EE4830 DIP lec #12

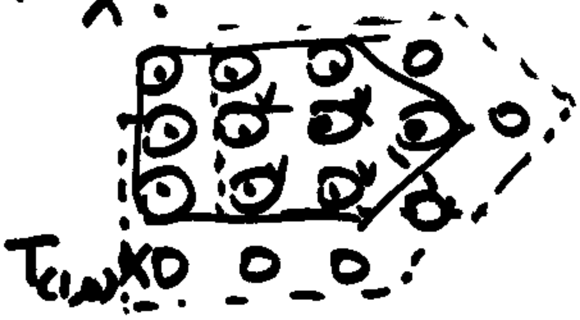
HW#5 Due May 5th.
send by email or deposit to TA Mbox.

In actual implementation.

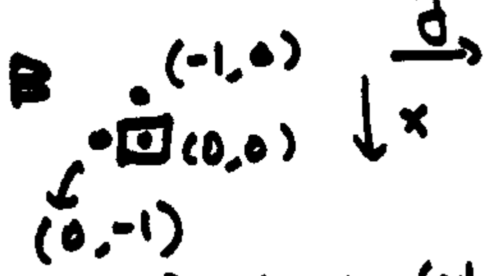
$$X \oplus B = \bigsqcup_{m,n \in B} T_{m,n}(X)$$

$$X \ominus B = \bigsqcap_{m,n \in -B} T_{m,n}(X)$$

e.g. X :



B

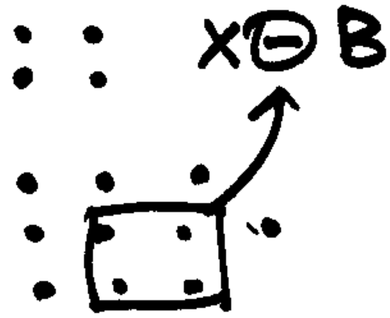


$-(m,n) = (0,0),\ (-1,0),\ (0,-1)$

$(0,0),\ (1,0),\ (0,1)$

$$X \oplus B = \bigsqcup_{m,n \in -B} T_{m,n}(X)$$

$$X \ominus B = \bigsqcap_{m,n \in -B} T_{m,n}(X)$$



If X has $N \times N$ points ("1")

$X_{m,n}$

$(N+1)(N)$

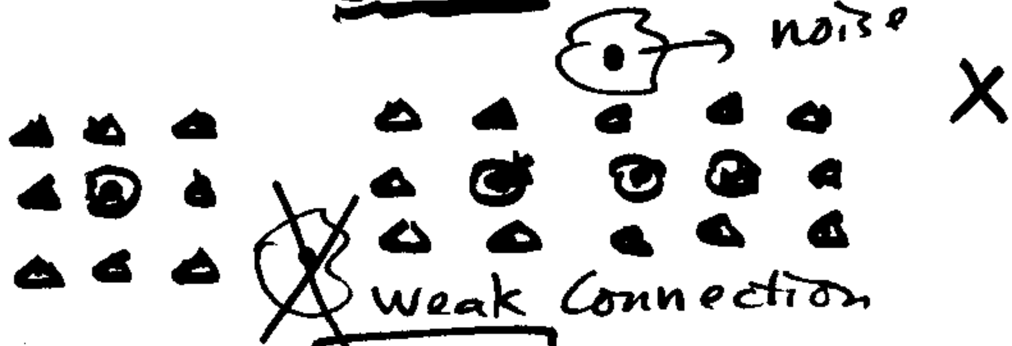
$(N)(N+1)$

$(N+1) \times (N+1)$

Open:

