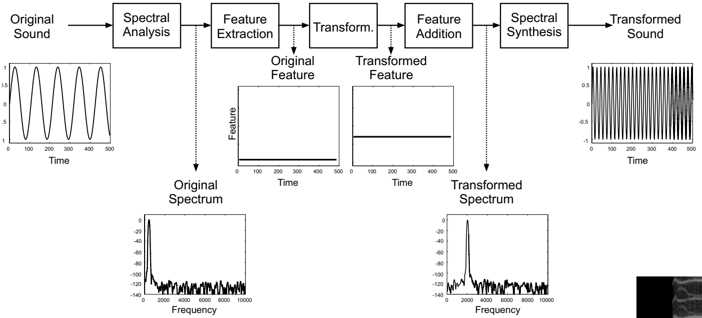
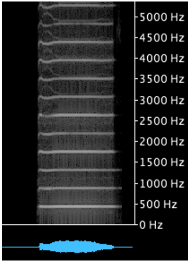
E85.2607: Lecture 9 – Sinusoidal Modeling

Sinusoidal Modeling/Spectral Modeling Synthesis (SMS)



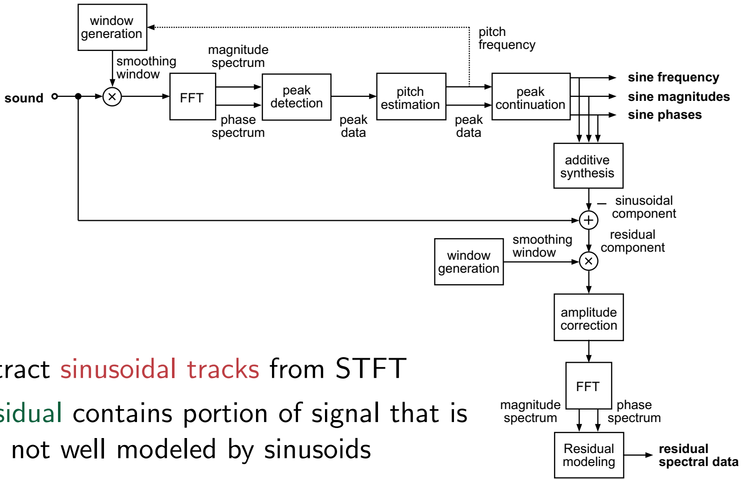


- Similar to phase vocoder, but assumes that signal is sum of sinusoids with smoothly varying parameters:

$$x[n] \approx \tilde{x}[n] = \sum_k a_k[n] \cos(\omega_k[n])$$

- Freq. domain representation analogous to Fourier series
- Flexible representation for transformations...

SMS overview



Analysis

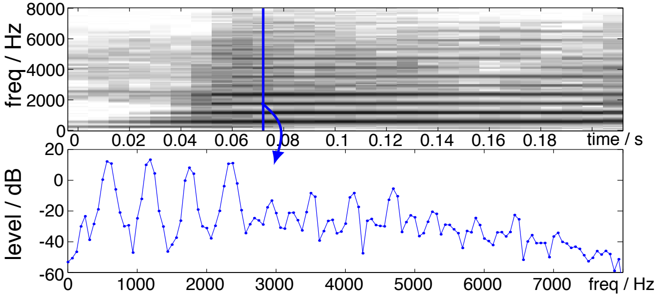
Extract sinusoidal tracks from STFT

Residual contains portion of signal that is not well modeled by sinusoids

Synthesis

Each track controls an oscillator

Analysis overview



1. Break signal into frame ala STFT
   - Be aware of time-frequency tradeoff
2. Pick peaks in each frame
   - Often expect to find peaks in harmonic series due to common pitch
   - Search for common factor
3. Organize spectral peaks into time-varying sinusoidal tracks
   - Expect sinusoids to drift in amplitude and frequency

