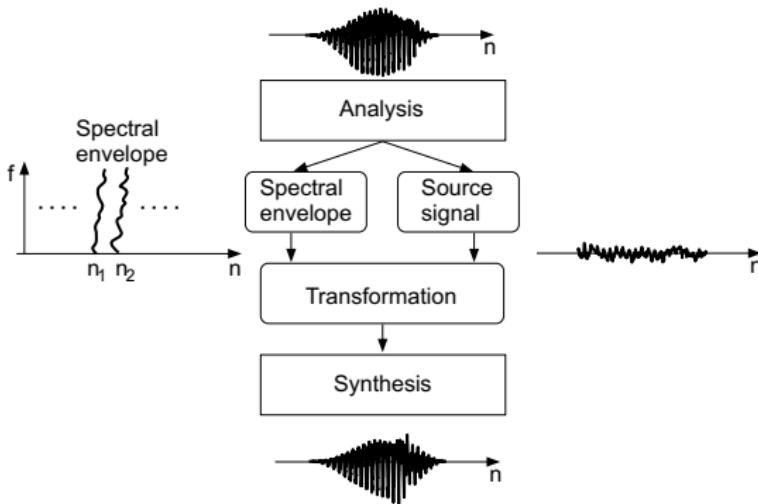


E85.2607: Lecture 8 – Source-Filter Processing

Source-filter analysis/synthesis

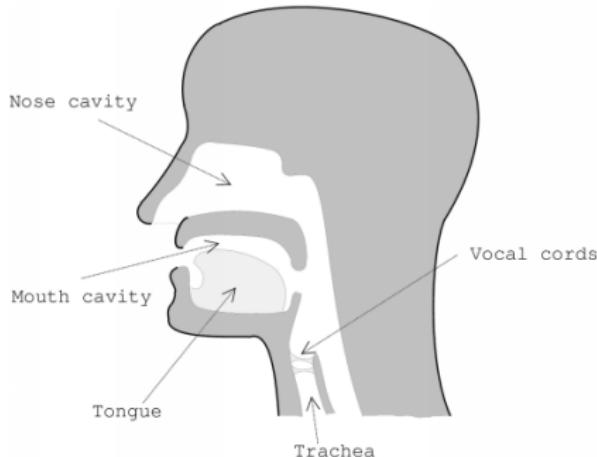


- Separate
 - Source/excitation fine time/frequency structure (e.g. pitch)
 - Filter broad spectral shape (resonances)
- Similar to subtractive synthesis
- Satisfying physical interpretation for real-world signals
- Easier to make sense of than e.g. phase

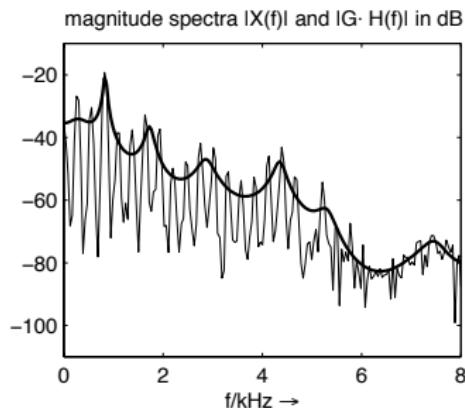
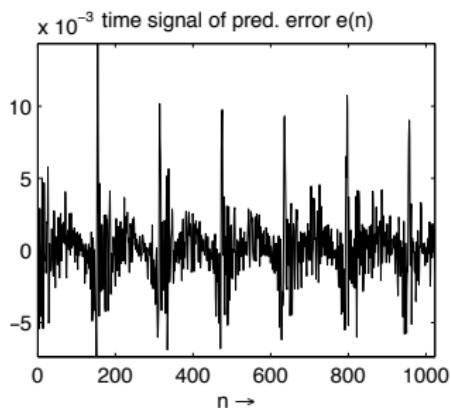
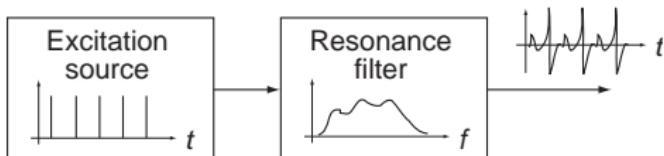
Human speech production