$X \circ B$

$= \underline{(X \ominus B)} \oplus B$



B = 

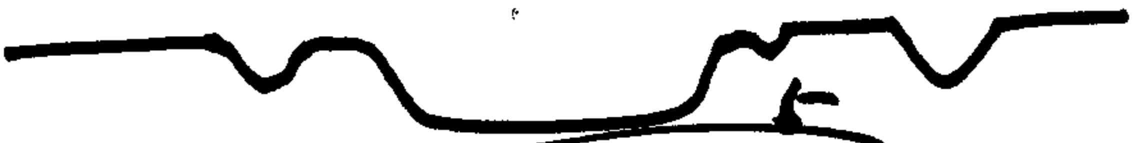
Close

$\underline{(X \oplus B)} \ominus B$

$(X \circ B) \bullet B$

$(X \bullet B) \circ B$

$\boxed{\text{imregionalmin}(I) = BW}$



e.g. $I = \begin{pmatrix} 10 & 10 & 10 & 10 & 10 & 10 & 10 \\ 10 & 7 & 7 & 8 & 10 & 6 & 10 \\ 10 & 7 & 8 & 9 & 10 & 6 & 10 \\ 10 & 10 & 10 & 10 & 10 & 10 & 10 \end{pmatrix}$

BW : binary image

$\underline{BW} = \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$

BW = imextendedmin($\underline{I}$, $\underline{22}$) .

Some Properties

$\oplus$: dilation $\quad$ B : design.

$\ominus$: erosion.

$(X \ominus B) \oplus B$ $\quad$ Open

$(X \oplus B) \ominus B$ $\quad$ Close

1. Translation Invarian. $\quad$ B 

~~B ⊕ T(X~~

$$X \underset{\ominus}{\oplus} T(B) = T(\underset{\ominus}{\underline{X \oplus B}})$$

2. Commutable

$$\underline{X \oplus B} = \underline{B \oplus X} \quad \text{(up to some shift)}$$

$$B \ominus X \neq X \ominus B$$

3. Distibutive

$$X \oplus (B_1 \cup B_2) = (X \oplus B_1) \cup (X \oplus B_2)$$

Set Theory

$$X \oplus B = B \oplus X = B \oplus (X_1 \cup X_2)$$

$$\underline{X \ominus (B_1 \cup B_2)} = (X \ominus B_1) \cap (X \ominus B_2)$$

B_1  $\quad$ B_2 

$B_1 \cup B_2$

4. Duality $\oplus$, $\ominus$

$$\underline{X \ominus B} = (X^c \oplus B)^c$$

$$X \oplus B = (X^c \ominus B)^c$$

5. Duality $\circ$, $\bullet$

Open Close

~~$B \bullet X$~~

$$(X \bullet B) = (X^c \circ B)^c$$

$$X \circ B = (X^c \bullet B)^c$$

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hit-miss operator

X

str. el. = $B = \{B_1, B_2\}$

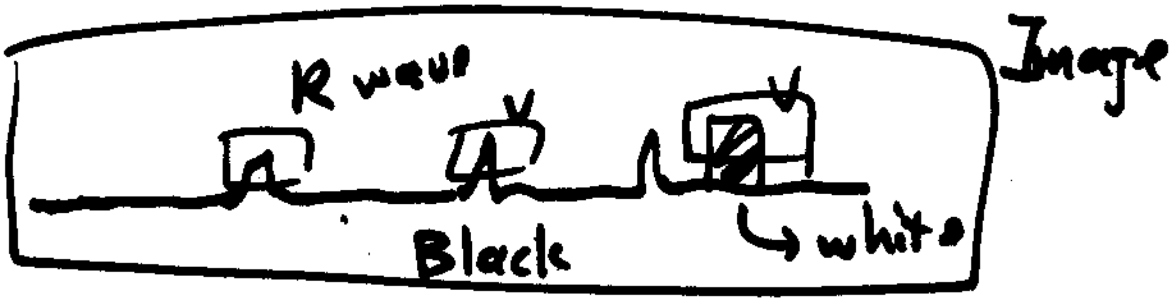
B_{ob}, B_{bk}

$$X \circledast B = \underline{(X \ominus B_1)} \,/\, \underline{(X \oplus B_2)}$$

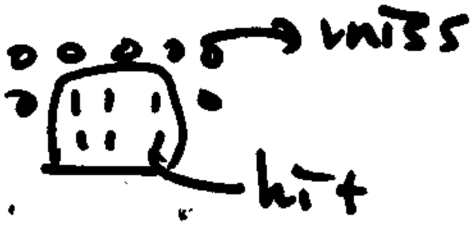
/ : "Difference" op.