Review: Time-frequency tradeoff

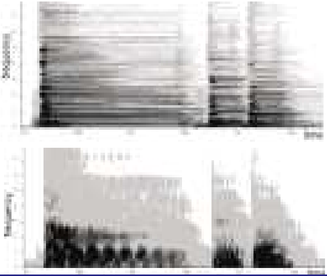
$$X[k,m] = \sum_{n=0}^{N-1} x[n+mL]w[n]e^{-j\frac{2\pi kn}{N}}$$

DFT length N

- Window determines freq. resolution
- Must be long enough to resolve harmonics $\sim 2\times$ longest pitch period ($\sim 50-100$ ms)
- Too long $\rightarrow$ blurred $a_k[n]$

Hop size L

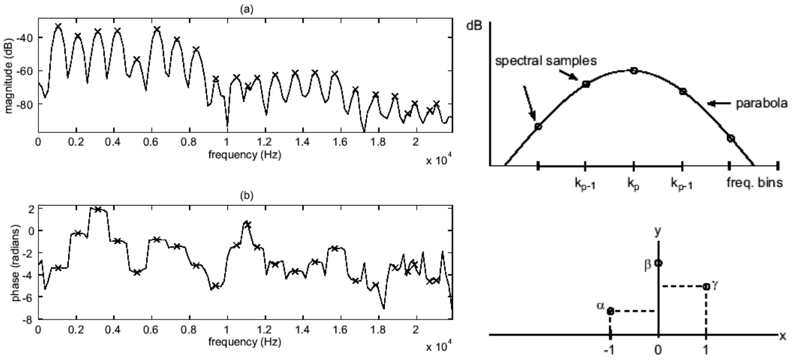
- typically $N/2$ or $N/4$
- small hop $\rightarrow$ simpler interpolation over time



good freq. resolution
bad time resolution

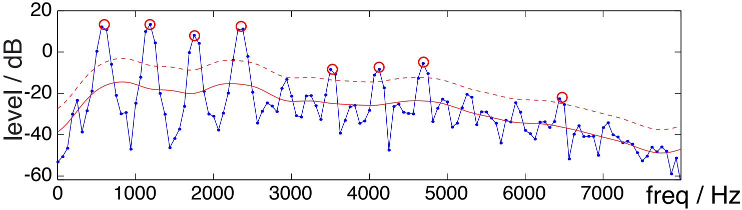
bad freq. resolution
good time resolution

Peak picking



- Find local maxima in each DFT frame
- Accuracy of peak detection is limited to half a sample
  - Can zero pad, but can be expensive
- Alternative: Frequency resolution smaller than one bin using quadratic interpolation

Peak picking 2

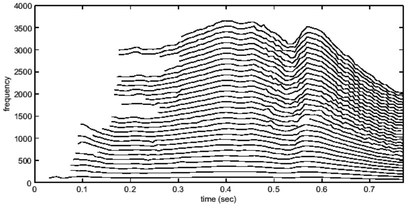


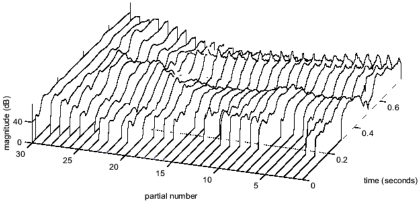
Not all peaks correspond to a stable sinusoid

- Only retain peaks larger than some threshold
- Only retain peaks consistent with harmonic series of underlying pitch
    - need pitch tracker, input must be monophonic
    - pitch-synchronous analysis?
- Look for stable parameters in adjacent frames
    - e.g. stable phase derivative in time and frequency

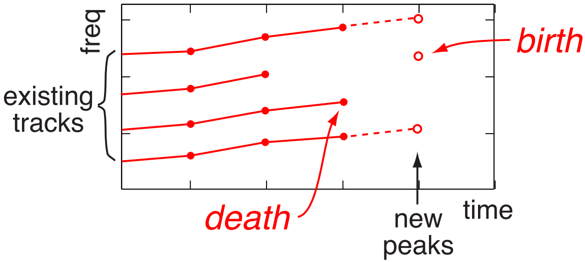
Peak continuation

- Connect peaks in adjacent frames to track sinusoid trajectory
- Lots of different approaches
  - e.g. Macaulay-Quatieri approach: Greedily attach peak in current frame to closest peak in next frame
- Ambiguous if large frequency changes



