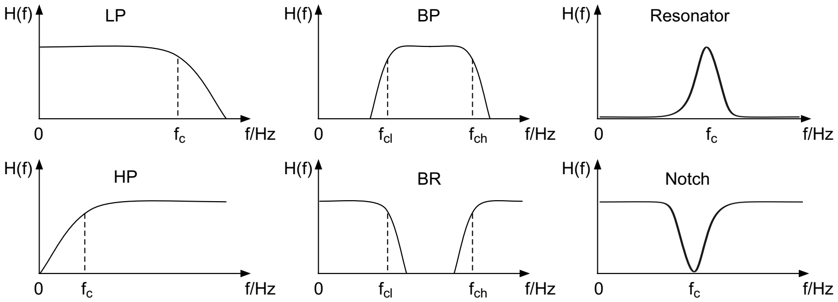
E85.2607: Lecture 2 – Filters

Basic filters



f_c cutoff or center frequency

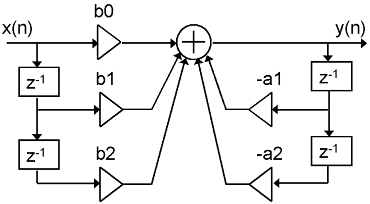
f_b bandwidth

Q "quality factor" $Q = \frac{f_b}{f_c}$

IIR filter structures: Digital biquad

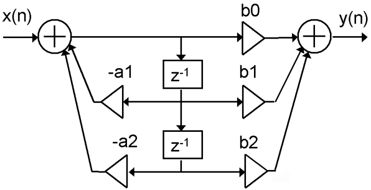
$$H(z) = \frac{B(z)}{A(z)} = \frac{b_0 + b_1\, z^{-1} + b_2\, z^{-2}}{1 + a_1\, z^{-1} + a_2\, z^{-2}}$$

Direct form 1



zeros on left, poles on right

Direct form 2



canonical (minimum delays)

Building blocks: Allpass filter

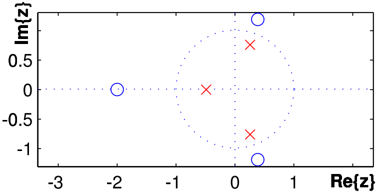
$$H(z) = \frac{B(z)}{z^{-N}B(z^{-1})}$$
$$= \frac{b_0 + b_1\,z^{-1} + \ldots + b_{N-1}\,z^{N-1} + b_N\,z^{-N}}{b_N + b_{N-1}\,z^{-1} + \ldots + b_1\,z^{N-1} + b_0\,z^{N}}$$

Where are the poles and zeros?

Building blocks: Allpass filter

$$H(z) = \frac{B(z)}{z^{-N} B(z^{-1})}$$
$$= \frac{b_0 + b_1 z^{-1} + \ldots + b_{N-1} z^{N-1} + b_N z^{-N}}{b_N + b_{N-1} z^{-1} + \ldots + b_1 z^{N-1} + b_0 z^{N}}$$

Where are the poles and zeros?



What about the frequency response?

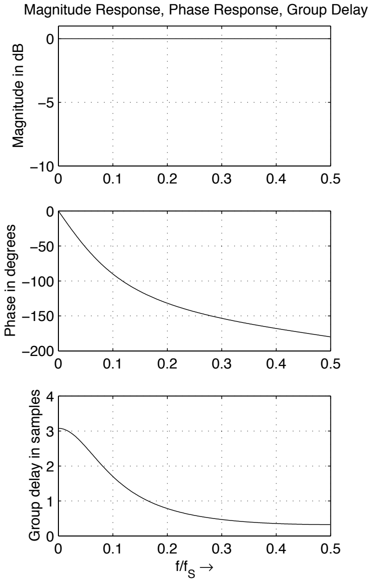
Parametric first order allpass filter

$$A(z) = \frac{c + z^{-1}}{1 + c\, z^{-1}}$$

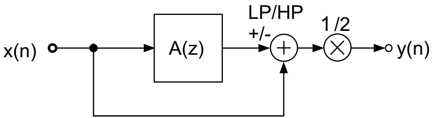
$$c = \frac{\tan(\pi f_c / f_s) - 1}{\tan(\pi f_c / f_s) + 1}$$

