COLUMBIA UNIVERSITY

SPRING SEMESTER-2008

ELEN E6909

Modern Digital Modulation Techniques

FIGURES FOR THE NYQUIST REVIEW

4 February 2008

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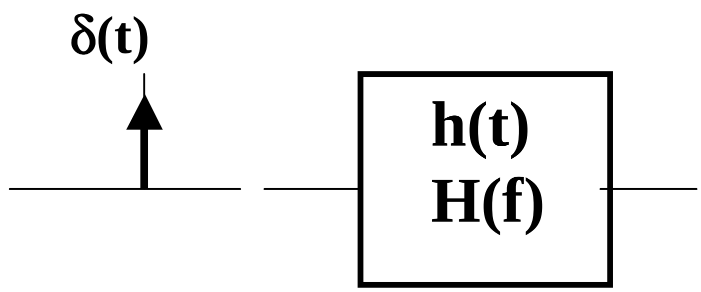
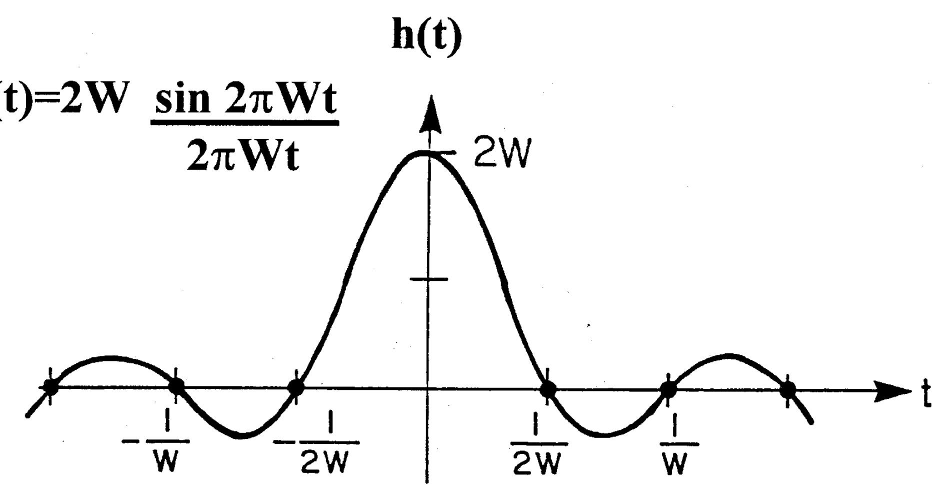
The figures below should help you review Nyquist signaling

