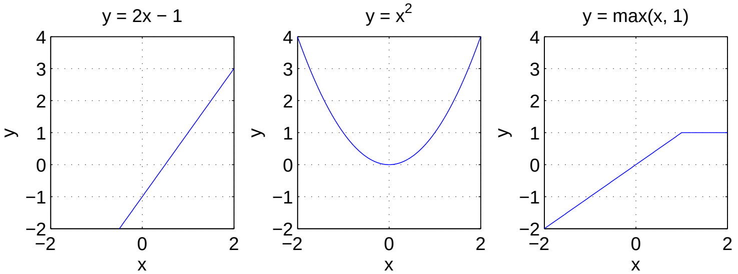
E85.2607: Lecture 11 – Nonlinear Processing

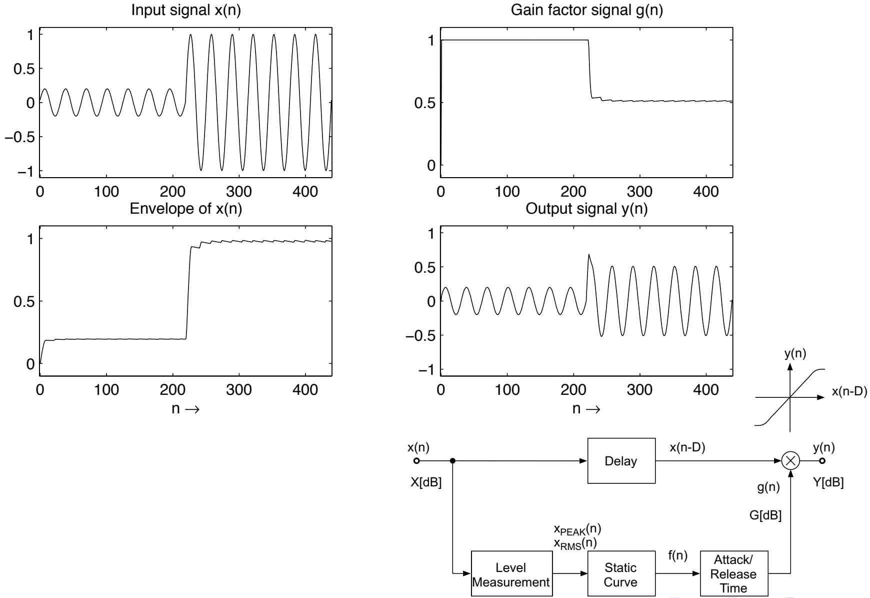
Nonlinear systems



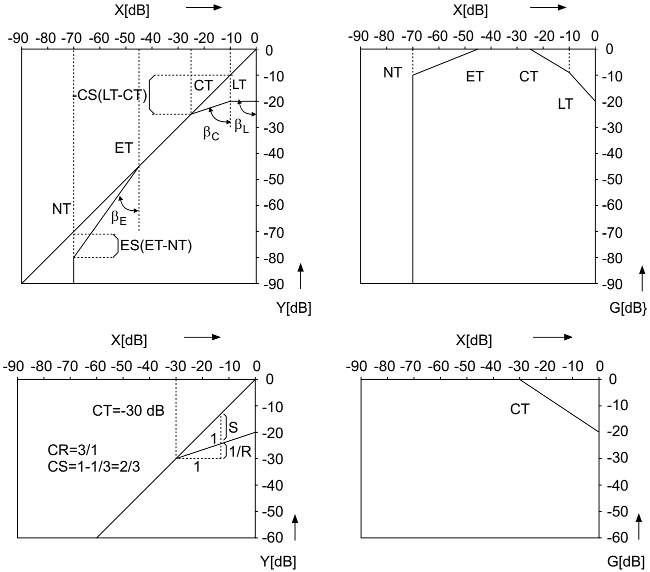
Which one of these is not like the other?

- Given input output pairs (x_1, y_1), (x_2, y_2)
- Linear system: input $ax_1 + bx_2$ results in $ay_1 + by_2$
- Nonlinear systems...
  - don't obey scaling and superposition
  - are hard to analyze
  - can make interesting sounds

