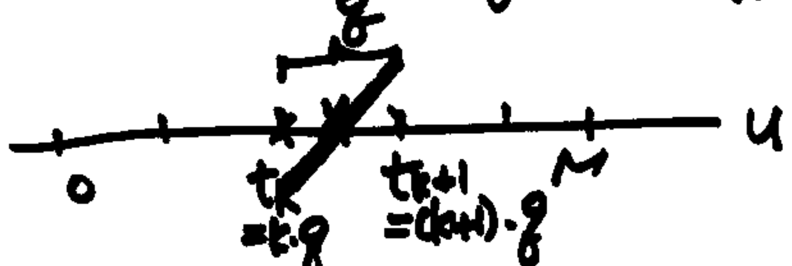
① $p(u)$ is uniform



$M = L \cdot q$

Opt. MSE Quantizer for uniform $p(u)$
is uniform Quantizer
Linear Quantizer

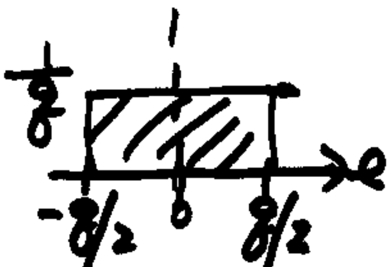
$$t_k = k \cdot q = k \cdot \frac{M}{L}$$



$$E(e^2) \stackrel{\text{MSEL}}{=} \sum_{k=1}^{L} \int_{t_k}^{t_{k+1}} e^2 \cdot \frac{1}{M} \cdot du \qquad M \to L \cdot q$$

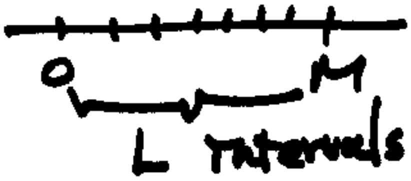
$$= \text{Variance of } e \quad \to \int_{-q/2}^{q/2} e^2 \frac{1}{q} \cdot de$$

$$= q^2/12$$



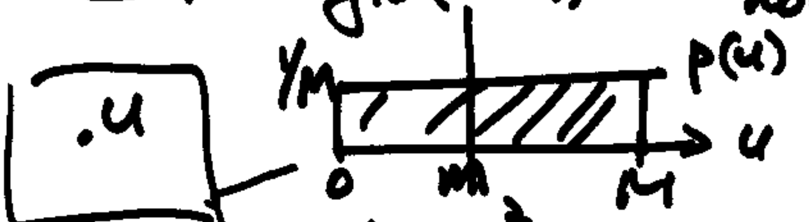
Mean Sq. Error $= E[e^2] = q^2/12$

$q = M/L$



SNR : signal noise ratio

$\triangleq 10 \cdot \log_{10}\left(\dfrac{\text{energy of signal}}{\text{'' noise}}\right)$



$E((u-m)^2) = \text{Variance}$

$\text{Var} = ?\quad M^2/12$

$= 10 \cdot \log_{10}\left(M^2/q^2\right)$ $\qquad M = L \cdot q$

$\qquad\qquad\qquad\qquad\qquad$ B : # bits

$= 20 \cdot \log_{10}(L)$ $\qquad L = 2^B$

$= 20 \cdot \log 2^B \simeq 6B$ (dB)

36dB

B=5 $\quad$ 5 bits/symbol $\qquad$ SNR = $\boxed{30\ dB}$

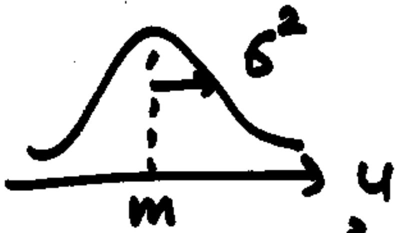
B=4 $\qquad\qquad\qquad\qquad$ = 24dB

B=1. $\qquad\qquad\qquad\qquad$ SNR = 6dB uniform quantizer $\Rightarrow$ opt. ~~of~~ for uni. distribution

what if u does not have uniform dist ?

Typical dist.

Gaussian dist.

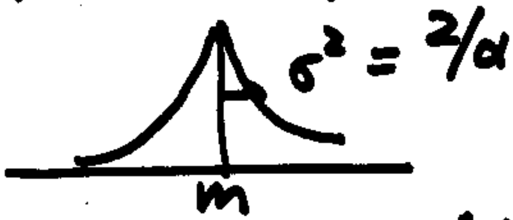


$$p(u) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(u-m)^2}{2\sigma^2}\right)$$

Jain, Table 4.1

L, t_k, r_k are computed

Laplacian Dist



$$p(u) = \frac{\alpha}{2} \exp(-\alpha|u-m|)$$

Jain Table 4.2

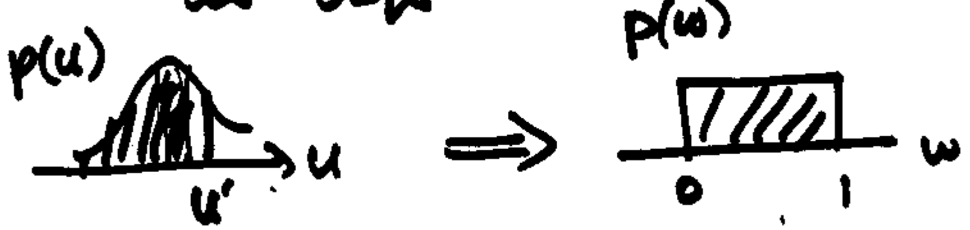
Variable Transformation

$$u \rightarrow \boxed{f(\cdot)} \xrightarrow{w} \boxed{Q} \xrightarrow{w'} \boxed{f^{-1}(\cdot)} \rightarrow u'$$