A/B

$$= (X \ominus B_1) \cap \underbrace{(X^c \ominus B_2)}$$



B_1



Boundary Extraction

$A / (A \ominus B)$

~~A~~

Convex Hull

X

hit-miss

$$B^1 = \begin{matrix} \bullet & \times & \times \\ \bullet & \bullet & \times \\ \bullet & \times & \times \end{matrix}$$

don't care

B_{bk}

B_{ob}

$$B^2 = \begin{matrix} \times & \times & \bullet \\ \times & \bullet & \bullet \\ \times & \times & \bullet \end{matrix}$$

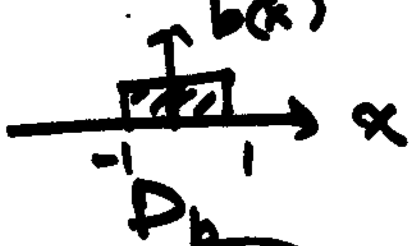
$$\boxed{X_0 = X}$$

$$X_{i+1} = (X_i \circledast B) \cup X_0$$

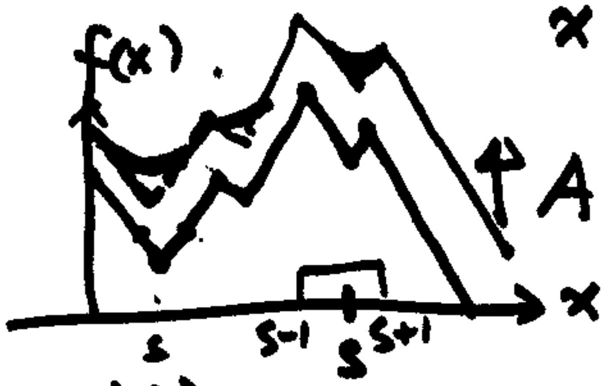
Grey-Scale Morphological Operation

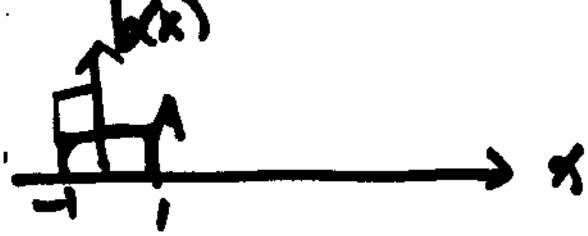
$f(x)$: Intensity function
defined over domain D_f

$b(x)$: structure function

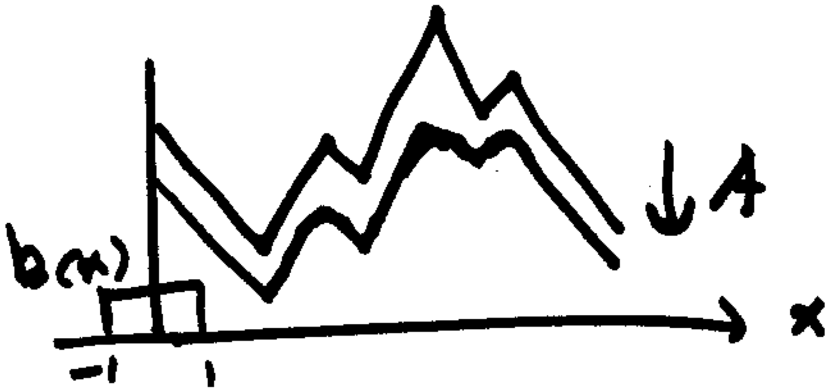


$$(f \oplus b)(s) = \max_{x} \left(f(x+s) + b(x) \;\middle|\; \begin{array}{l} x+s \in D_f \\ x \in D_b \end{array} \right)$$





$$(f \ominus b)(s) = \min_{x} \left(f(x+s) - b(x) \;\middle|\; \begin{matrix} x+s \in D_f \\ x \in D_b \end{matrix} \right)$$



$$((f \oplus b) \ominus b)(s)$$ Close

Top hat

$$f - (f \circ b)$$

Bottom hat $\quad f \bullet b - f$