E85.2607: Lecture 11 – Physical Modeling

The physics of music

- Synthesize realistic notes by modeling the mechanical and acoustic behavior of a musical instrument
- Sound produced by waves traveling through some medium
  - Common math for different physical phenomena: gas, solids, EM
- Waves transfer energy without permanent displacement of matter

- e.g. guitar string, cymbal

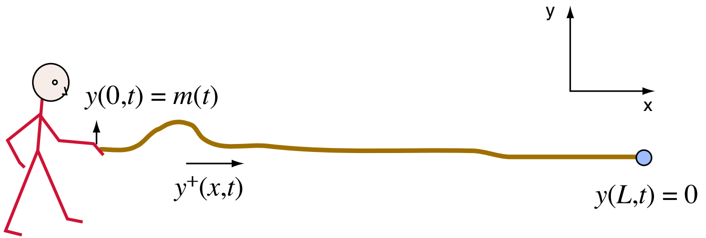
Some scary math: The wave equation

- Lossless string in a 1-D medium with displacement $y(x,t)$:

$$c^2 \frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}$$

curvature *acceleration*

$c = \sqrt{\frac{K}{\varepsilon}}$ (wave speed), K = string tension, ε = density

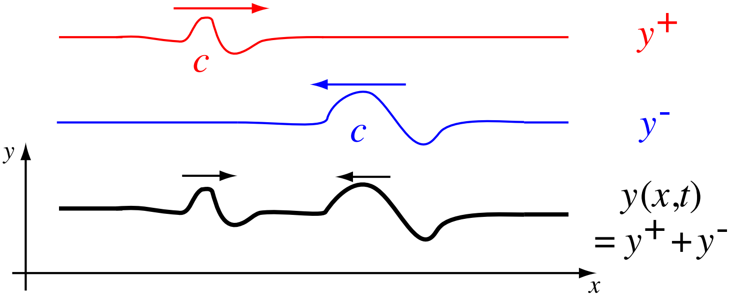


Solving the wave equation

- d'Alembert's solution (1747):

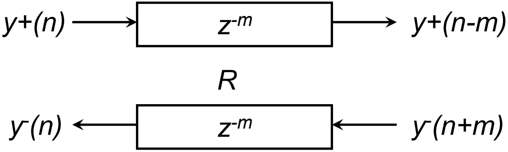
$$y(x,t) = y^{+}(x-ct) + y^{-}(x+ct)$$

- Sum of left-moving (y^{+}) and right-moving (y^{-}) traveling waves
- Shape doesn't change (set by initial conditions)



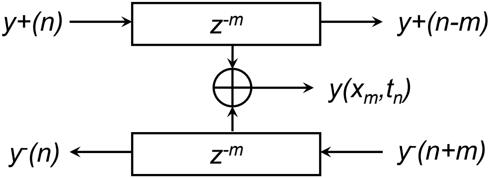
Digital waveguides

- Represent each traveling wave using a delay line



  - String length determines length of delay line m
  - wave impedance R

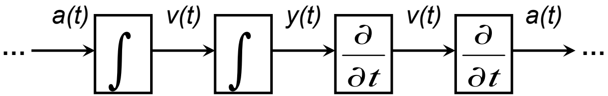
- Compute solution to wave equation by sampling delay line and summing contribution of each traveling wave



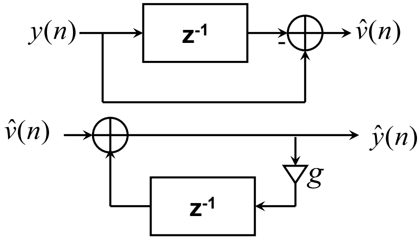
Physical outputs

- Can work with other physical variables (acceleration, velocity)
- Derived from displacement:

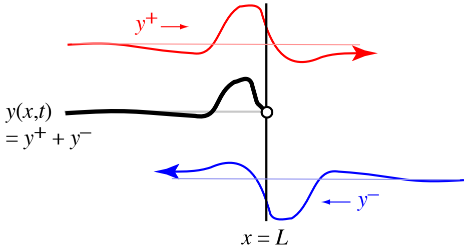
$$v = \frac{\partial y}{\partial t} = \text{velocity} \qquad a = \frac{\partial^2 y}{\partial t^2} = \text{acceleration}$$