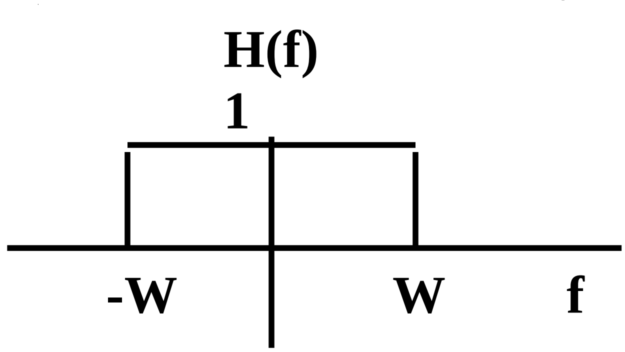
NYQUIST I SIGNALS

- **The Sampling Pulse**
- **No Intersymbol Interference (ISI)**



$$h(t) = 2W \frac{\sin 2\pi W t}{2\pi W t}$$



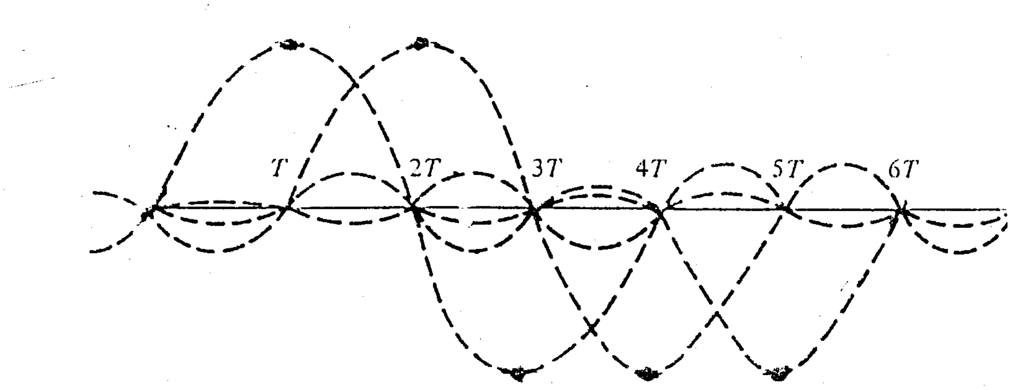


ENERGY=2W

SIGNALING WITH NYQUIST PULSE

•Minimum Bandwidth

(T=1/2W seconds)



• Send a Nyquist signal every T=1/2W seconds

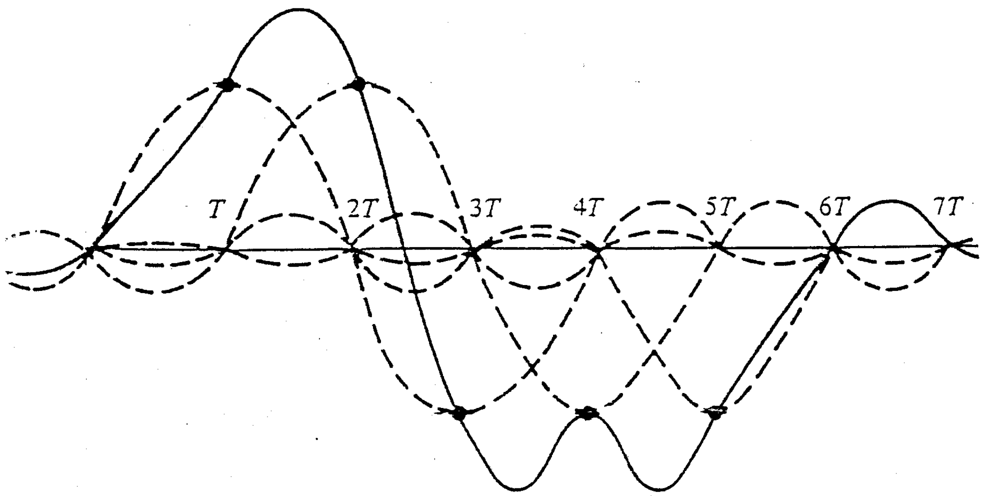
• If transmitted signal is sampled at the receiver at the correct times, there will be no intersymbol interference (ISI).

We can transmit and receive
2W independent pulses (or values)/sec

Each pulse may have any amplitude!

