Modern Digital Modulation Techniques

ELEN E6909

Columbia University
Spring Semester- 2008

Problem Sets # 5 -7
(Revised Version)
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PROBLEM SET # 5

Due Date: 2 April 2008 (New Due Date)

Problem #1

Assume that a BPSK signal is transmitted over a slow flat Rayleigh fading channel with additive WGN. We use one antenna at the transmitter and one antenna at the receiver (SISO).

a. **Calculate the average bit error probability, $Pr_b\{\varepsilon\}$ for BPSK (we actually started this in class), as a function of the average E_b/N_0.**
b. **Repeat (a) for the symbol error probability, $Pr_s\{\varepsilon\}$,for QAM (ignoring the Q^2 terms in the error probability equation), as a function of the average E_s/N_0..**

<u>Problem #2</u>

Assume that a BPSK signal is transmitted over a slow flat Rayleigh fading channel with additive WGN, in a SISO system.

a. **Calculate the Outage Probability for BPSK (we actually did this in class), as a function of the average E_b/N_0 and $E_{b,req}/N_0$, for this SISO system**
b. **For BPSK, find the required average E_b/N_0, if the outage probability, Pr{out} is equal to 10^{-3}, and if the desired instantaneous, $Pr_b\{\varepsilon\}$ equals 10^{-5} .**
c. **Compare the value found in (b) with the average E_b/N_0, required when the <u>average $Pr_b\{\varepsilon\}$</u>$=10^{-5}$.**
d. **Repeat parts (b) and (c) if the desired instantaneous $Pr_b\{\varepsilon\}$ remains the same (i.e., equals 10^{-5}) but the Pr{out}equals 10^{-1} and also 10^{-5}.**

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PROBLEM SET # 6 (New Set Number)

Due Date: 2 April 2008

Read the following articles on BLAST and MIMO

The theoretical background behind MIMO.

1. **G.J. Foschini and M.J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas", Wireless Personal Communications, Vol. 6, No. 3, 1998, pp. 311-335.**

The BLAST Algorithm

2. **P. W. Wolniansky, G. J. Foschini, G. D. Golden, R. A. Valenzuela, "V-BLAST: An Architecture for Realizing Very High Data Rates Over the Rich-Scattering Wireless Channel" , Invited Paper, Proc. ISSSE-98, Pisa, Italy, Sept. 29, 1998.**

These articles may be downloaded from the following website

http://www1.bell-labs.com/project/blast/

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PROBLEM SET # 7 (New Set Number)

Due Date: 9 April 2008 (New Due Date)

Problem #1

This problem concerns Maximal Ratio Combining (MRC) –SIMO techniques

a) The modulation technique is BPSK. Find the outage probability at the output of a maximal-ratio combining receiver (with two receiving antennas) as a function of the average received energy per bit per antenna, divided by the noise spectral density, $E_{b, avg, ant}/N_0$, and the required instantaneous, $E_{b, req, ant}/N_0$, for the required instantaneous probability of error. Assume that the receiving antennas receive independent signals of the same average power.

b) Now find the outage probability of a maximal-ratio combining receiver (with two receiving antennas) as a function of the total average received energy per bit at both antennas divided by the noise spectral density, $E_{b, avg, total}/N_0$. Assume that the antennas receive independent signals of the same average power.

c) For BPSK, compare the results of (a) and (b) with those for a single receiving antenna at an outage probabilities, of 10^{-3} and 10^{-1}, if the desired instantaneous $Pr_b\{\varepsilon\}=10^{-5}$. How many dB have been gained in each case by using MRC-SIMO techniques?

Problem #2

This problem concerns MRC techniques when the number of receiving antennas is "L".

For BPSK, show that the probability density function, f(x), for the combined received signal for L antennas, with maximal-ratio combining, is given by the equation below.

$$f(x)= \frac{1}{(L-1)!\,(2\sigma^2)^L} x^{L-1} \exp\{-x/2\sigma^2\}; \qquad x \geq 0$$

(where $x = x_1 + x_2 + \ldots.. x_L$) ; $x_i = r_i^2$.

The variable, r_i represents the random Rayleigh variable at each receiving antenna.

Hint: This is similar to what we did in class for two receiving antennas.