Reasonable approximation to speech signals:

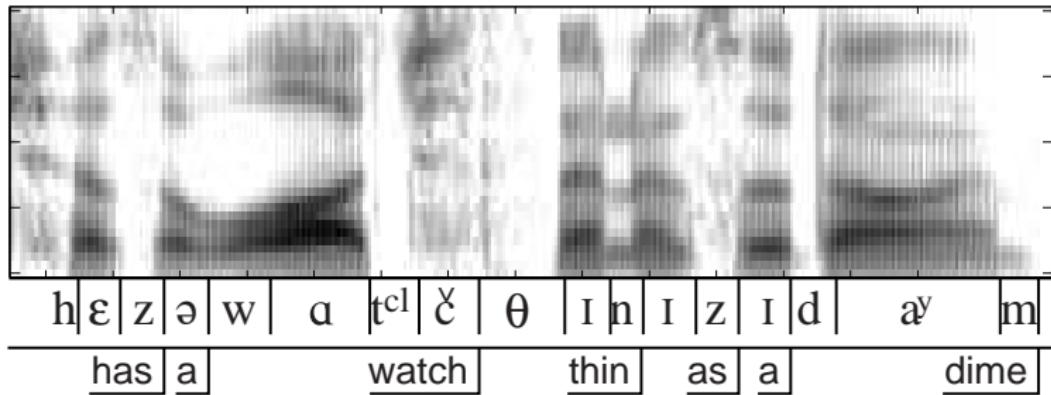
- Source is oscillation of vocal chords
 - e.g. normal speech (varying pitches) vs whispering
- Filtered by vocal tract (throat + tongue + lips)
 - e.g. “oooh” vs “aaah”
 - resonances = **formants**
- Both are time-varying



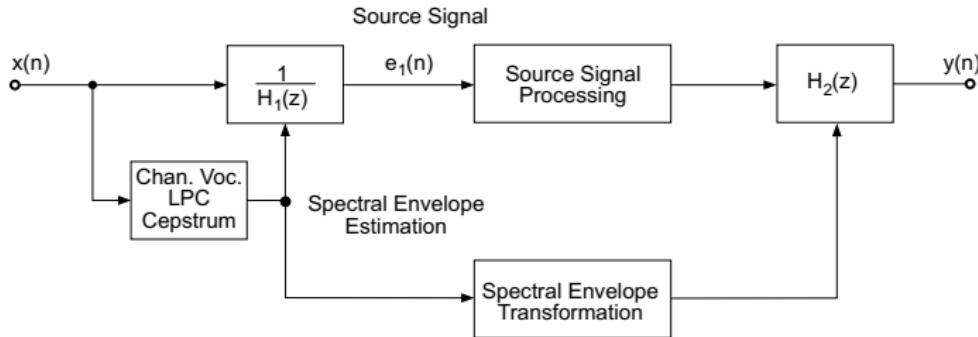
Source filter model



Formants in speech



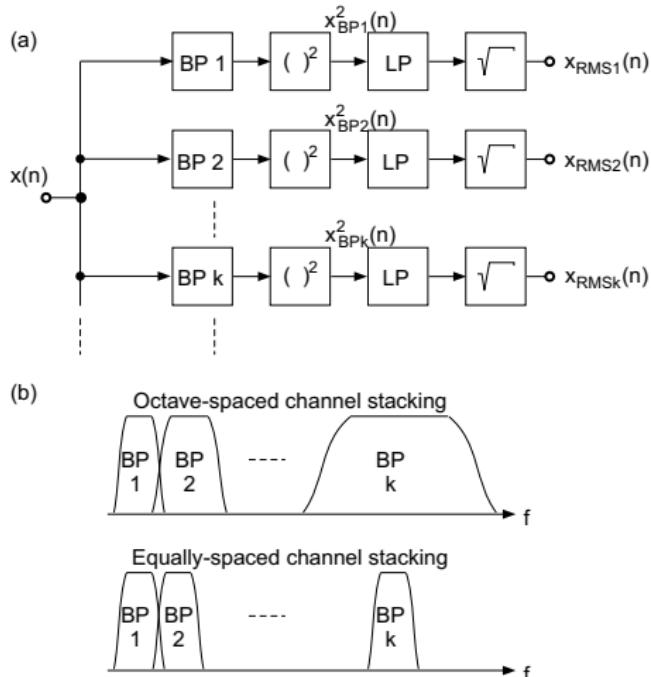
How to separate the source and filter?