Flat magnitude response, but introduces phase distortion

Group delay $= -\frac{\partial}{\partial \omega} \angle H(e^{j\omega})$



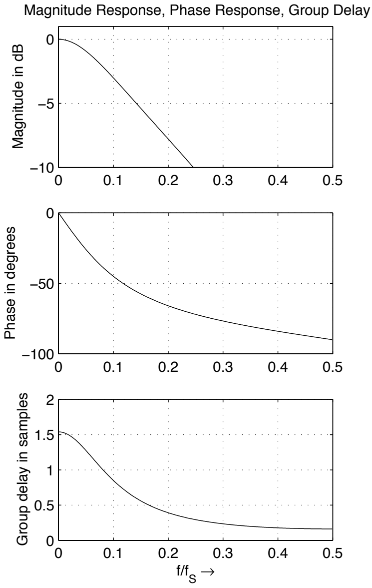
Tunable lowpass/highpass filters

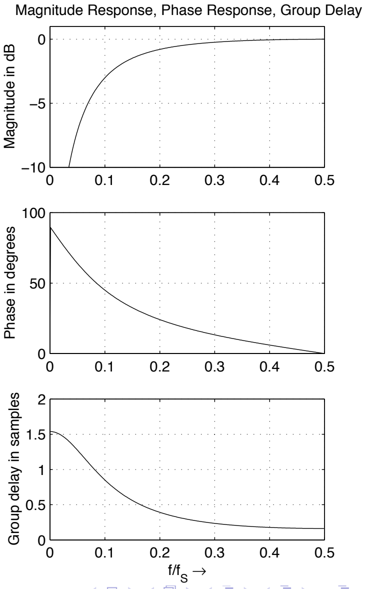


$$
\begin{aligned}
H(z) &= \frac{1}{2}(1 \pm A(z)) \quad \text{(LP/HP +/-)} \\
A(z) &= \frac{z^{-1} + c}{1 + cz^{-1}} \\
c &= \frac{\tan(\pi f_c/f_s) - 1}{\tan(\pi f_c/f_s) + 1},
\end{aligned}
$$

Why does this work?

LP/HP frequency response



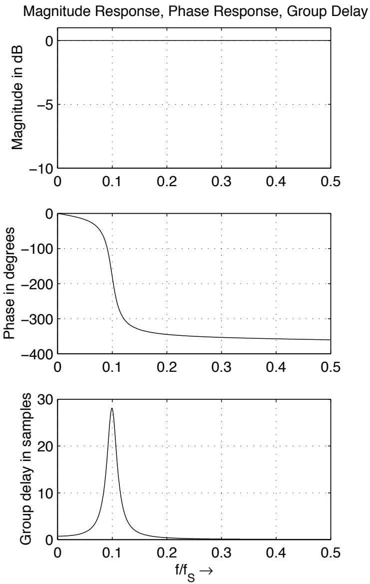


Second order allpass

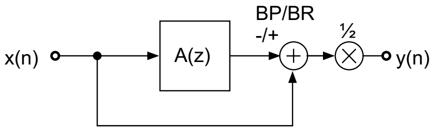
$$A(z) = \frac{-c + d(1-c)z^{-1} + z^{-2}}{1 + d(1-c)z^{-1} - cz^{-2}}$$
$$c = \frac{\tan(\pi f_b/f_s) - 1}{\tan(\pi f_b/f_s) + 1}$$
$$d = -\cos(2\pi f_c/f_s).$$