SIGNALING WITH NYQUIST PULSES

• Minimum Bandwidth

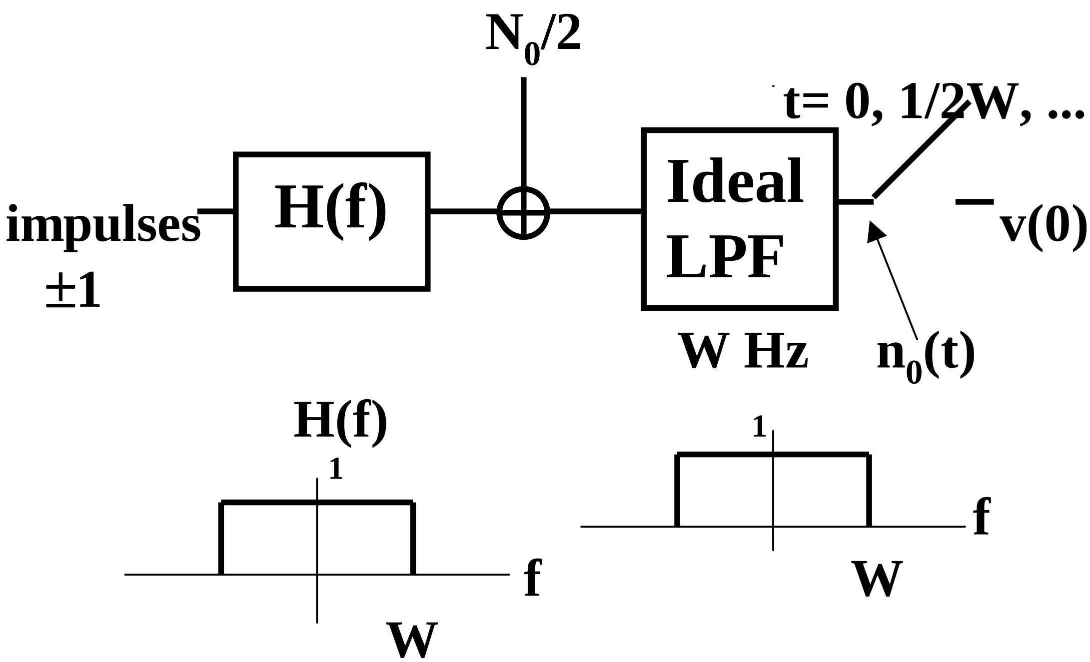


• The actual transmitted signal is the sum of all the Nyquist signals.

• The overshoots, between the sampling times at T, 2T, 3T, ..., theoretically may reach infinity!

There is a <u>tremendous peak-to-average ratio</u>. We solve the problem using Nyquist signals with Raised-Cosine filtering with rolloff factors greater than zero.

DETECTABILITY PERFORMANCE
BINARY NYQUIST SIGNALS



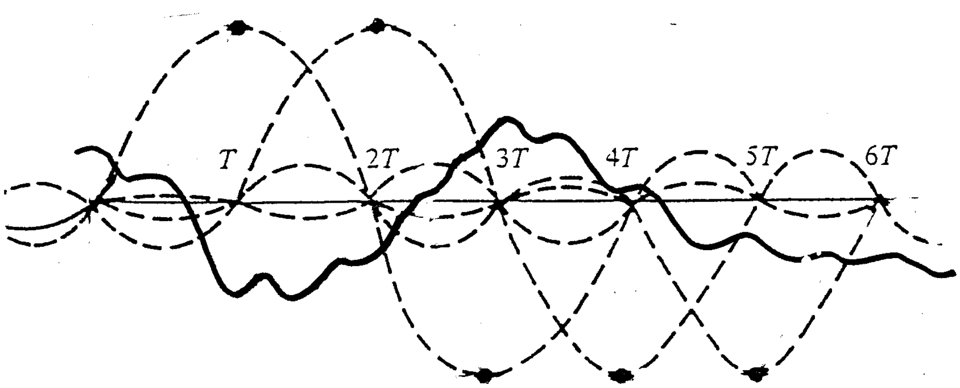
***At t=0, the output voltage, v(0), due to the signal is ±2W**

