E85.2607: Lecture 1 – Introduction

Course overview

- *Advanced* Digital Signal Theory
- Design, analysis, and implementation of audio effects and synthesizers
  - EQ, reverb, chorus, phase vocoder, sinusoidal modeling, FM synthesis, ...
- Emphasis on practical implementation, building complete systems.
- Course web page: `http://www.ee.columbia.edu/~ronw/adst`

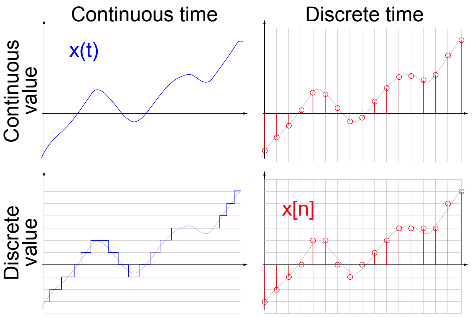
whoami

- PhD, Electrical Engineering, Columbia University
- Research interests: Source separation, speech recognition, music information retrieval
- `http://www.ee.columbia.edu/~ronw`

Sound familiar?

- Periodic and aperiodic signals
- Discrete Fourier Transform
- Convolution, filtering
- Linear time-invariant systems
- Impulse response
- Frequency response
- z-transform

Digital signals



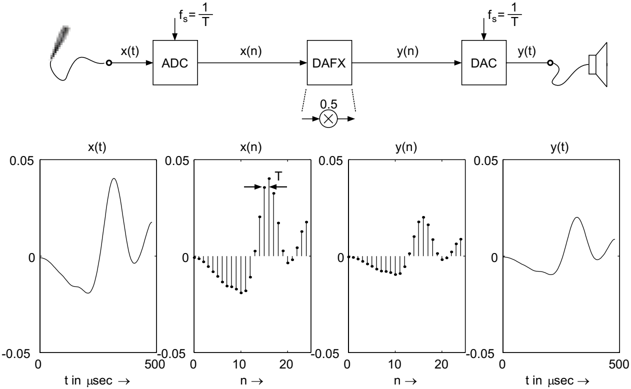
Discrete-time signal sequence of *samples*

- e.g. $x[n] = [0, -2.3, -1.3, 20, 4.2, \ldots]$

Digital signal discrete-time signal that has been quantized

- Discrete on both axes: samples can only take on a limited set of values (quantization levels)
- Quantization introduces *noise*

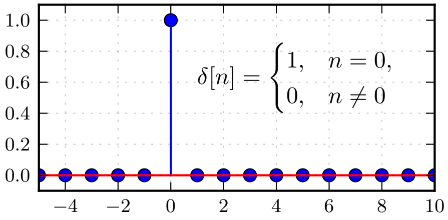
Sampling



$$t = n\,\frac{1}{f_s}$$

t	time in seconds (real number)
n	time in samples (integer)
f_s	sampling rate $\left(\text{e.g. } 44100\ \frac{\text{samples}}{\text{second}}\right)$

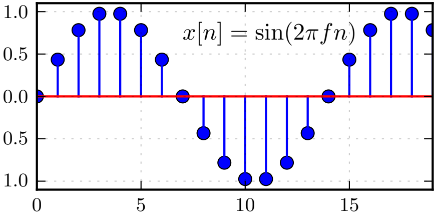
Important signals: impulse



Think of all discrete time signals as a sequence of scaled and time-shifted impulses.

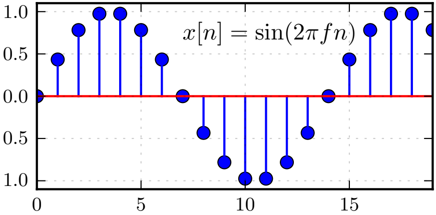
$$\begin{aligned} x[n] &= [0, -2.3, -1.3, 20, 4.2, \ldots] \\ &= 0\ \delta[n] - 2.3\ \delta[n-1] - 1.3\ \delta[n-2] + 20\ \delta[n-3] + 4.2\ \delta[n-4] + \ldots \end{aligned}$$

Important signals: sinusoid



- $\sin(2\pi fn + \phi)$, frequency $= f \ \frac{\text{cycles}}{\text{sample}}$, phase $= \phi$
- Period: $N = \frac{1}{f}$ samples
- But what will it sound like?

Important signals: sinusoid



- $\sin(2\pi f n + \phi)$, frequency $= f \ \frac{\text{cycles}}{\text{sample}}$, phase $= \phi$
- Period: $N = \frac{1}{f}$ samples
- But what will it sound like?
  - Convert from samples: $f \ \frac{\text{cycles}}{\text{sample}} \times \frac{1}{f_s} \ \frac{\text{samples}}{\text{second}}$
  - How many samples in one period of a 440 Hz tone sampled at 44.1 kHz?

