E85.2607: Lecture 10 – Modulation

Modulation

Modulation Use an audio signal to vary the parameters of a sinusoid

$$y_{\text{mod}}[n] = m[n] \cos\left(2\pi f_c n + \phi[n]\right)$$

$m[n]$, $\phi[n]$ modulating signals

$\cos(f_c n)$ carrier signal with carrier freq. f_c

Used for:

- Transmitting radio signals
- Tremolo, vibrato, other effects
- Synthesizing complex harmonic series

ile phones)

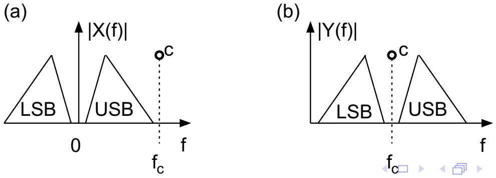
Ring modulation

$$y_{\mathrm{RM}}[n] = m[n]\cos(2\pi f_c n)$$

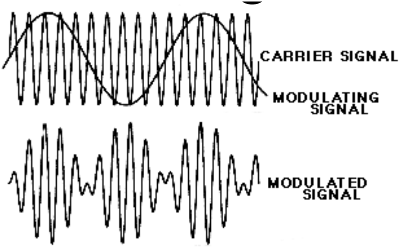
- Shifts spectrum of modulating signal to be centered around f_c
- e.g. let $m[n] = \cos(2\pi f_m n)$:

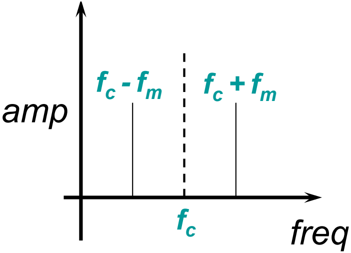
$$\begin{aligned} y_{\mathrm{RM}}[n] &= \cos(2\pi f_m n)\ \cos(2\pi f_c n) \\ &= \frac{1}{2}\cos\left(2\pi(f_c - f_m)\right) + \cos\left(2\pi(f_c + f_m)\right) \end{aligned}$$

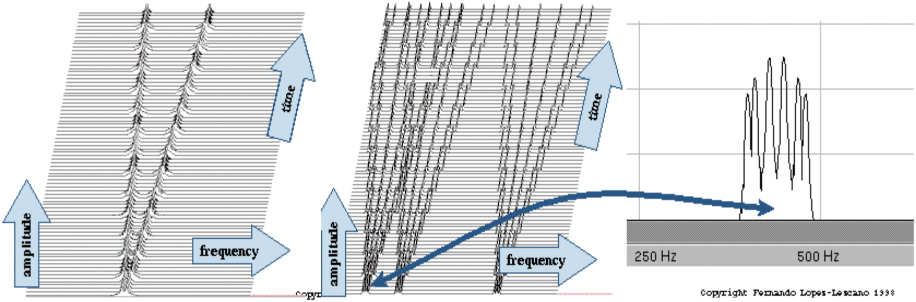
Using trigonometric identity: $\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$



Ring modulation







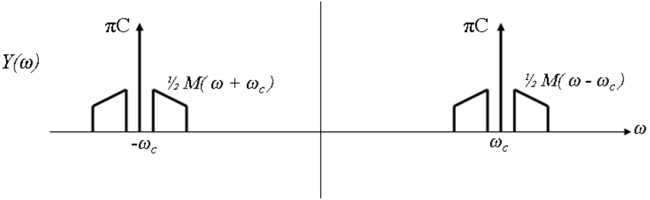
Amplitude modulation

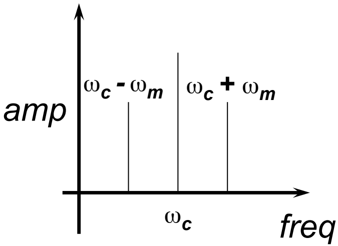
- Like ring modulation, but with DC offset added to modulating signal

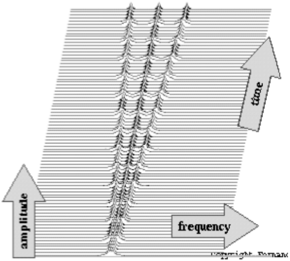
$$y_{\mathrm{AM}}[n] = (1 + \alpha m[n]) \cos(2\pi f_c n)$$