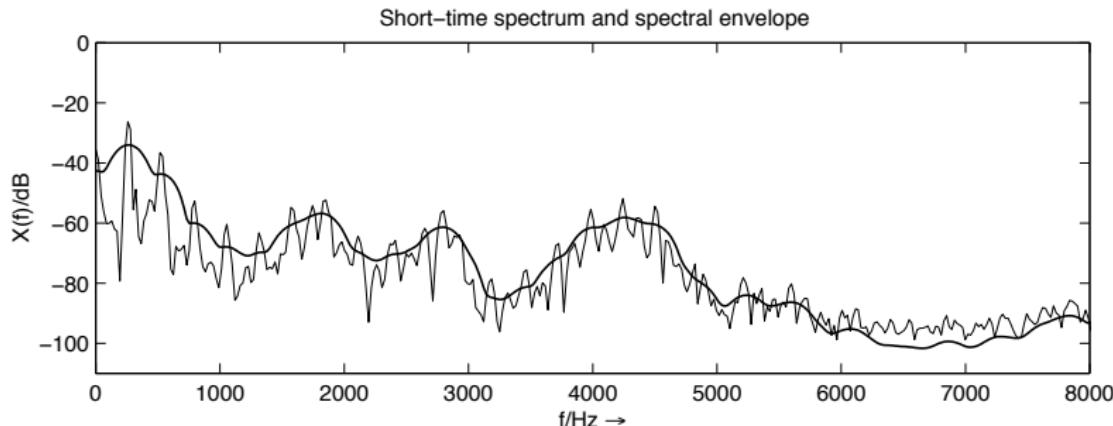
- Short-time analysis
- For each frame, estimate spectral envelope (filter response)
 - ① Channel vocoder (frequency-domain)
 - ② Linear Predictive Coding (LPC) (time-domain)
 - ③ Cepstral analysis
- Source signal is what's left over (residual) after “whitening”

Channel vocoder

- Wideband STFT filterbank
 - but using relatively few filters
 - Linearly spaced with equal bandwidth (STFT)
 - Logarithmically spaced (constant-Q filter bank)
- Take RMS energy in each frequency band



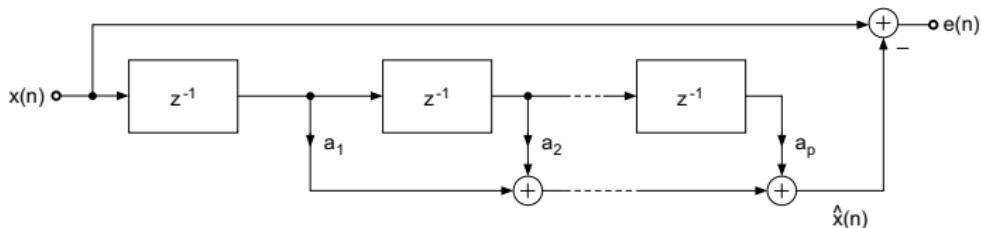
Channel vocoder using FFT



- Lowpass filter magnitude of each STFT frame
 - i.e. filter columns of the spectrogram

Linear predictive coding

- Predict next input sample as linear combination of previous samples



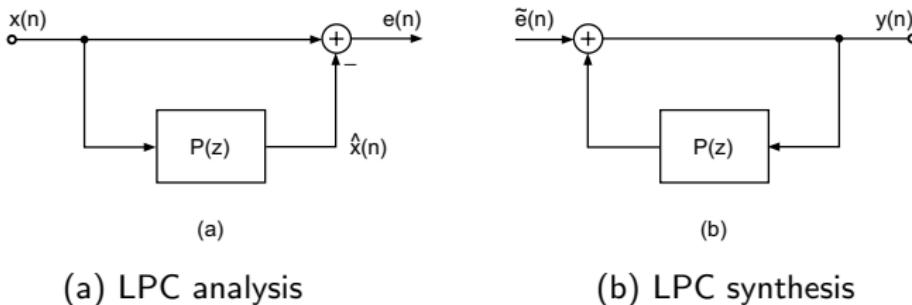
- Filter is described by a few filter coefficients for each frame