***The average mean-square noise power, $P_{n,out}$, at the output is given below**

$$P_{n,out} = N_0 W \text{ watts} \quad (= \sigma^2)$$

***The noise signal, $n_0(t)$, at the output of the LPF has gaussian statistics.**

THE OUTPUT SIGNAL

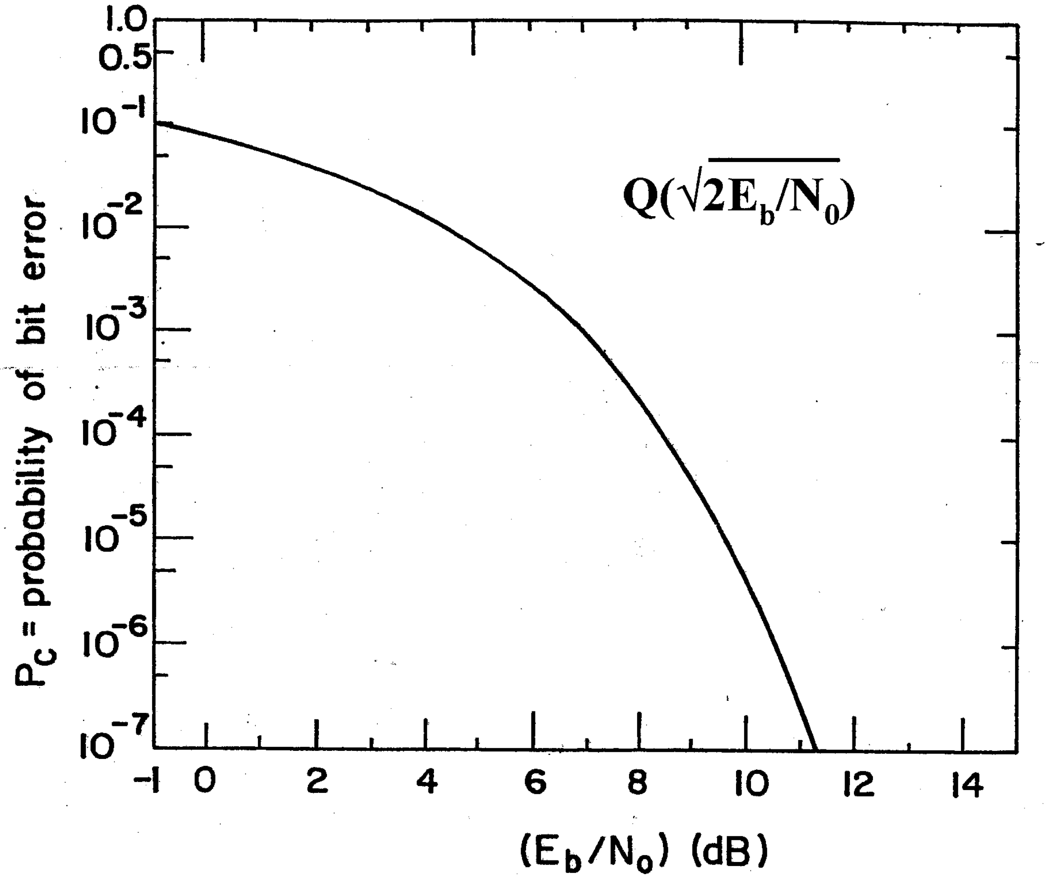


$$\frac{S_{out}}{N_{out}} = \frac{(2W)^2}{N_0 W} = \frac{4W}{N_0} = \frac{2(2W)}{N_0}$$

$$\frac{S_{out}}{N_{out}} = \frac{2E_b}{N_0}$$

E_b = Energy per bit

DETECTABILITY PERFORMANCE NYQUIST SIGNALS



The Probability of Error, $Pr_b\{\varepsilon\}$, is

$$Pr_b\{\varepsilon\} = Q(\sqrt{2E_b/N_0})$$

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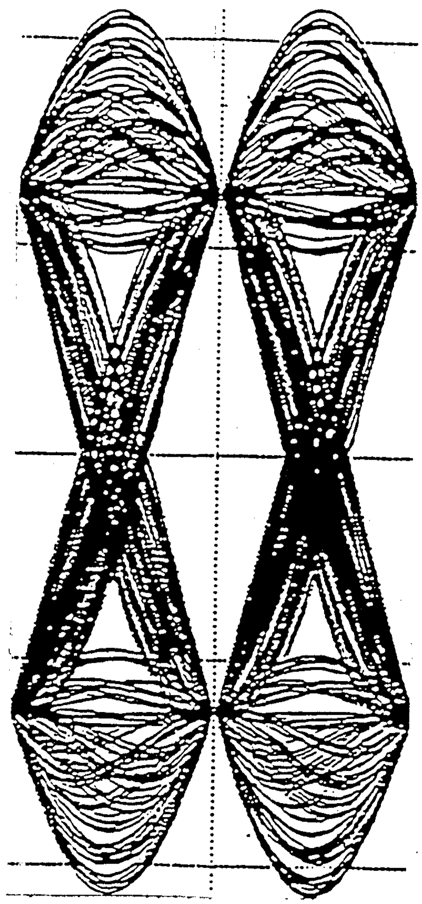
EYE PATTERN