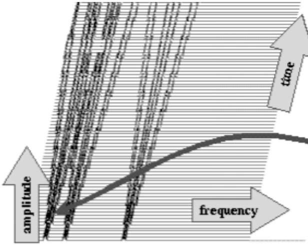
- Receiver (demodulator) is easier to build
- e.g. let $m[n] = \cos(2\pi f_m n)$:

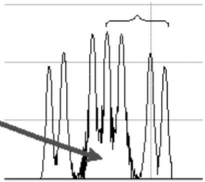
$$\begin{aligned} y_{\mathrm{AM}}[n] &= (1 + \alpha \cos(2\pi f_m n))\ \cos(2\pi f_c n) \\ &= \cos(2\pi f_c n) + \frac{\alpha}{2} \cos\left(2\pi(f_c - f_m)\right) + \cos\left(2\pi(f_c + f_m)\right) \end{aligned}$$




amp
$\omega_c - \omega_m$
$\omega_c + \omega_m$
ω_c
freq


time
amplitude
frequency


time
amplitude
frequency



Amplitude modulation in the time domain

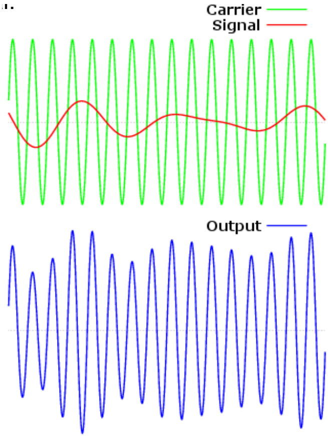
- Demodulate using an envelope detector

  = rectifier + LPF
- or product detector

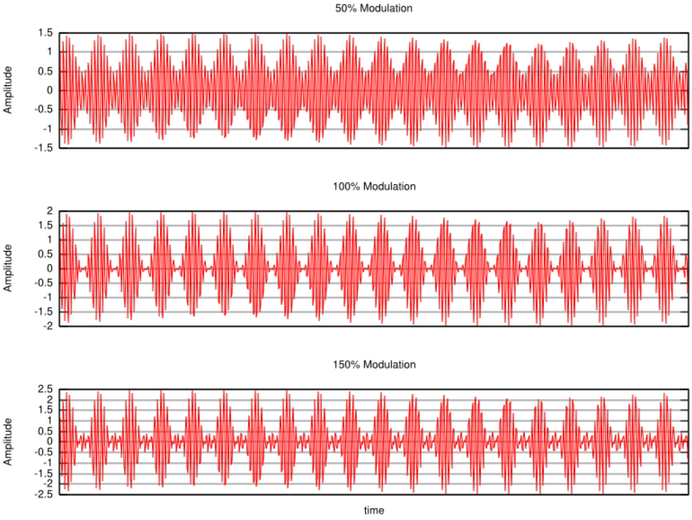
  = coherent ring modulation + LPF

$$
\begin{aligned}
y_{AM}[t]\cos(2\pi f_c) \\
&= (1 + \alpha m[n])\cos(2\pi f_c n)\cos(2\pi f_c n) \\
&= (1 + \alpha m[n])\left(\frac{1}{2} + \frac{1}{2}cos(2\pi 2 f_c n)\right)
\end{aligned}
$$

  - Also works for ring modulation

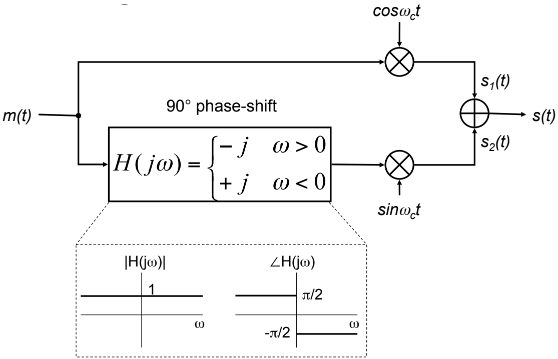


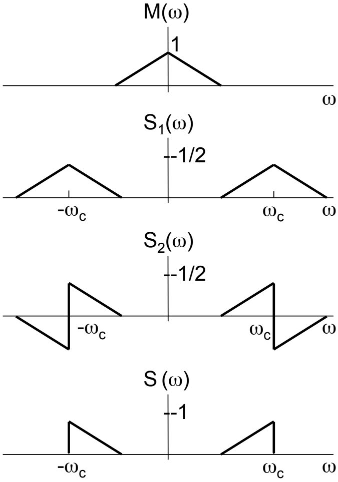
Effect of modulation index (α)