$$x^m[n] \approx \hat{x}[n] = \sum_{k=1}^p a_k x[n - k]$$

- Excitation is what's left after filtering (residual aka prediction error)

$$e[n] = x[n] - \hat{x}[n] = x[n] - \sum_{k=1}^p a_k x[n - k]$$

LPC analysis/synthesis



(a) LPC analysis

(b) LPC synthesis

- $P(z)$ is just an FIR filter: $P(z) = \sum_{k=1}^p a_k z^{-k}$
- Excitation is still a filtered version of the input:

$$E(x) = X(z)(1 - P(z))$$

- For synthesis, pass (approximate) excitation through the inverse filter:

$$Y(z) = \tilde{E}(z)H(z)$$

$$H(z) = \frac{1}{1 - P(z)}$$

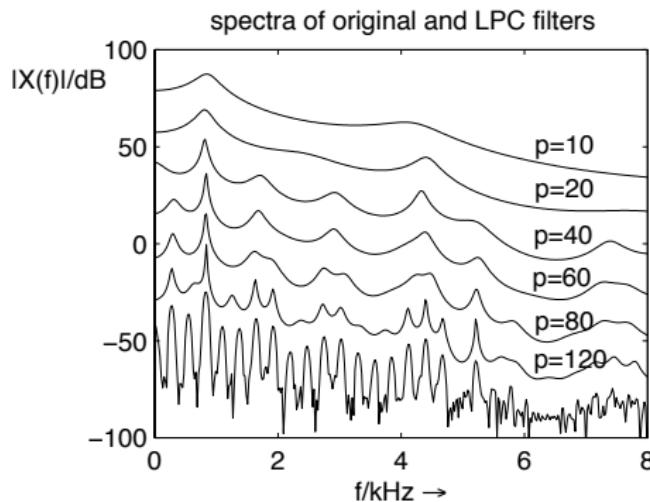
- all-pole “autoregressive” (AR) modeling

LPC - varying filter order

- LPC filter $H(z)$ models the spectrum of $x[n]$
- Minimizing the energy of the residual $e[n]$ gives optimal coefficients

$$\{a_k\} = \underset{\mathbf{a}_k}{\operatorname{argmin}} \sum_n \left(x[n] - \sum_k a_k x[m-k] \right)^2$$

- The approximation improves with increasing filter order p



Estimating LPC parameters

- Set derivative of $\sum_n e^2[n]$ w.r.t. a_k zero and solve for a_k :

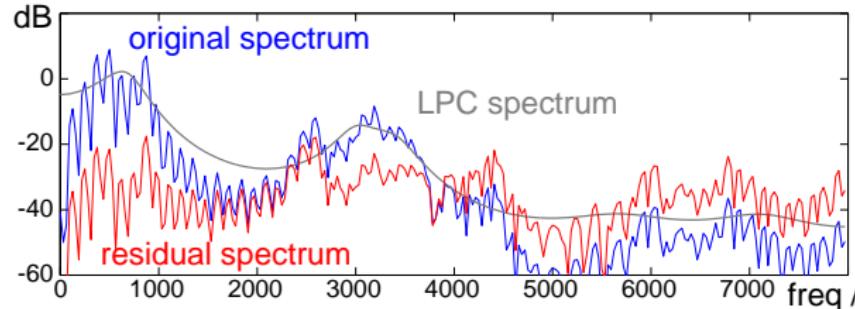
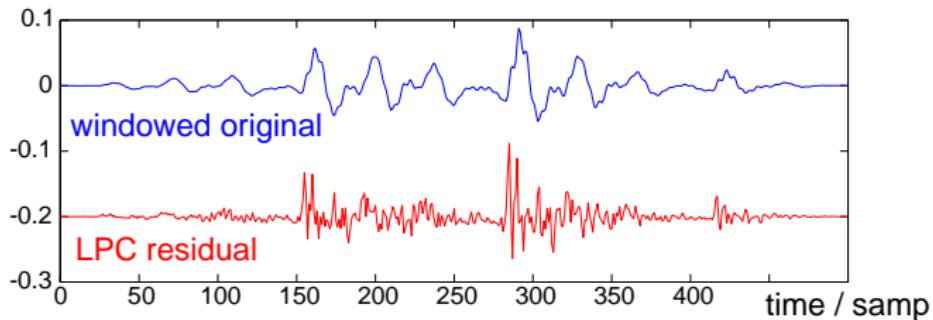
$$\frac{\partial}{\partial a_k} \sum_n e^2[n] = 0$$