c controls bandwidth

d controls cut-off



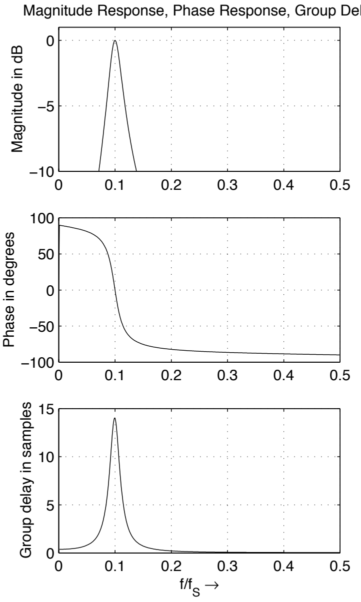
Tunable bandpass/bandreject filters

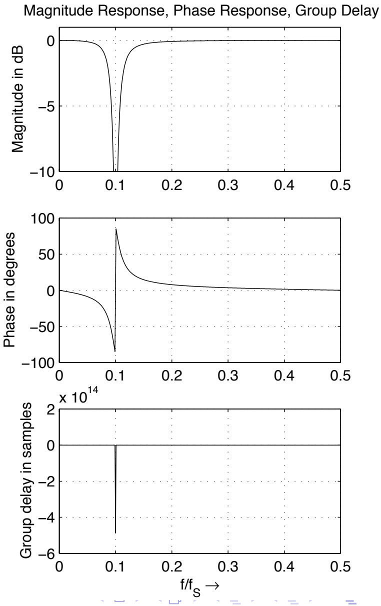


$$
\begin{aligned}
H(z) &= \frac{1}{2}\left[1 \mp A(z)\right] \quad \text{(BP/BR -/+)} \\
A(z) &= \frac{-c + d(1-c)z^{-1} + z^{-2}}{1 + d(1-c)z^{-1} - cz^{-2}} \\
c &= \frac{\tan(\pi f_b/f_s) - 1}{\tan(2\pi f_b/f_s) + 1} \\
d &= -\cos(2\pi f_c/f_s),
\end{aligned}
$$

typo

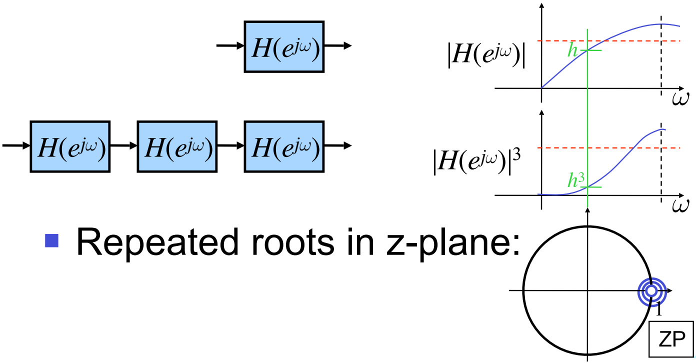
BP/BR frequency response





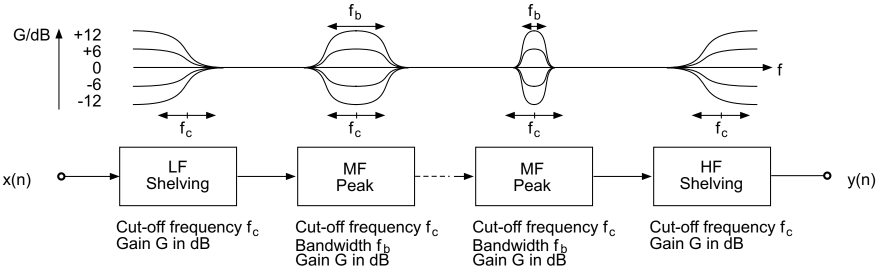
Cascading filters

Make the frequency response sharper by passing the signal through the same filter multiple times



- Repeated roots in z-plane:

Equalizers



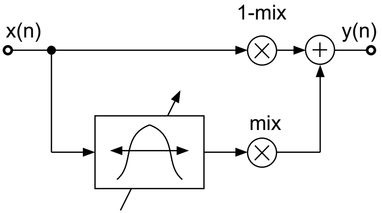
- Chain of simple filters to shape spectrum

  Low/high shelf $H(z) = 1 + (10^{G/20} - 1)\, H_{LP/HP}(z)$

  Peak $H(z) = 1 + (10^{G/20} - 1)\, H_{BP}(z)$

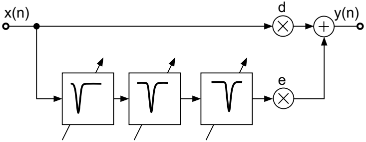
- Parameters: Gain, center frequency, bandwidth (Q)
- Applications: mixing, compensate for room acoustics, ~~genre controls~~

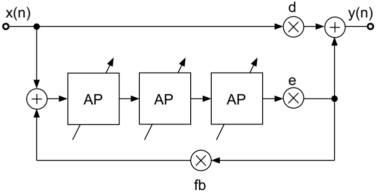
Time-varying filters: Wah



- Bandpass filter with time varying center frequency
- Mimics formant resonances in speech

Time-varying filters: Phaser





- Notches with time varying center frequency
- Controlled by a low frequency oscillator

Reading

- *Introduction to Digital Filters*
  - Elementary Audio Digital Filters
- *DAFX*, Chapter 2 (if you have it)