Single Sideband (SSB) modulation

AM and RM waste bandwidth (and power) in redundant sidelobes

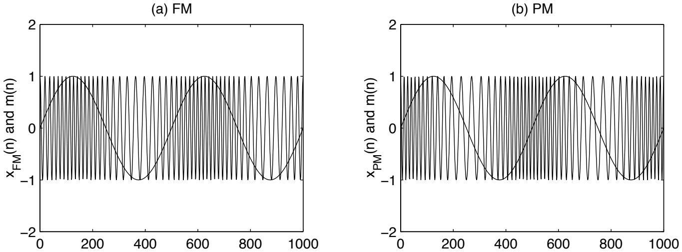




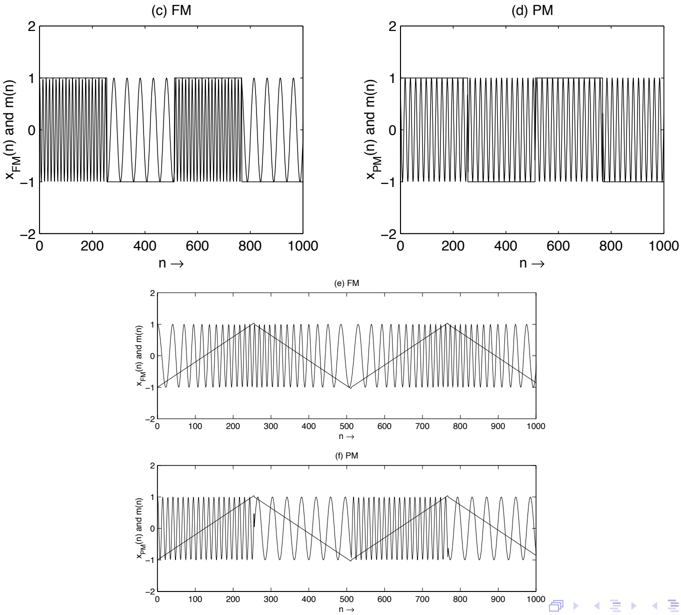
Angle modulation

$$y_{\mathrm{PM/FM}}[n] = \cos(2\pi f_c n + \beta\,\phi_{\mathrm{PM/FM}}[n])$$
$$\phi_{\mathrm{PM}}[n] = m[n]$$
$$\phi_{\mathrm{FM}}[n] = 2\pi \int_{-\infty}^{n} m[\tau]\,d\tau$$

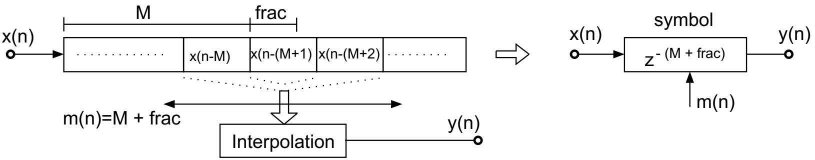
- Looks like phase is being modulated, but they're really the same
    - instantaneous frequency $= \frac{\partial}{\partial n}(2\pi f_c n + \beta\phi[n])$
    - ("FM" often used to refer to phase modulation)



FM vs PM

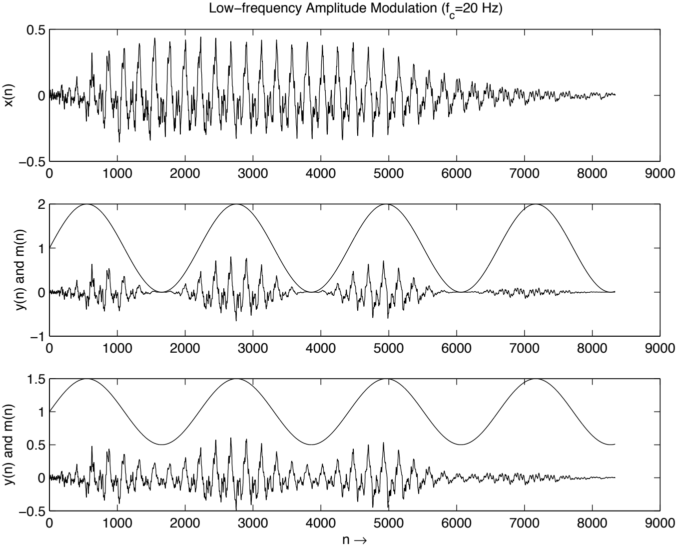


Implementing angle modulation



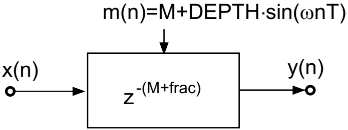
- Just index into carrier using time-varying delay
- Interpolate as necessary

