

3.55

The DFT of a real sequence is periodic conjugate symmetric, so we can fill in the rest of the DFT as: $X[k] = \{12, -1 + 3j, 3 + 4j, 1 - 5j, -2 + 2j, 6 + 3j, -2 - 3j, 10, -2 + 3j, 6 - 3j, -2 - 2j, 1 + 5j, 3 - 4j, -1 - 3j\}$.

(a) $x[0] = \frac{1}{14} \sum_{k=0}^{13} x[k] = \frac{16}{7}$.

(b) $x[7] = \frac{1}{14} \sum_{k=0}^{13} x[k] W_{14}^{-7k}$, where

$$W_{14}^{-7k} = e^{-j \frac{2\pi}{14} \cdot 7k} = e^{j\pi k} = \begin{cases} -1 & k \text{ odd} \\ 1 & k \text{ even} \end{cases}$$

So

$$x[7] = \frac{1}{14} \sum_{k \text{ even}} x[k] - \frac{1}{14} \sum_{k \text{ odd}} x[k] = \frac{-12}{14}.$$

(c)

$$\begin{aligned} \sum_0^{13} x[n] &= \sum_{n=0}^{13} \frac{1}{14} \sum_{k=0}^{13} X[k] W_{14}^{-nk} \\ &= \frac{1}{14} \sum_{k=0}^{13} X[k] \sum_{n=0}^{13} W_{14}^{-nk} \end{aligned}$$

but

$$\sum_{n=0}^{13} W_{14}^{-nk} = \begin{cases} N & k = rN, \text{ r integer} \\ 0 & \text{otherwise} \end{cases}$$

so $\sum_0^{13} x[n] = \frac{1}{N} X[0]N = X[0] = 12$.

(d) $e^{j \frac{4\pi n}{7}} = e^{j \frac{2\pi}{14} 4n} = W_N^{-4n}$, so

$$\begin{aligned} \sum_0^{13} e^{j \frac{4\pi n}{7}} x[n] &= \sum_0^{13} x[n] W_N^{-4n} \\ &= X[\langle -4 \rangle_{14}] = X[10] = -2 - 2j. \end{aligned}$$

(e) $\sum_0^{13} |x[n]|^2$ is the energy of the sequence $x[n]$. By Parseval's theorem, the energy in the time domain is related to the energy in the frequency domain by

$$\sum_{n=0}^{13} |x[n]|^2 = \frac{1}{14} \sum_{k=0}^{13} |X[k]|^2 = \frac{498}{14}.$$

3.64

(a) The linear convolution $y_L[n] = g[n] * h[n] = \{-6160 - 1924\}$.

(b) Zero-padding to length 4, $g_e[n] = \{-3240\}$, and the circular convolution of length 4 is:

$$\begin{aligned} y_c[0] &= -3 \cdot 2 + 2 \cdot 1 + 4 \cdot 0 + 0 \cdot -4 = -4 \\ y_c[1] &= -3 \cdot -4 + 2 \cdot 2 + 4 \cdot 1 + 0 \cdot 0 = 20 \\ y_c[2] &= -3 \cdot 0 + 2 \cdot -4 + 4 \cdot 2 + 0 \cdot 1 = 0 \\ y_c[3] &= -3 \cdot 1 + 2 \cdot 0 + 4 \cdot -4 + 0 \cdot 2 = -19 \end{aligned}$$

(c) First, finding the DFTs of $g[n]$ and $h[n]$:

$$G[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} -3 \\ 2 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -7 - 2j \\ -1 \\ -7 + 2j \end{bmatrix}$$

$$H[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ -4 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 + 5j \\ 5 \\ 2 - 5j \end{bmatrix}$$

Multiplying and taking the IDFT,

$$\begin{aligned} y_c[n] &= \frac{1}{4} D_4^*(G[k]H[k]) \\ &= \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} -3 \\ -4 - 39j \\ -5 \\ -4 + 39j \end{bmatrix} = \begin{bmatrix} -4 \\ 20 \\ 0 \\ -19 \end{bmatrix}, \end{aligned}$$

which is the same result as doing it in time domain.