- End up with p linear equations involving autocorrelations of x :

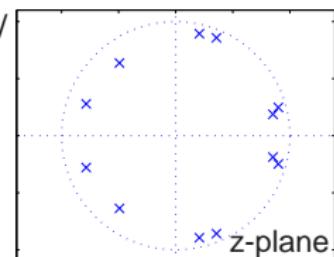
$$\sum_m x[m]x[m - k] = \sum_i a_k \sum_m x[m - i]x[m - k]$$

- Solve using **Levinson-Durbin recursion**

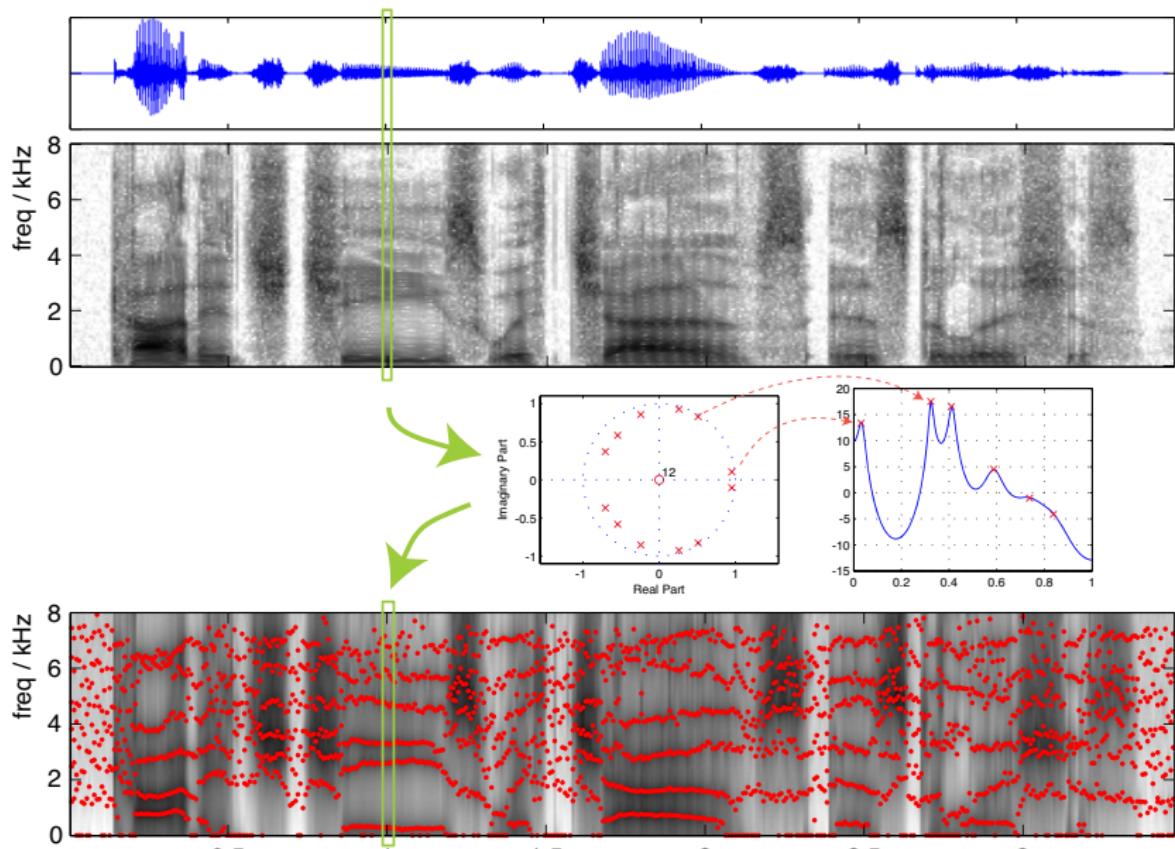
LPC example



Filter poles



Short-time LPC analysis