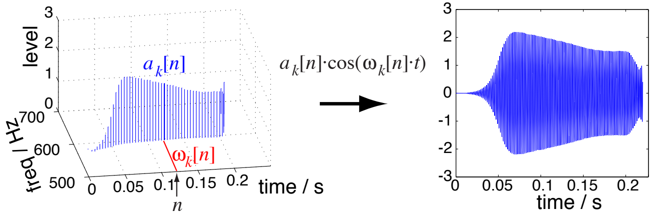


Track formation heuristics

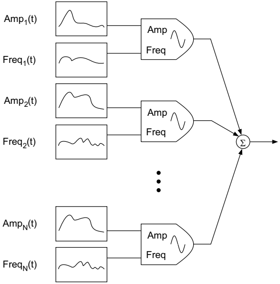


- Unclaimed peak → create new track
- No continuation of track → termination
- Lots of other potential rules:
  - min track length to avoid spurious peaks
  - max allowable frequency deviation between adjacent frames
  - max silent gap length
  - max number of tracks per frame
- Tricky to implement . . .

Sinusoidal synthesis

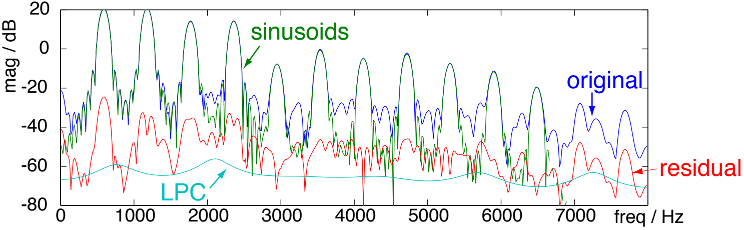


- Each sinusoid track $\{a_k[n], \omega_k[n]\}$ drives an oscillator
- Interpolate parameters to avoid clicks at frame boundaries
- Faster method: synthesize DFT frames, then overlap-add



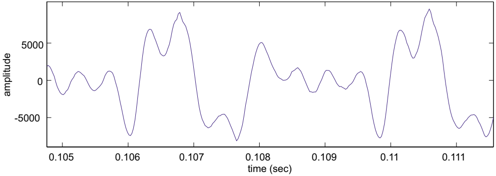
- Sinusoids is not a good fit for all types of signals (e.g. noise)
- Sometimes want to retain residual in addition to sinusoids