EYE PATTERN

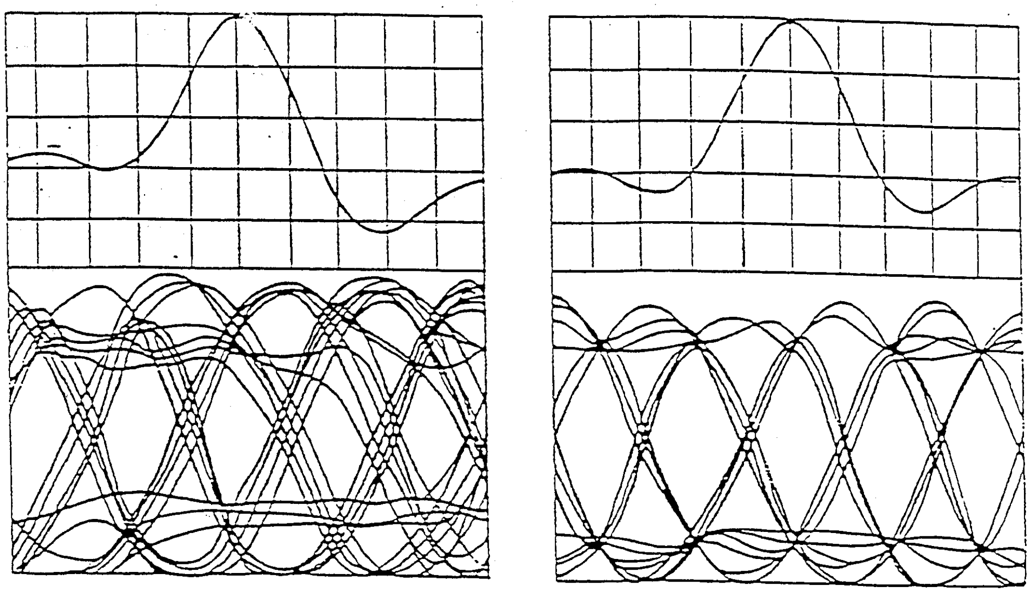


Fig. 3-17. Sketch of oscillograms of received eye pattern and after autoequalization. (From "Common Carrier Data Communication" by R.W. Lucky; Chapter 5 in *Computer-Communication Networks*, F.F. Kuo, Editor, Prentice Hall, 1973.)

"BAD"-CLOSED **"GOOD"- OPEN**

NYQUIST FIRST (I) CONDITION

Problems with sin $2\pi Wt / 2\pi Wt$

- **Brickwall Filter- hard to build**

- **sin x/x decays very slowly- (1/x)**
  -This may result in very large overshoots.
  The ISI may also be very big. if not sampling times are not correctly synchronized.

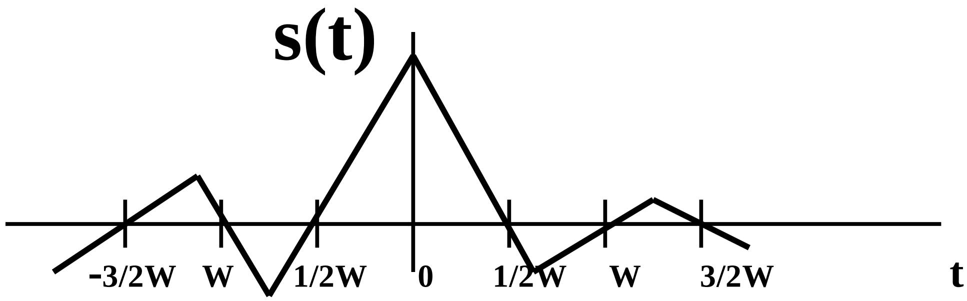
- **A lot of energy near W.**

H. Nyquist, "Certain topics in telegraph transmission theory", Trans. AIEE, Vol. 47, pp. 617-644, Apr. 1928.

NYQUIST I CONDITION

- **FOR NO ISI**

$$s(n/2W) = \begin{cases} 2W; & \text{if } n=0 \\ 0; & \text{if } n\neq 0 \end{cases}$$