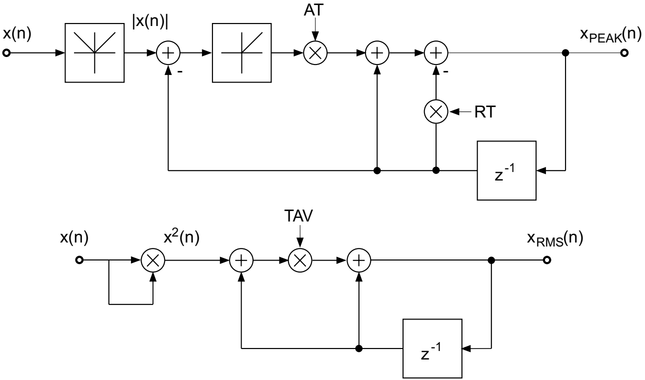
Dynamics processing



Static curves



Level measurement



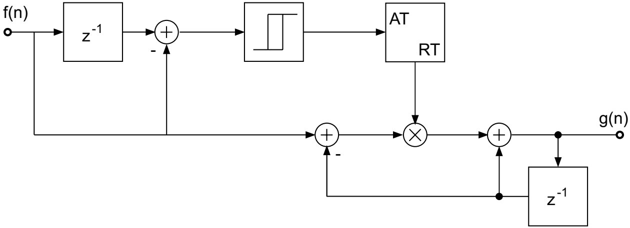
Time constant: $\quad AT = 1 - e^{-2.2/(f_s\ t_{AT})}$

Peak detector $x_{PEAK}[n] \approx x_{PEAK}[n-1] - \max(|x[n]| - y_{PEAK}[n-1], 0)$

RMS detector $y_{RMS} \approx x^2 \rightarrow \text{LPF} \rightarrow \text{sqrt}$

Attack/release time adjustment

Check if $f[n]$ (gain) is increasing or decreasing to select time constant

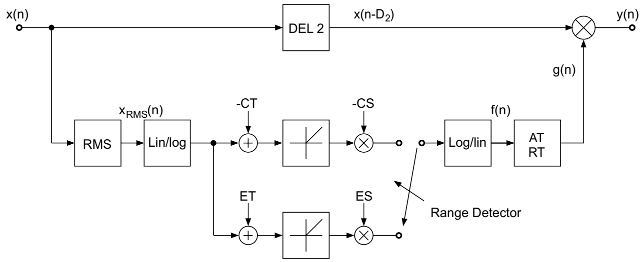


Then pass through LPF as in envelope followers

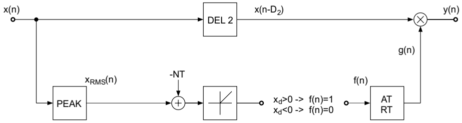
$$g[n] = \text{TC}\, f[n] + (1 - \text{TC})\, g[n-1]$$

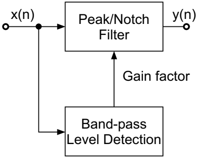
Dynamics processors

Limiter/Compressor/Expander Same idea, different parameters



Noise gate Attenuate signal if level is below threshold





De-esser control notch gain using BPF level measurement (2-6 kHz)

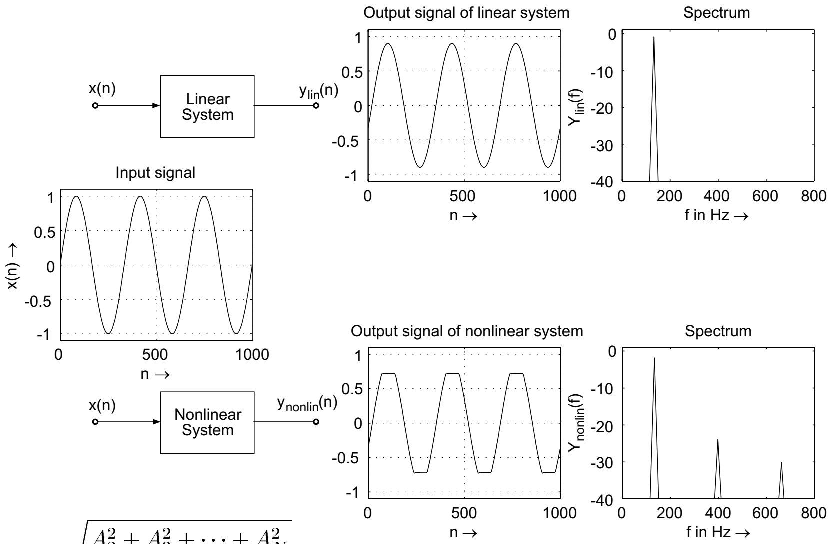
Distortion effects

- Simplest implementation is memoryless
  - Pass each sample through the same nonlinearity e.g. $y[n] = x^2[n]$
- Can get more sophisticated using Volterra modeling

$$y[n] = \sum_{k_1} h_1[k_1]x[n-k_1] + \sum_{k_1}\sum_{k_2} h_2[k_1,k_2]x[n-k_1]x[n-k_2] + \dots$$

- Different nonlinearities have different effects:
  - Modeling amplifiers/simulating
  - Distortion/overdrive
  - Tape saturation

Harmonic distortion



$$\mathrm{THD} = \sqrt{\frac{A_2^2 + A_3^2 + \cdots + A_N^2}{A_1^2 + A_2^2 + \cdots + A_N^2}},$$

Harmonic distortion - why?

