Problem Set #8 Solutions

1. a. Old telephone line $P/N_0W \geq 28db$, $W = 3khz$, $28db = 10^{2.8} = 630.96$, then the minimum capacity is

$$C_{min} \geq 3 \times 10^3 \log_2 [1 + 630.96] = 27.911kbps.$$

b. $W \to \infty, P/N_0W \to 0$, then

$$\begin{aligned} C_\infty &= \lim_{W \to \infty} W \log_2 \left(1 + \frac{P}{N_0W}\right) \\ &= \frac{P}{N_0} \frac{1}{\log 2}. \end{aligned}$$

$P/N_0W = 3 \times 10^3 \times 630.96 = 1.893MHZ$, so $C_\infty = 2.73Mbps$. Compared with the 3khz channel, we gain capacity by $2730 - 27.9 = 2702.1kbps$

c. $0.99 \times 2.73 \times 10^6 = W \log_2 \left[1 + \frac{1.893 \times 10^6}{W}\right]$, assuming W big we get
$W = \frac{(1.893 \times 10^6)^2}{2 \log 2 \times 27.308 \times 10^3} = 94.64Mhz.$
From the results, we can see that in order to improve the capacity, we have to increase the BW. But the capacity tends asymptotically to the value $P/(N_0 \log 2)$.

a. When $N_RP/(N_0W) \gg 1$,

$$\begin{aligned} C_a &= W \log_2 \left(1 + \frac{N_RP}{N_0W}\right) bps \\ &\approx W \log_2 \left(\frac{N_RP}{N_0W}\right). \end{aligned}$$

b. When $P/(N_0W) \gg 1$,

$$\begin{aligned} C_b &= M_TW \log_2 \left[1 + \frac{N_RP}{M_TN_0W}\right] \\ &\approx W \log_2 \left[\frac{N_RP}{M_TN_0W}\right]. \end{aligned}$$

c. If $M_t = N_R = N$,

$$\begin{aligned} C_b &= NW \log_2 \left(\frac{P}{N_0W}\right), \\ C_a &= W \log_2 \left(\frac{NP}{N_0W}\right), \end{aligned}$$

C_b is clearly linear in N, C_a is a logarithmic function of N. When $P/(N_0W) \gg 1$ constant, the linear function grows much faster than the log function. In fact, we have $\lim_{N\to\infty} C_b/C_a = +\infty$,i.e. $C_b \gg C_a$ when $N \to \infty$.

d. When $N_RP/N_0W \ll 1$,

$$\begin{aligned} C_a &= W\log_2\left(1+\frac{N_RP}{N_0W}\right) \\ &= \frac{W}{\log 2}\log\left(1+\frac{N_RP}{N_0W}\right) \\ &\simeq \frac{N_RP}{N_0\log 2}. \end{aligned}$$

When $P/N_0W \ll 1$,

$$\begin{aligned} C_b &= M_TW\log 2\left[1+\frac{N_RP}{M_TN_0W}\right] \\ &\simeq \frac{N_RP}{\log 2N_0}. \end{aligned}$$

At least in first order approximation $C_a = C_b$. For $P/N_0W \ll 1$, MIMO does not affect any improvement over SIMO.