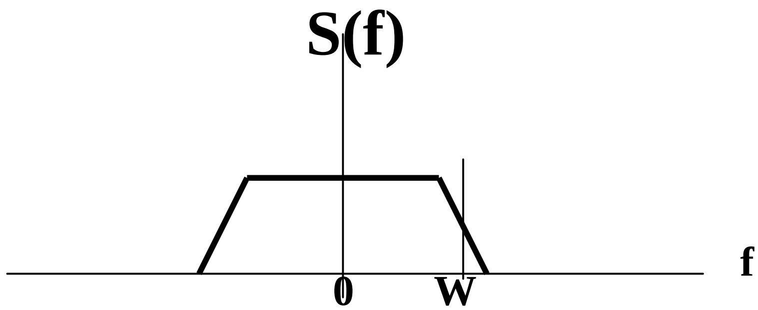


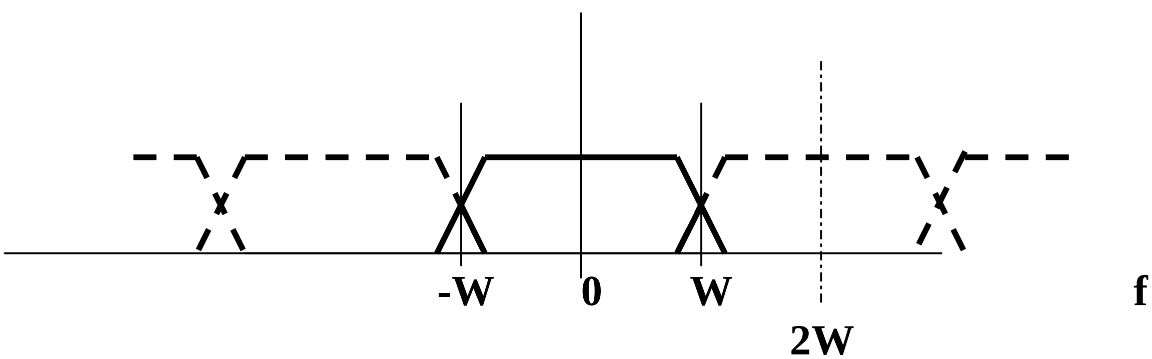
The condition (or requirement) on the spectrum S(f) to guarantee no ISI is:

$$\sum_{n=-\infty}^{n=\infty} S(f+n2W)=1; \quad \text{for } |f| \leq W$$

J. Proakis, "Digital Communications", Fourth Edition, McGraw-Hill, New York, 2001, pp. 556-559.

SIGNAL SPECTRUM WHICH SATISFIES NYQUIST I





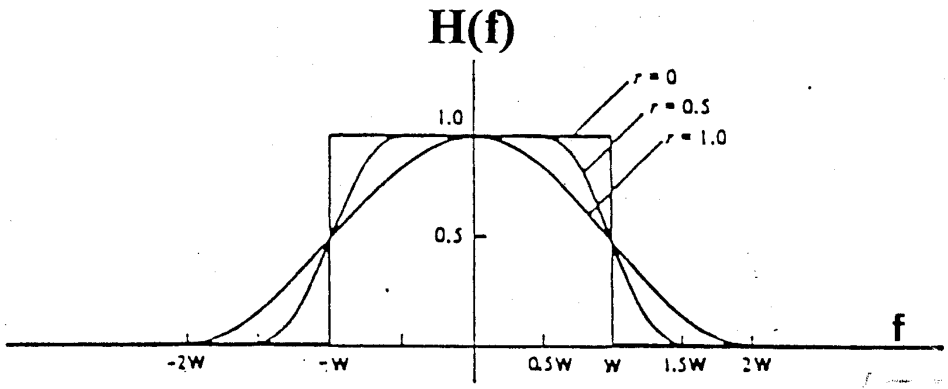
$$\sum_{n=-\infty}^{n=\infty} S(f+n2W)=1; \quad \text{for } |f| \leq W$$