3.84

$$G(z) = \frac{(z + .4)(z - .91)(z^2 + .3z + .4)}{(z^2 - .6z + .6)(z^2 + 3z + 5)}$$

The poles of $G(z)$ are $z = \frac{-3 \pm j\sqrt{11}}{2}$ and $z = \frac{.6 \pm j\sqrt{2.04}}{2}$. The first pair of poles have magnitude $\sqrt{5}$, and the second pair has magnitude $\sqrt{.6}$, so there are three regions of convergence: $|z| < \sqrt{.6}$, which holds for a left-sided sequence, $\sqrt{.6} < |z| < \sqrt{5}$, which holds for a two-sided sequence, and $|z| > \sqrt{5}$, which holds for a right-sided sequence.

3.89

Properties of the z-transform.

(a) Linearity.

$$\begin{aligned}\sum_{-\infty}^{\infty}(\alpha g[n] + \beta h[n])z^{-n} &= \alpha \sum g[n]z^{-n} + \beta \sum h[n]z^{-n} \\ &= \alpha G(z) + \beta H(z).\end{aligned}$$

Both sequences $G(z)$ and $H(z)$ must converge, so the region of convergence is the intersection of R_g and R_h .

(b) Time-reversal. Substituting $m = -n$,

$$\sum_n g[-n]z^{-n} = \sum_m g[m]z^m = \sum_m g[m](z^{-1})^{-m} = G(z^{-1}).$$

$G(z^{-1})$ converges for z if $G(z)$ converges for $\frac{1}{z}$, so the region of convergence is $R_{g-} < \frac{1}{|z|} < R_{g+}$, or $R_{g+} < |z| < R_{g-}$, which we denote by $\frac{1}{R_g}$.

(c) Time-shift. By changing variables $m = n - n_0$,

$$\begin{aligned}\sum_n g[n - n_0]z^{-n} &= \sum_m g[m]z^{-m-n_0} \\ &= z^{-n_0} \sum_m g[m]z^{-m} = z^{-n_0} G(z)\end{aligned}$$

This will converge wherever $G(z)$ converges, except we have added n_0 new poles at $z = 0$, so that is excluded.

(d) Multiplication by exponential.

$$\sum_n \alpha^n g[n]z^{-n} = \sum_n g[n](\alpha^{-1}z)^{-n} = G\left(\frac{z}{\alpha}\right).$$

If $G(z)$ converges for z , then $G\left(\frac{z}{\alpha}\right)$ converges for $R_{g-} < \frac{|z|}{|\alpha|} < R_{g+}$, or $|\alpha|R_{g-} < |z| < |\alpha|R_{g+}$, so the ROC is $|\alpha|R_g$.

(e) Differentiation.

$$\begin{aligned}\frac{dG(z)}{dz} &= \sum -ng[n]z^{-n-1} \\ &= -z^{-1} \sum ng[n]z^{-n} = -z^{-1}F(z),\end{aligned}$$

where $f[n] = ng[n]$. So we see that

$$ng[n] \Leftrightarrow -z \frac{d}{dz} G(z).$$

This converges when $\sum ng[n]z^{-n}$ exists, but I can't see why that's the same as the ROC of $G(z)$...

(f) Convolution.

$$\begin{aligned} g[n] * h[n] &= \sum_{k=-\infty}^{\infty} g[k]h[n-k] \Leftrightarrow \sum_{n=-\infty}^{\infty} \left[\sum_{k=-\infty}^{\infty} g[k]h[n-k] \right] z^{-n} \\ &= \sum_k g[k] \sum_n h[n-k] z^{-n} \\ &= \sum_k g[k] z^{-k} H(z) \\ &= G(z)H(z) \end{aligned}$$

For this to converge, note that both $G(z)$ and $H(z)$ must converge, so the ROC is the intersection.