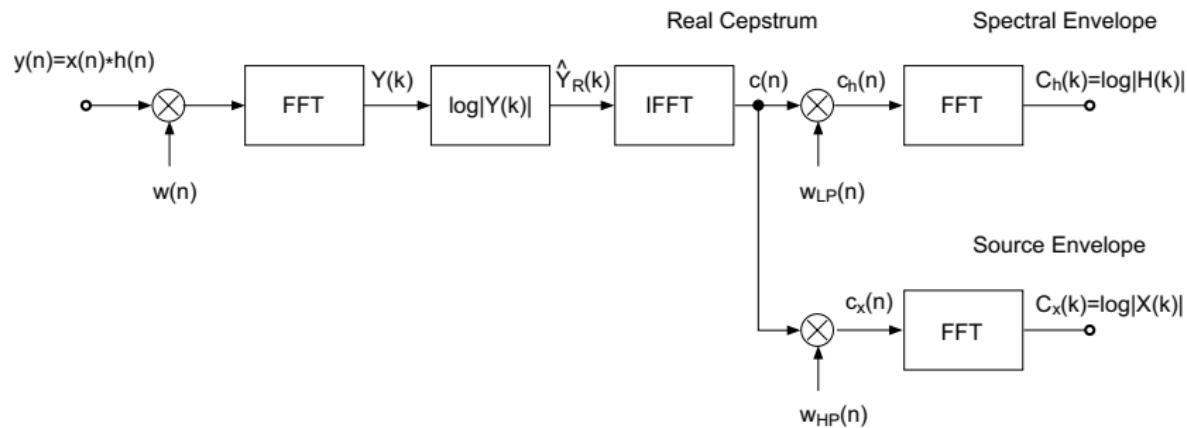
Cepstral analysis

- cepstrum = `String.reverse("spec") + "trum"`
 - Entire lexicon of funny anagrams
- Insight: source and filter **add** in the **log** spectral domain

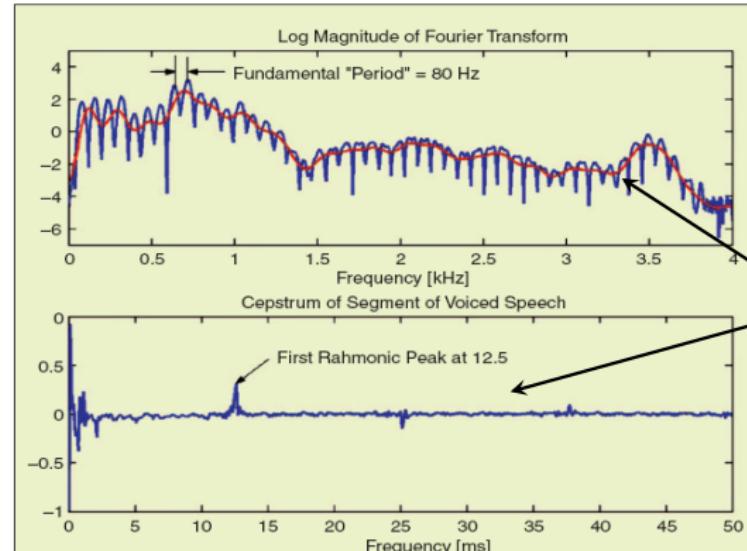
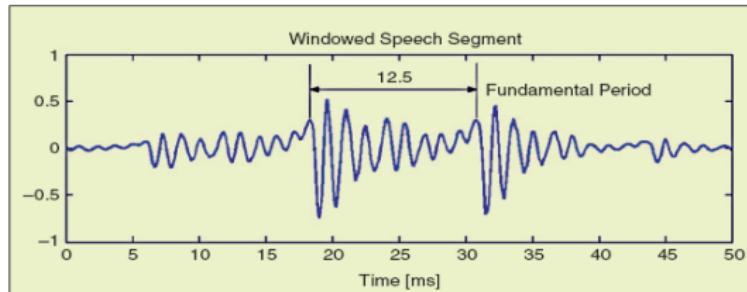
$$X(z) = E(z)H(z)$$