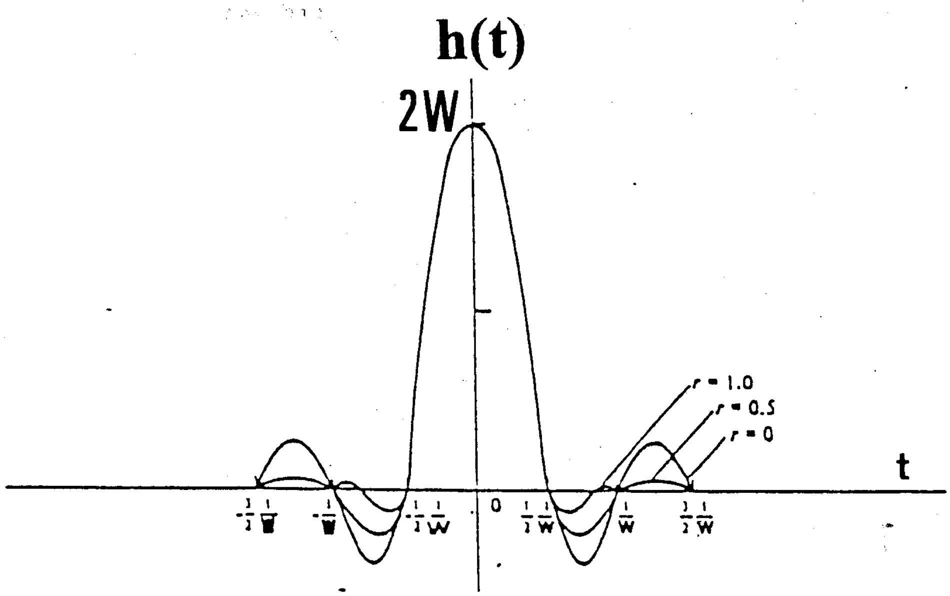
NYQUIST I FILTERS

• Raised Cosine Filter

(r = rolloff factor, e.g., 50%)

The higher the rolloff factor, the smaller the peak-to-average ratio but the bigger the bandwidth!