Discrete Fourier Transform

Decompose any **periodic** signal into sum of re-scaled sinusoids

Inverse DFT

$$x[n] = \frac{1}{N} \sum_{n=0}^{N-1} X[k]\, e^{j2\pi nk/N} \quad k = 0, \ldots, N-1$$

Forward DFT

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j2\pi nk/N}$$

Note that $X[k]$ are complex: $X[k] = X_R[k] + jX_I[k]$

Magnitude and Phase spectra

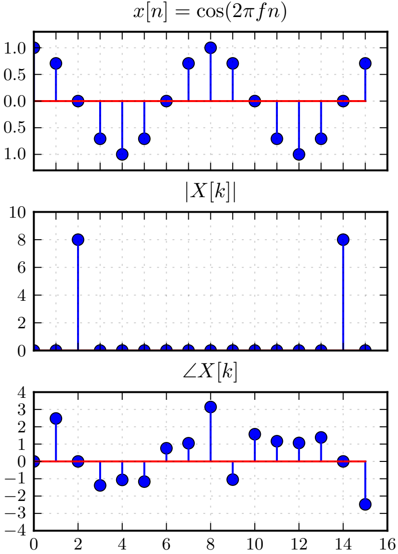
$$X[k] = |X[k]| \, e^{j\angle X[k]}$$

- Magnitude: amount of energy at each frequency

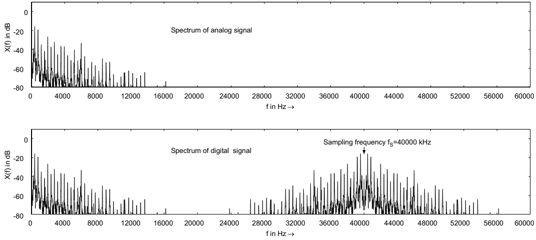
$$|X[k]| = \sqrt{X_R^2[k] + X_I^2[k]}$$

- Phase: delay at each frequency

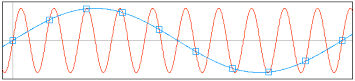
$$\angle X[k] = \arctan \frac{X_I[k]}{X_R[k]}$$



DFT symmetry and aliasing

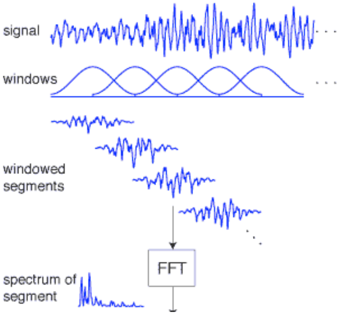


- The spectrum of a discrete time signal is periodic with period f_s
- The spectrum of a real valued signal is symmetric around $f_s/2$
- Any energy at frequencies greater than $f_s/2$ will wrap around

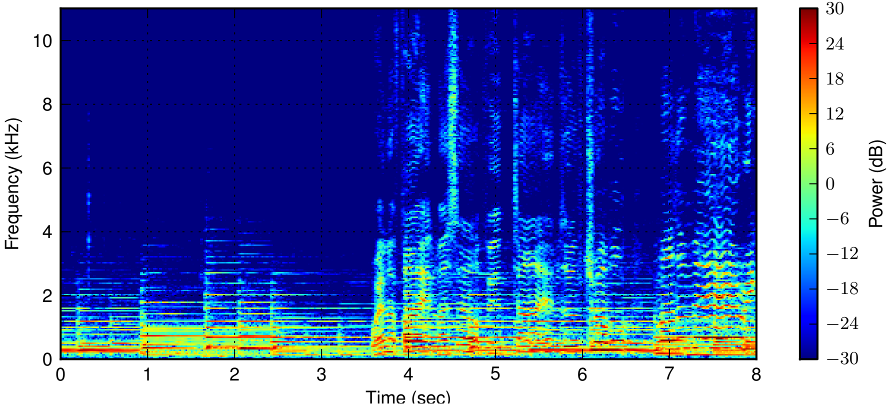


Short-time Fourier Transform

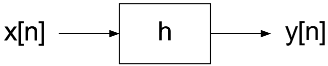
- What if frequency content varies with time?
- Break signal up into short (optionally overlapping) segments
- Multiply by window function
- Take DFT of each segment



STFT example



Linear time-invariant systems



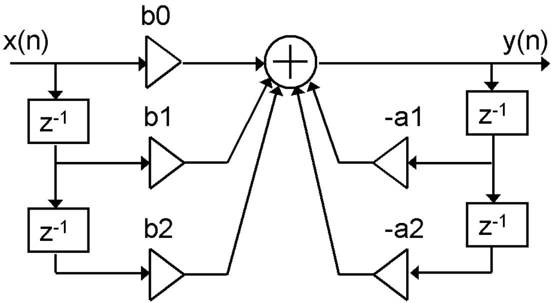
- Process input signal using delays, multiplications, additions
- Describe using a difference equation, e.g.

$$y[n] = b_0\, x[n] + b_1\, x[n-1]$$

- Can also have feedback, e.g.

$$y[n] = b_0\, x[n] + b_1\, x[n-1] + a_1\, y[n-1] + a_2\, y[n-2]$$

LTI systems – Block diagram



LTI systems – Properties

Given:

$x[n] \longrightarrow \boxed{h} \longrightarrow y[n]$

$x2[n] \longrightarrow \boxed{h} \longrightarrow y2[n]$

1. Linearity

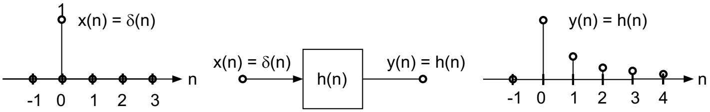
$a\,x[n] \longrightarrow \boxed{h} \longrightarrow a\,y[n]$ Scaling

$x[n] + x2[n] \longrightarrow \boxed{h} \longrightarrow y[n] + y2[n]$ Superposition

2. Time-invariance

$x[n-k] \longrightarrow \boxed{h} \longrightarrow y[n-k]$

Impulse response and convolution



- Characterize LTI system in time-domain by its impulse response $h[n]$
  - An LTI system is just another signal
- Given input signal $x[n]$, output is convolution with $h[n]$

Convolution

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

- But how do we compute the impulse response from a difference equation?

z-transform

z-transform

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$$

- Maps discrete-time signal to a continuous function of a complex variable
- Incredibly useful for analyzing LTI systems
  - Turns difference equations into polynomials:

$$x[n-m] \overset{z}{\longleftrightarrow} z^{-M} X(z)$$

  - Convolution becomes multiplication:

$$x[n] * h[n] \overset{z}{\longleftrightarrow} X(z)\, H(z)$$

- If $z = e^{j\Omega}$, get discrete-time Fourier transform (DTFT)

Transfer function

- Transfer function $H(z)$ is the z-transform of the impulse response
- Can read off z-transform from difference equation

$$y[n] = b_0\, x[n] + b_1\, x[n-1] + a_1\, y[n-1] + a_2\, y[n-2]$$

Transfer function

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$$
\begin{aligned}
y[n] &= b_0\, x[n] + b_1\, x[n-1] + a_1\, y[n-1] + a_2\, y[n-2] \\
&\xleftrightarrow{z} Y(z) = (b_o + b_1\, z^{-1})\, X(z) + (a_1\, z^{-1} + a_2\, z^{-2})\, Y(z)
\end{aligned}
$$

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$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_o + b_1\, z^{-1}}{1 - a_1\, z^{-1} - a_2\, z^{-2}}$$

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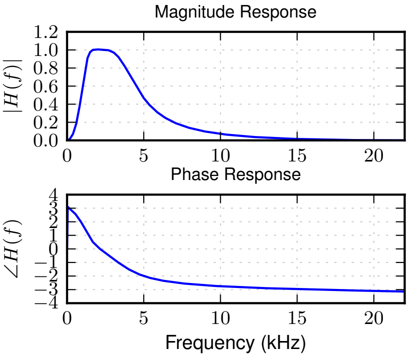
$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_o + b_1\, z^{-1}}{1 - a_1\, z^{-1} - a_2\, z^{-2}}$$

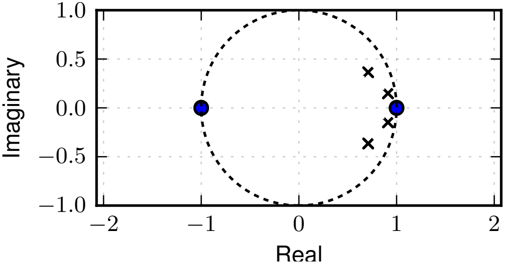
- But why? $\delta[n] \stackrel{z}{\longleftrightarrow} 1$
- Compute impulse response analytically by finding inverse z-transform of $H(z)$ (lots of algebra)

Frequency response



- Often more intuitive to analyze system in the frequency-domain
- $H(e^{j\Omega})$ DTFT of impulse response
    - Slice of z-transform corresponding to the unit circle
    - DFT (discrete freq) is just sampled DTFT (continuous freq)

Poles and zeros



- Zeros: roots of numerator of $H(z)$
  - Correspond to valleys in frequency response
- Poles: roots of denominator of $H(z)$
  - Correspond to peaks in frequency response
- System is **unstable** if it has poles outside of (or on) the unit circle
  - Impulse response goes to infinity

FIR and IIR filters

- Finite Impulse Response
  - No feedback $\Rightarrow$ all zeroes $\Rightarrow$ always stable*
    $*$ if coefficients are finite
  - Easy to design by drawing frequency response by hand, then using inverse DFT to get impulse response
  - Often needs a long impulse response $\Rightarrow$ expensive to implement
- Infinite Impulse Response
  - Feedback $\Rightarrow$ has poles $\Rightarrow$ can be unstable
  - Can implement complex filters using fewer delays than FIR
  - But harder to design

Fun with Matlab

Signal generation	`linspace, rand, sin`
I/O	`wavread, wavwrite, soundsc`
Plotting	`plot, imagesc`
Transforms	`fft, ifft`
Filtering	`conv, filter`
Analyzing filters	`freqz, zplane, iztrans`

Reading

- Review your DST notes
- Skim *Introduction to Digital Filters*
  - Linear Time-Invariant Filters
  - Transfer Function Analysis
  - Frequency Response Analysis
- *DAFX*, Chapter 1 (if you have it)