$$\log X(z) = \log E(z) + \log H(z)$$

- Makes them easy to separate



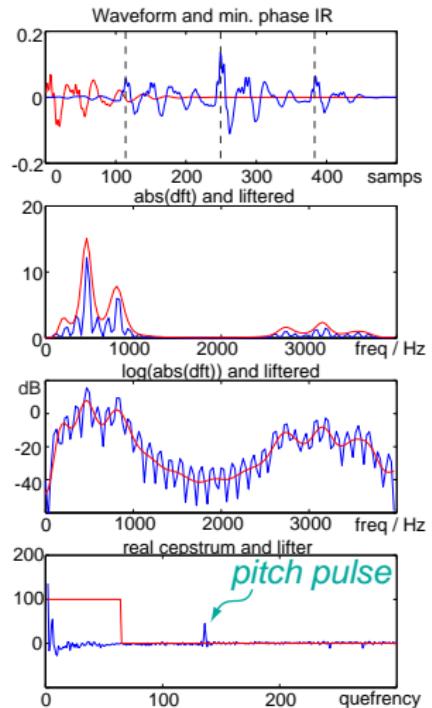
Liftering example



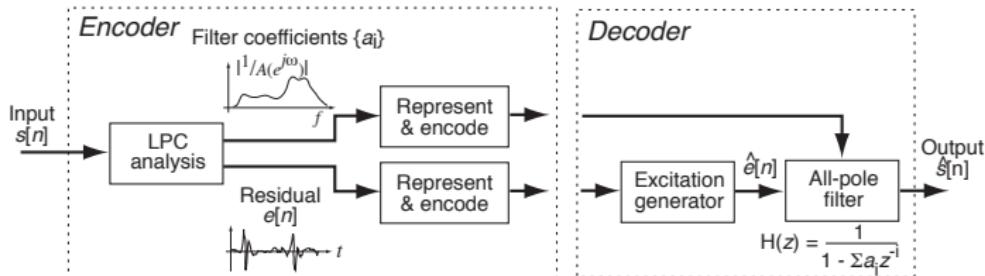
By low-pass "liftering" the cepstrum we obtain the spectral envelope of the signal

Liftering example 2

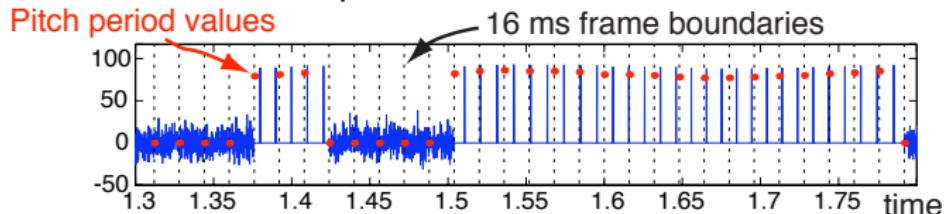
- Original waveform has excitation fine structure convolved with resonances
- DFT shows harmonics modulated by resonances
- Log DFT is sum of harmonic ‘comb’ and resonant bumps
- IDFT separates out resonant bumps (low quefrency) and regular, fine structure (‘pitch pulse’)
- Selecting low-n cepstrum separates resonance information (deconvolution / ‘liftering’)