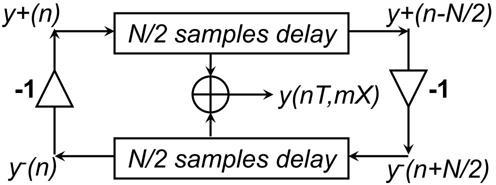
- Implement using digital filters



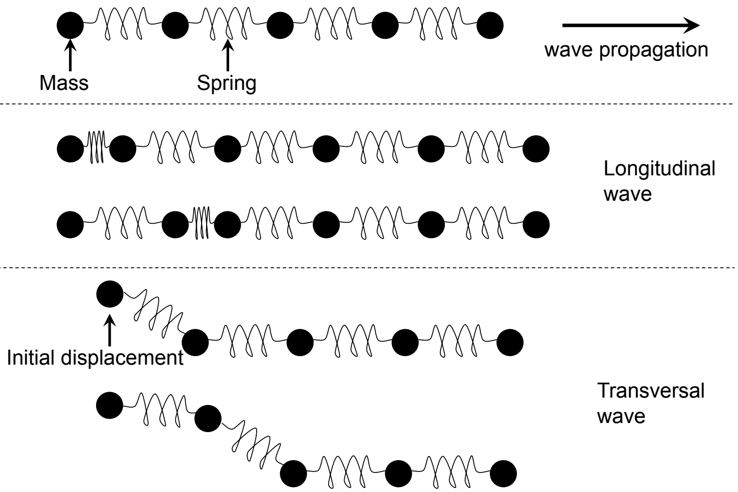
- Waves in musical instruments aren't the Energizer bunny . . .
- Solution to wave equation must match constraints
  - leads to reflections at rigid terminations



- Easy to incorporate into digital waveguide

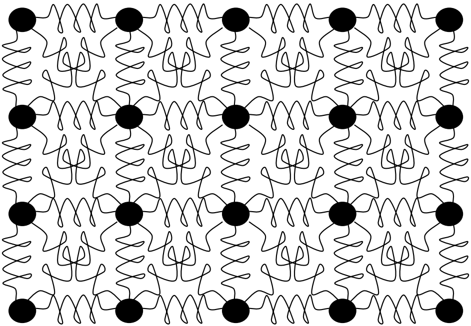


Alternative interpretation: Mass-spring (lumped) model

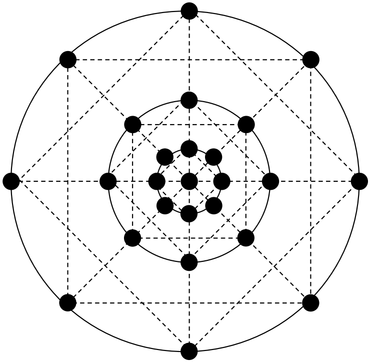


Real strings have losses (e.g. friction within springs) . . .

(More sophisticated: 2-D mass-spring)



A 2-D square surface

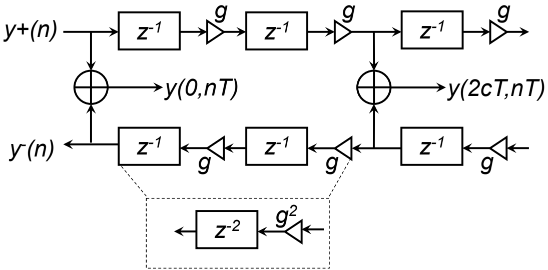


A drum membrane

Lossy 1-D wave

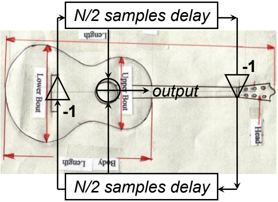
- Simple model: constant loss at each "spring":

$$y(x,t) = g^x y^+(x - ct) + g^{-x} y^-(x + ct)$$

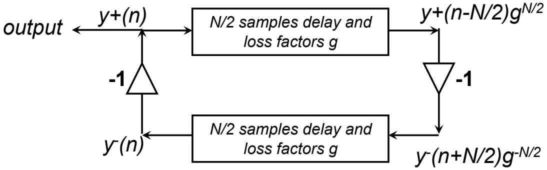


- Consolidate delays and losses where possible
- More realistic: frequency-dependent losses
  - Replace g with filter

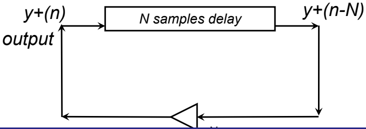
- We almost have a guitar string:



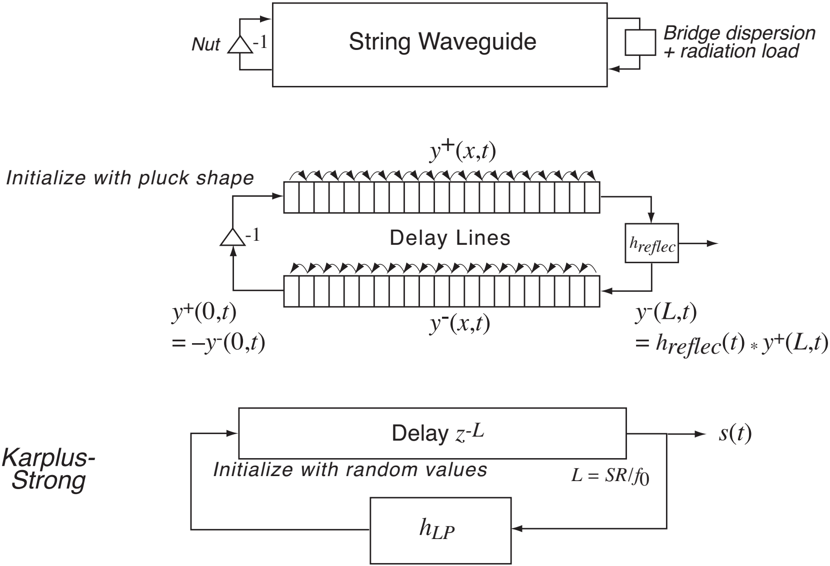
- But, real strings have losses
  - exponentially decaying traveling waves



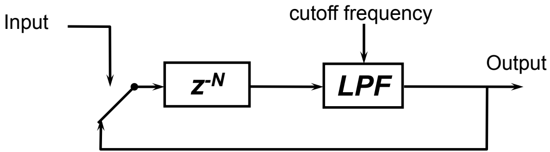
- Because there is no input/output coupling, can consolidate all delays and loses at a single point in the loop:



Digital waveguide review

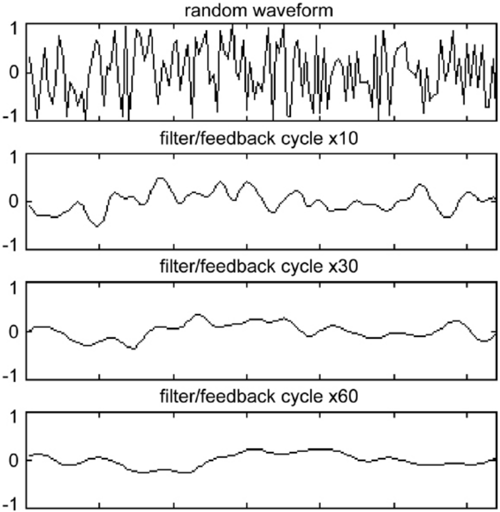


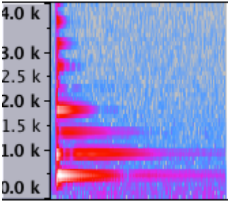
The Karplus-Strong algorithm (1978)



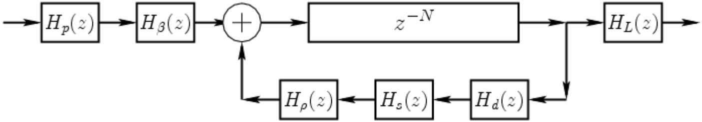
- Initialize the waveguide with random noise
- Noise "wave" will propagate through the loop
  - decaying as it passes through the filter
- Pitch is proportional to length of delay line: $f = \frac{f_s}{N}$
- Does this look familiar?
  - it's just an IIR comb filter ...
    - with an LPF in the loop instead of a fixed gain
  - pass a short noise burst in instead of long term noise

Karplus-Strong examples





Strings: Drums:



$$N \;=\; \text{pitch period } (2\times \text{ string length}) \text{ in samples}$$

$$H_p(z) \;=\; \frac{1-p}{1-p\,z^{-1}} = \text{pick-direction lowpass filter}$$

$$H_\beta(z) \;=\; 1 - z^{-\beta N} = \text{pick-position comb filter, } \beta \in (0,1)$$