Modulate amplitude of audio signal with low frequency sinusoid



More effects

Vibrato modulate phase of audio signal with low frequency sinusoid



Detuning SSB modulation to shift spectrum up or down in frequency

Applications: synthesizing notes

AM synthesis change carrier frequency to change pitch

- e.g. simple synthesizer with 3 harmonics by modulating sinusoidal carrier with sinusoidal signal:

$$(1 + \cos(2\pi f_m n))\ \cos(2\pi f_c n)$$

- easy to implement
- but, limited timbral possibilities ...

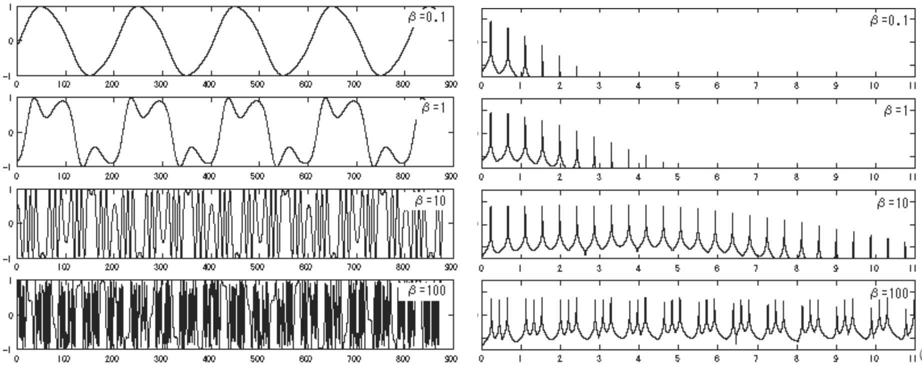
FM synthesis produce spectrally rich sounds with minimal effort

$$\cos(2\pi f_c n + \beta \sin(2\pi f_m n))$$

- need integer $\frac{f_c}{f_m}$ to make harmonic sounds
  - sidebands at $f_c \pm k f_m$
- introduced by John Chowning at Stanford in early 1970s
- commercialized by Yamaha in the 1980s (DX7)

FM modulation index

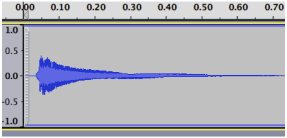
$$y[n] = \cos(2\pi\ 220\ n + \beta \sin(2\pi\ 440\ n))$$

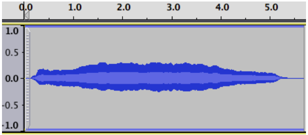


- FM signals theoretically have infinite bandwidth
- $\sim 2(\beta + 1)$ audible sidebands

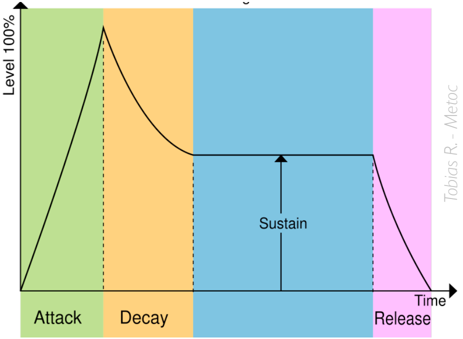
Note dynamics

- Real notes are time-limited
  - struck/plucked vs. bowed/blown



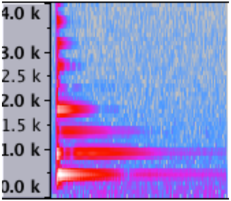


- simulate using ADSR envelope



Toward more realistic synthesis

- Amplitude modulation alone is not enough
  - real instruments have time-varying spectra
  - e.g. plucked string



- Model using LPF
  - high frequencies die away after initial transient
- Or just model the physics...

Reading

DAFX Chapter 4 - Modulators and Demodulators