$$x[n] = \sum_k a_k[n]cos(\omega_k[n]n) + e[n]$$

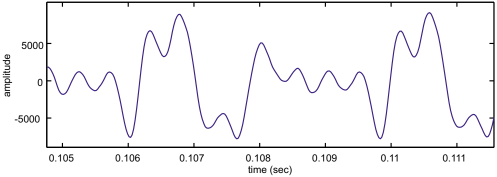


- Model residual as white noise passed through time-varying filter

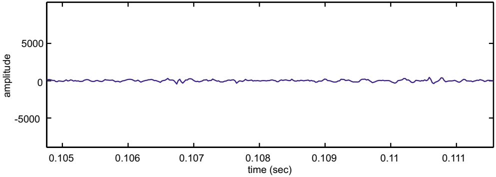
Sinusoidal subtraction



original sound
$x(n)$

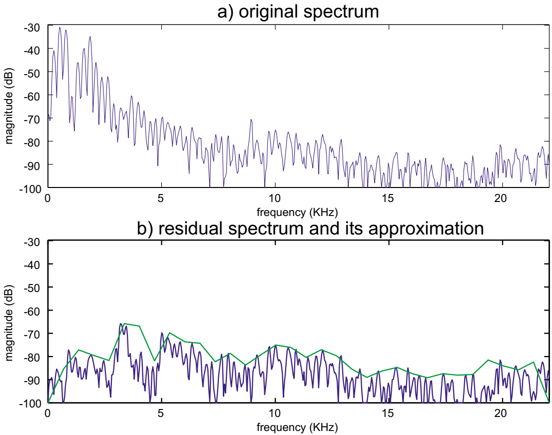


synthesized sound
with phase matching
$s(n)$



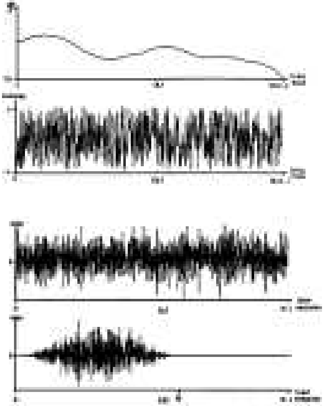
residual sound
$e(n) = w(n) \times (x(n) - s(n)),$
$n = 0, 1, \ldots, N - 1$

Modeling the residual



- Many options for modeling residual filter parameters
- Can use LPC to approximate shape of residual
- or simply smooth magnitude spectrum ala channel vocoder

Residual synthesis



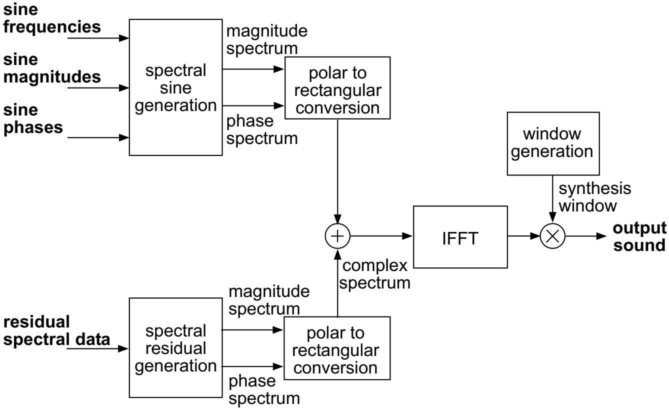
spectral magnitude
approximation of residual

random spectral phase

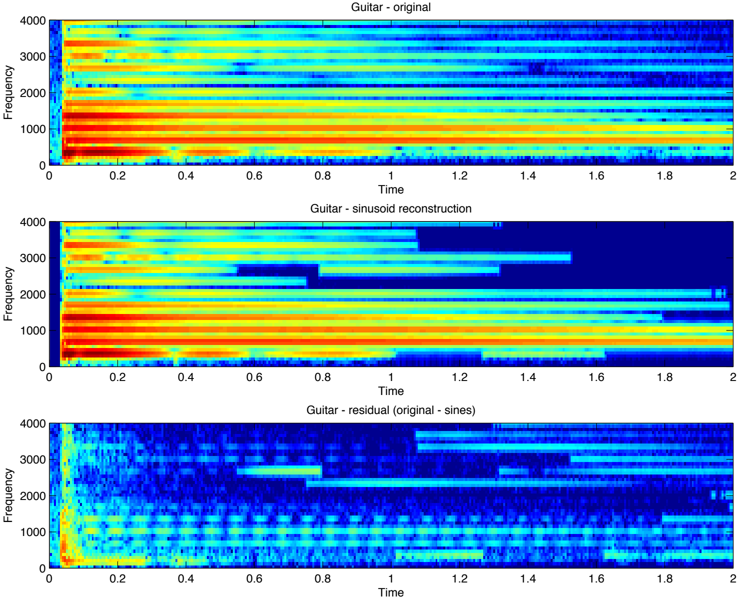
synthesized sound

synthesized sound
with window

Synthesis: Putting it all together



Sin + noise - example



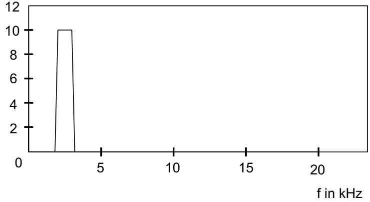
Examples

	Orig	Sin	Residual	SMS	Transformed
flute					
clarinet					
guitar					
drums					
water					