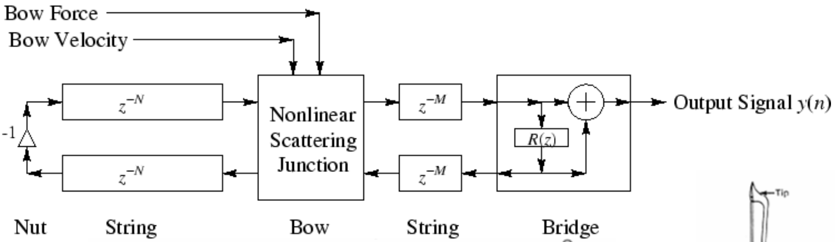
$$H_d(z) \;=\; \text{string-damping filter (one/two poles/zeros typical)}$$

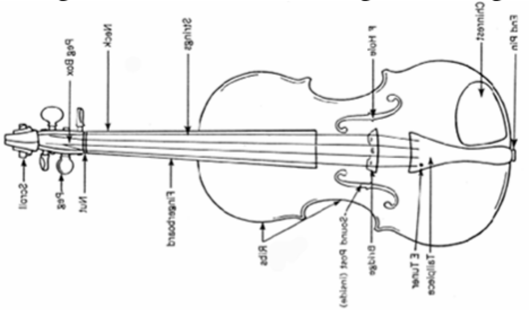
$$H_s(z) \;=\; \text{string-stiffness allpass filter (several poles and zeros)}$$

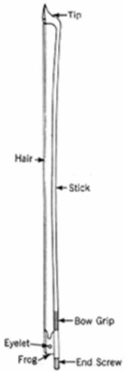
$$H_\rho(z) \;=\; \frac{\rho(N) - z^{-1}}{1 - \rho(N)\,z^{-1}} = \text{first-order string-tuning allpass filter}$$

$$H_L(z) \;=\; \frac{1-R_L}{1-R_L\,z^{-1}} = \text{dynamic-level lowpass filter}$$

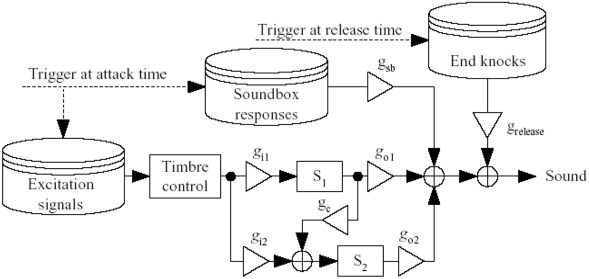
Bowed strings have more complex excitation

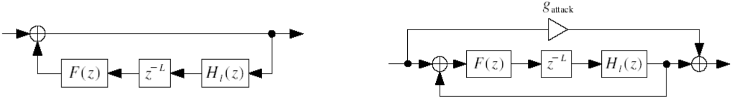






More strings: Clavichord (Valimaki et al, 2004)

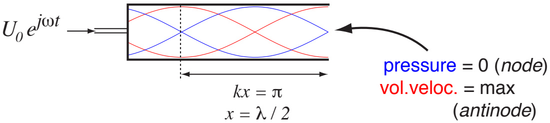


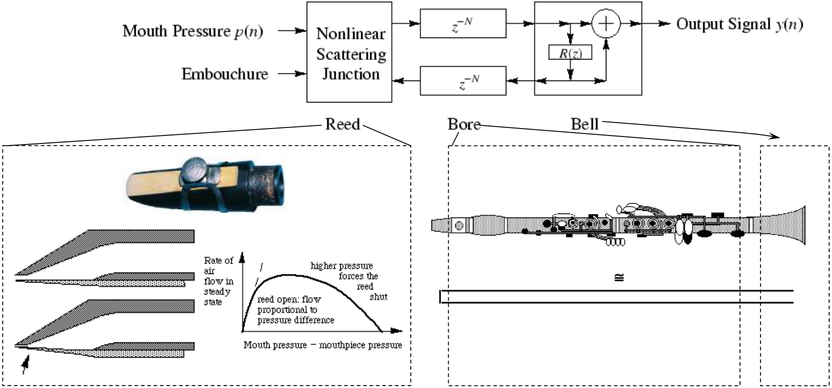


(a)

(b)

Woodwinds (Smith, 1986)





Physical Modeling: The Verdict

- Realistic synthesis of acoustic instruments
- Parameters based on the physical attributes of real instruments
    - less guesswork involved
- But expensive to implement
- Need different model for each instrument

Reading

- J. Smith, Physical Modeling using Digital Waveguides, Computer Music Journal, 1992.
- J. Smith, Virtual Acoustic Musical Instruments: Review of Models and Selected Research, WASPAA, 2005
- Much more at https://ccrma.stanford.edu/∼jos/wg.html