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EE 4830
HW-2

1. a) For an input signal u,

$$p(u) = \begin{cases} A+u & -1 \le u < 0, \\ A-u & 0 \le u \le 1, \\ 0 & otherwise. \end{cases}$$

is a probability density function, so the integral must sum to one, i.e. $\int p(u)du = 1$, so

$$\int_{-1}^{0} (A+u)du + \int_{0}^{1} (A-u)du = 1$$
$$2A - 1/2 - 1/2 = 1$$
$$A = 1$$

b) The error is $\xi = u - Q(u)$, where

$$Q(u) = \begin{cases} -1/2 & -1 \le u < 0, \\ 1/2 & 0 \le u \le 1. \end{cases}$$

So we have:

$$\xi = \begin{cases} u + 1/2 & -1 \le u < 0, \\ u - 1/2 & 0 \le u \le 1, \end{cases}$$

which is shown in Figure 1. The probability density $p(\xi)$ is, $\xi^{-1}p(u)$, which is shown in Figure 2. The domain of $p(\xi)$ is $[-1/2, 1/2]$, which is the range of ξ. $p(\xi)$ is the sum of two lines, because the inverse of ξ is not a function. As you can see, the two lines sum to constant probability, so ξ is uniformly distributed.

c) Find the distortion $D = E[\xi^2]$ where

$$\xi = \begin{cases} u + 1/2 & -1 \le u < 0, \\ u - 1/2 & 0 \le u \le 1. \end{cases}$$

So we have

$$D = E[\xi^2] = \int_{-1}^{0} (u+1/2)^2(1+u)du + \int_{0}^{1} (u-1/2)^2(1-u)du$$
$$= \int_{-1}^{0} (u^3 + 2u^2 + \frac{5}{4}u + \frac{1}{4})du + \int_{0}^{1} (\frac{1}{4} - \frac{5}{4}u + 2u^2 - u^3)du$$
$$= \frac{1}{12}$$