Applications - Speech coding

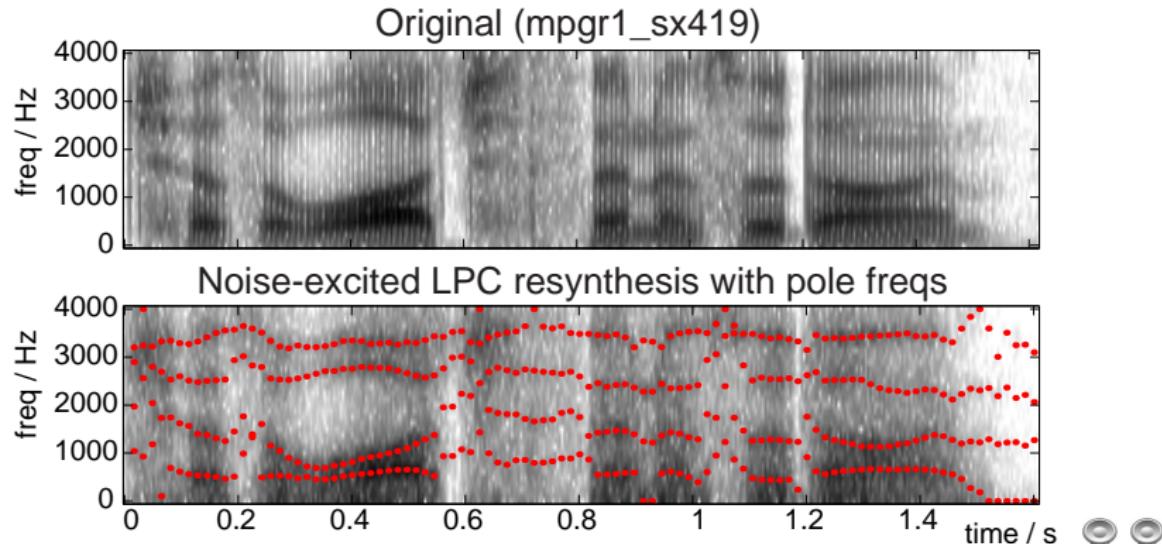


- Low bitrate speech codec used in cell phones is based on LPC
- Quantize LPC filter parameters, use crude approximation to residual
 - Many different ways to represent filter params: prediction coefficients $\{a_k\}$, roots of $1 - P(z)$, line spectral frequencies
 - Switch between noise and pulse train for excitation



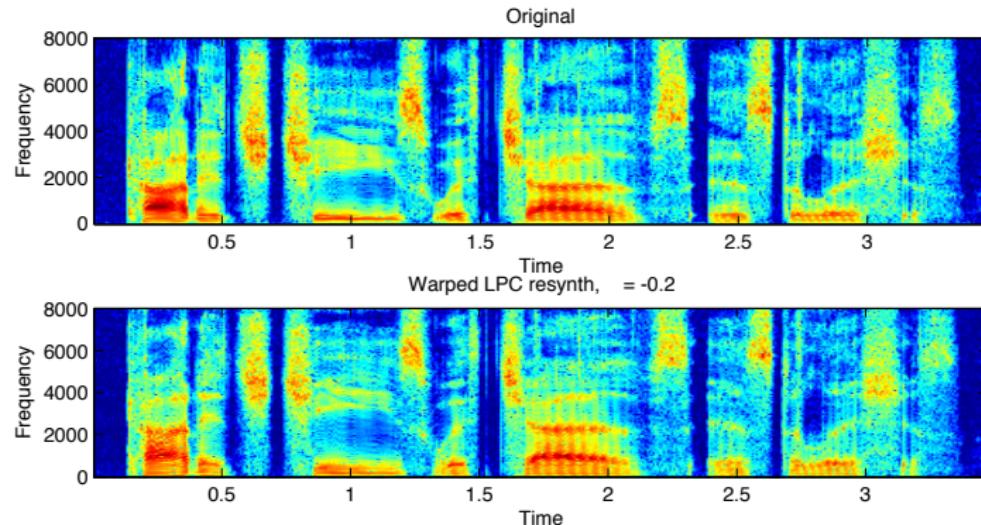
- Use codebook of excitations (CELP: Code Excited Linear Prediction)

Applications - Cross-synthesis/Vocoding



- Reconstruct using excitation from one sound and filter from another
- Whisperization: replace excitation with white noise

Still more applications



- Process formants independent of pitch
 - Pitch-shifting while preserving formants
 - Shift formants while preserving pitch
 - <http://www.ee.columbia.edu/~dpwe/resources/matlab/polewarp/>
- Voice transformation
- Pitch-analysis

Reading

DAFX 9.1 – 9.3 - Source-Filter Processing