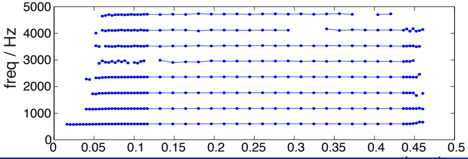
Applications

Loads of applications

- Similar to other TF analysis-synthesis techniques
- but parameters more convenient for some transformations
  - can treat spectral shape independent from harmonics
  - can treat different tracks independently
- Filtering with arbitrary time resolution

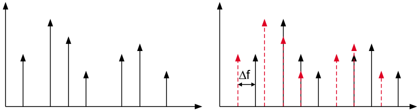


- Timescale modification

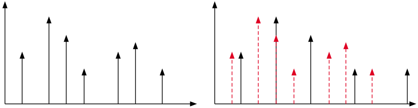


Applications: Frequency transformations

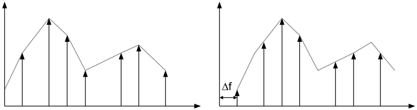
- Partial-dependent frequency scaling



  - e.g. pseudo inharmonicities of higher partials in piano sound
- Frequency stretching: $\hat{\omega}_k[n] = p * \omega_k[n]$



  - e.g. pitch-shift without preserving timbre
- Spectral shape shift



- Quantize pitch (autotune)

Still more applications

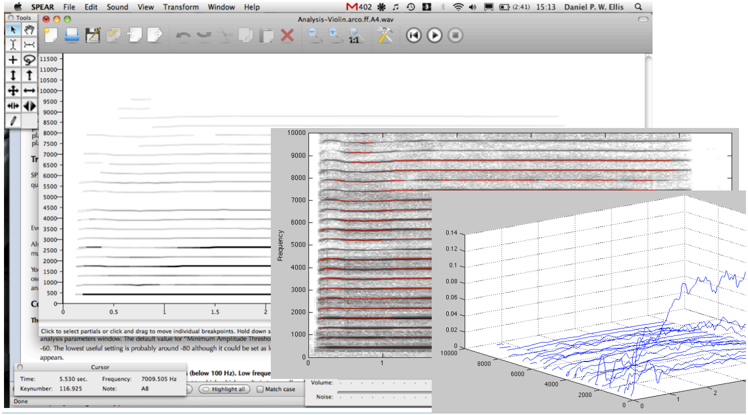
- Effects we've seen before:

Vibrato	modulate $\omega_k[n]$ with LFO
Tremolo	modulate $a_k[n]$ with LFO

- Hoarseness: boost residual relative to sinusoidal components
- Morphing between sounds
- Gender change: shift pitch and formants separately
- Singing voice synthesis/conversion (Vocaloid)
- Separate transients from steady-state harmonics
- ...

Tools: SPEAR

From Michael Klingbeil: http://www.klingbeil.com/spear/



Bringing it all together

PVOC $X[k,n] = A[k,n]\; e^{j\omega[k,n]}$

- Magnitude and phase of fixed oscillators

Source-filter $X[k,n] = E[k,n]\; H[k,n]$

- Filter captures spectral shape (smoothed $A[k,n]$)
- Source is whatever is left - typically pulse train corresponding to harmonic series

Sinusoids $X[k,n] = A_k[n] \cos(\omega_k[n]) + E[k,n]$

- Organize input into oscillators with time-varying frequency (harmonic tracks)
- Sinusoid magnitudes $A_k[n]$ approximate spectral shape
- Frequencies $\omega_k[n]$ encode source information

Reading

DAFX Chapter 10 - Spectral Processing