- *n*th order polynomial adds *n* harmonics
- Let $x[n] = \cos(\omega n)$:

$$\begin{aligned} x^2[n] &= \cos^2(\omega n) \\ &= \frac{1}{2} + \frac{1}{2}\cos(2\omega n) \\ x^3[n] &= \cos(\omega n)\cos^2(\omega n) = \frac{1}{2}\cos(\omega n) + \frac{1}{2}\cos(\omega n)\cos(2\omega n) \\ &= \frac{3}{4}\cos(\omega n) + \frac{1}{4}\cos(3\omega n) \end{aligned}$$

- Review: Maclaurin series:
  - Any function can be represented as a polynomial (of infinite order)

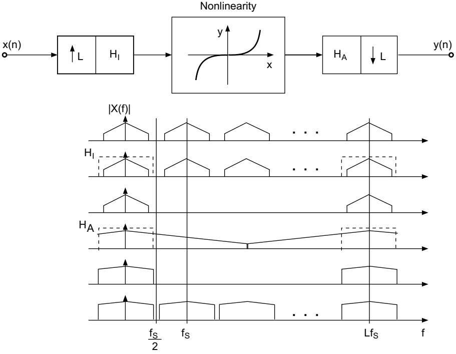
$$f(x) = f(0) + f'(0)x + f''(0)x^2 + f'''(0)x^3 + \ldots$$

  - Size of derivatives determines number of harmonics
- High-frequency harmonics lead to (inharmonic) aliasing...

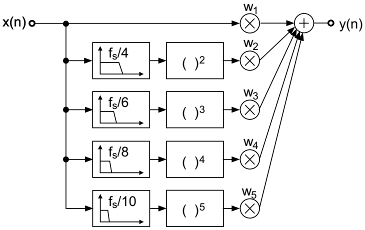
Aliasing BAD!

Two strategies to avoid aliasing distortion:

1. Upsample in front of nonlinearity, then LPF and downsample

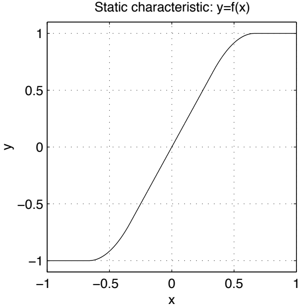


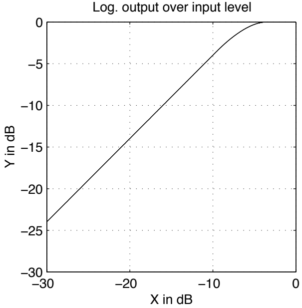
2. Limit input bandwidth



Overdrive and tube simulation

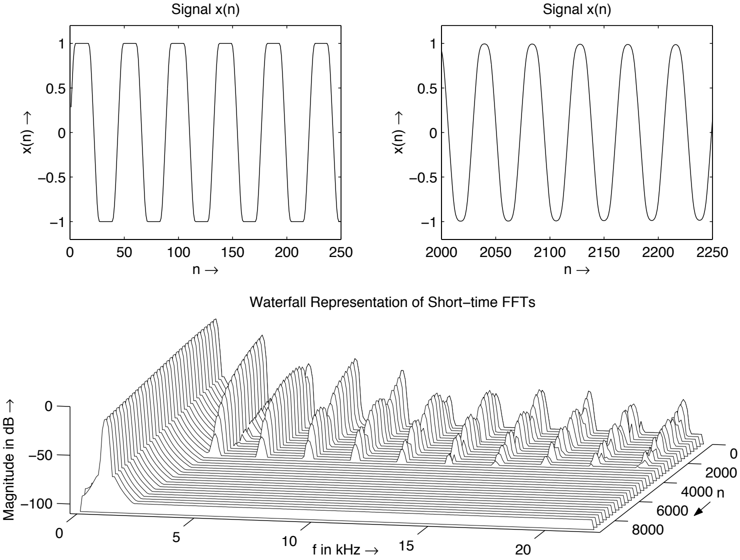
Symmetric soft clipping





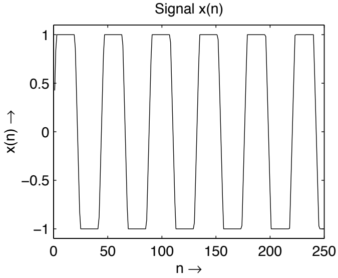
$$f(x) = \begin{cases} 2x & 0 \leq |x| < \frac{1}{3} \\ 1 - \frac{1}{3}(2-3x)^2 & \frac{1}{3} \leq |x| < \frac{2}{3} \\ 1 & \frac{2}{3} \leq |x| \leq 1 \end{cases}$$