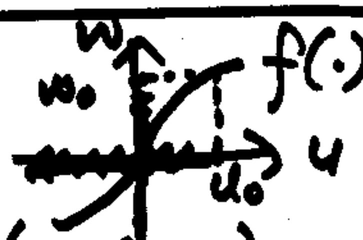
$$\underbrace{p(w) : \text{uniform}}_{?}$$

Solution :

$$\underset{\sim}{w} = \int_{-\infty}^{u} p(u')\,du'$$

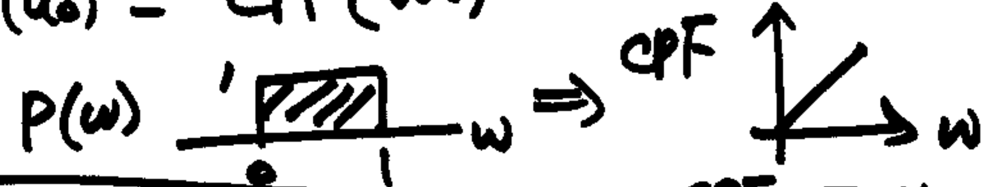


$\underline{w = f(u) \text{ monotonic}}$



$$\underbrace{\text{Prob}(u \le u_0)}_{\text{C.P.F.}} = \underbrace{\text{prob}(w \le w_0)}_{\text{Cumulative Prob. func.}}$$

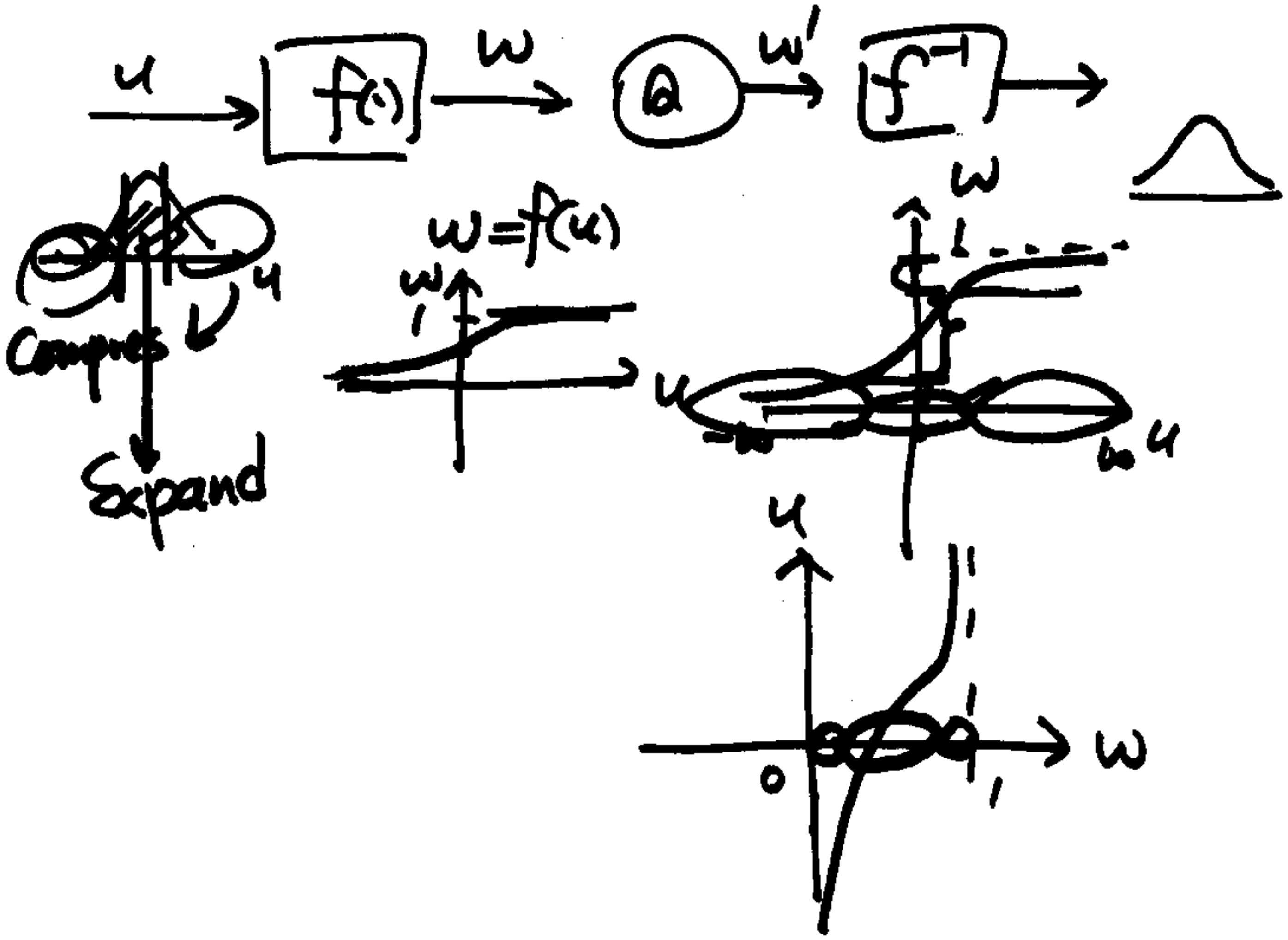
$$CPF(u_0) = CPF(w_0)$$



$CPF_w = w$

$$\boxed{\int_{-\infty}^{u} p(u')\,du' = w}$$

Compandor Quantizer


u
f(·)
w
Q
w'
f⁻¹
Compress
Expand
w=f(u)
w
u
1
-∞
∞
0