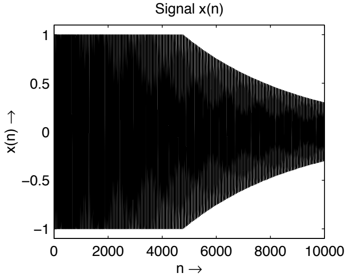
Soft clipping example

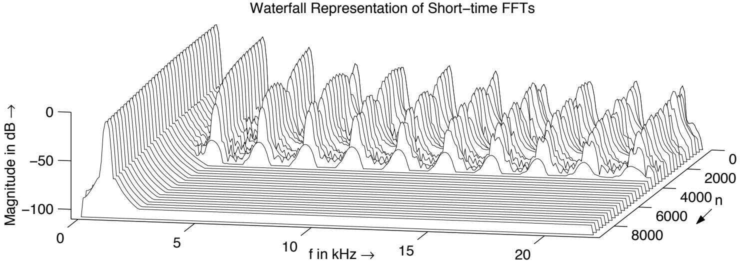


Adds odd harmonics, decay with increasing frequency

Hard clipping example

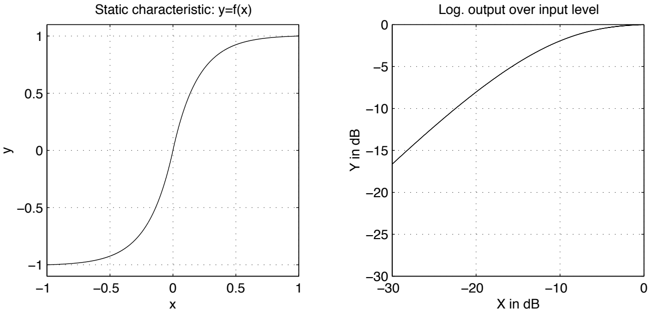






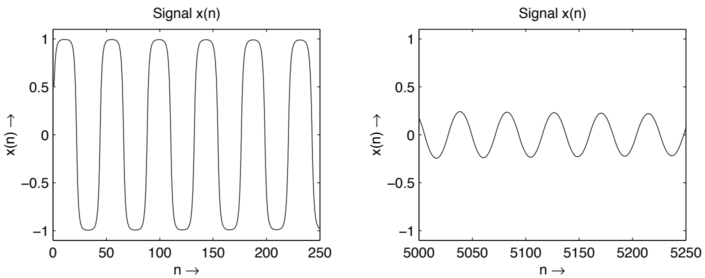
Infinite Maclaurin series → more energy in high frequency harmonics → aliasing

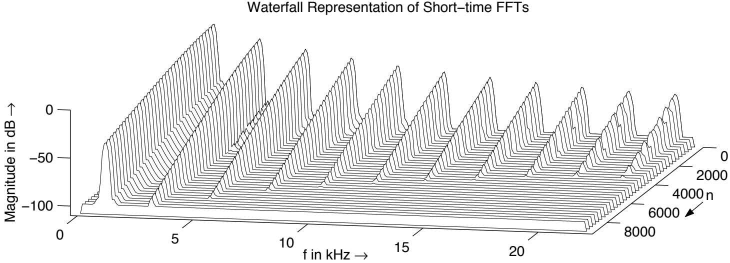
Exponential distortion



$$f(x) = \frac{x}{|x|}\left(1 - e^{x^2/|x|}\right)$$

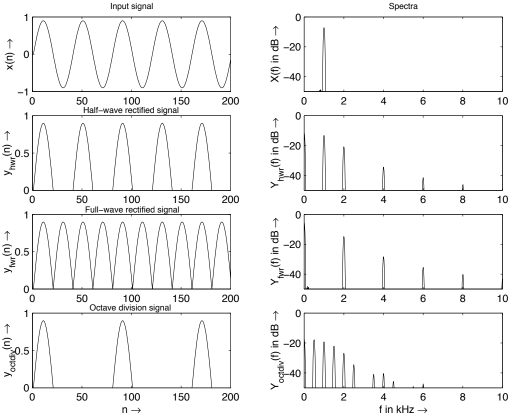
Exponential distortion example





Adds odd harmonics, decay with increasing frequency

More nonlinear effects



Octave effect

Simple nonlinearities add harmonics

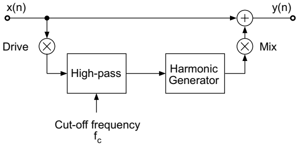
Exciter

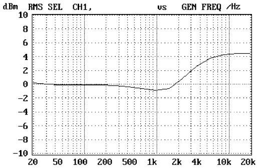
Increase "brightness" without EQ

Enhancer

More sophisticated EQ + harmonic generation

Idea in all cases is to generate subtle distortion





Reading

DAFX Chapter 5 - Nonlinear Processing