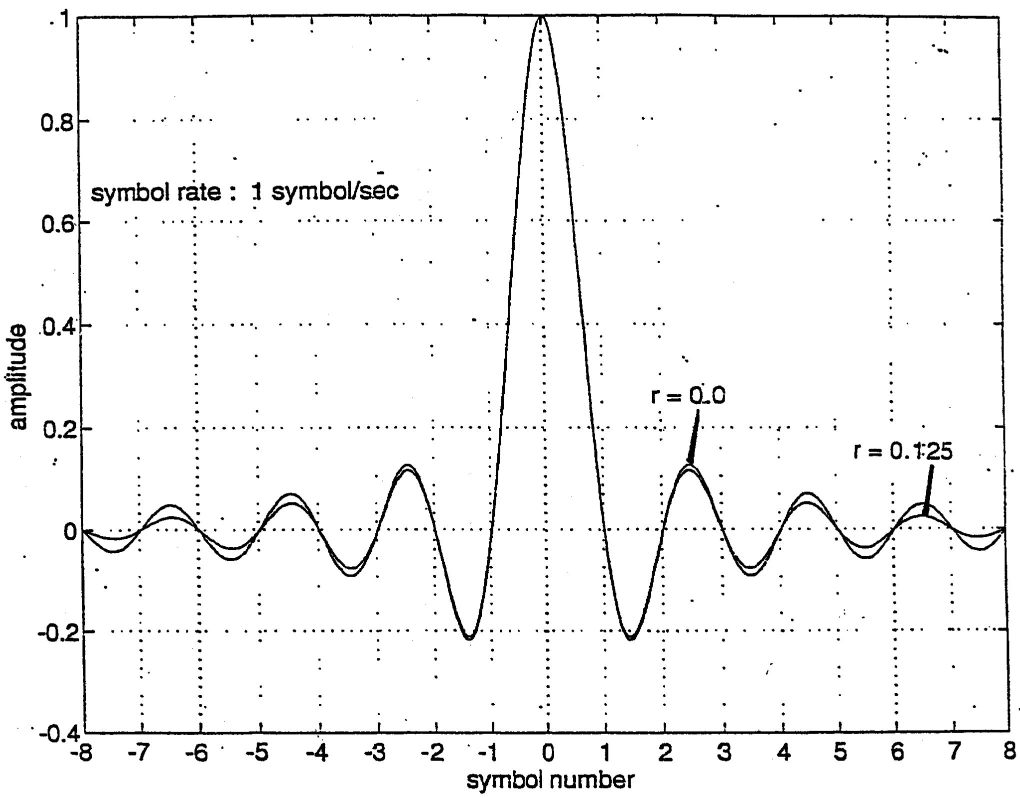




$$h(t)=2W\,\frac{\sin 2\pi Wt}{2\pi Wt}\,\frac{\cos 2\pi rWt}{1-4r^2(2Wt)^2}$$

RAISED COSINE PULSES

Rolloff Factor= 0%, 12.5%



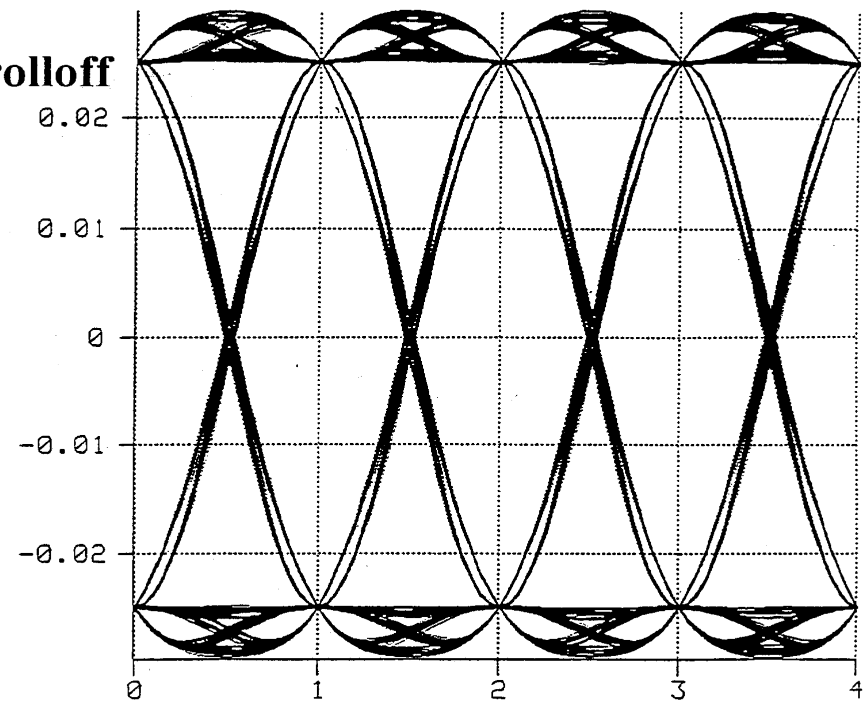
QAM SYSTEMS

System	Rolloff Factor	no.of bits / symbol	R_b (bps)
Modems (telephone)	**12.5%**	**4,6**	**9.6 Kbps 14.4 Kbps**
Intelsat	**40%**	**2**	**120 Mbps**
MSATX	**100%**	**2**	**4.8 Kbps**
IS-136(54)	**35%**	**2**	**48.6 Kbps**
VDSL	**20%**	**$\geq$ 6**	**$\geq$ 1.5 Mbps**
IS-95	**$\approx$ 0%**	**2**	**1.2288 Mcps**

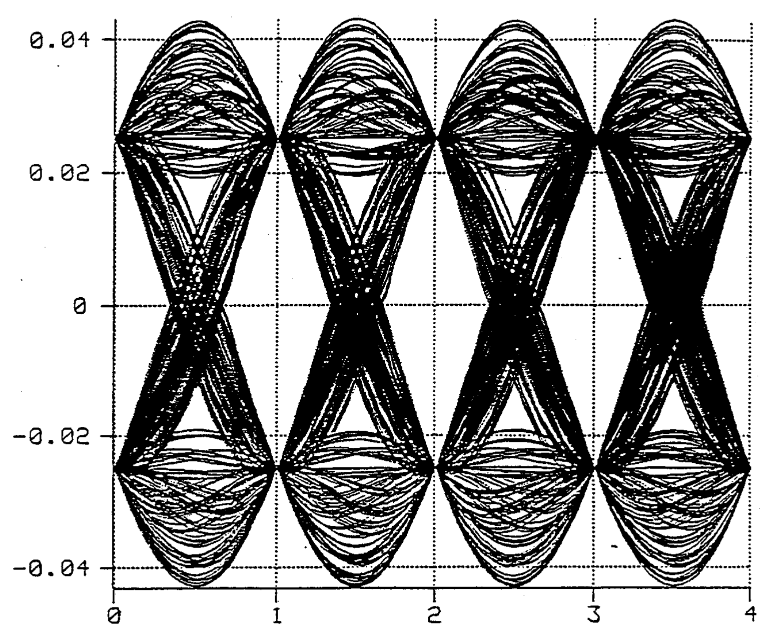
WCDMA-IMT2000 (r=22%)

EYE PATTERN

*80% rolloff



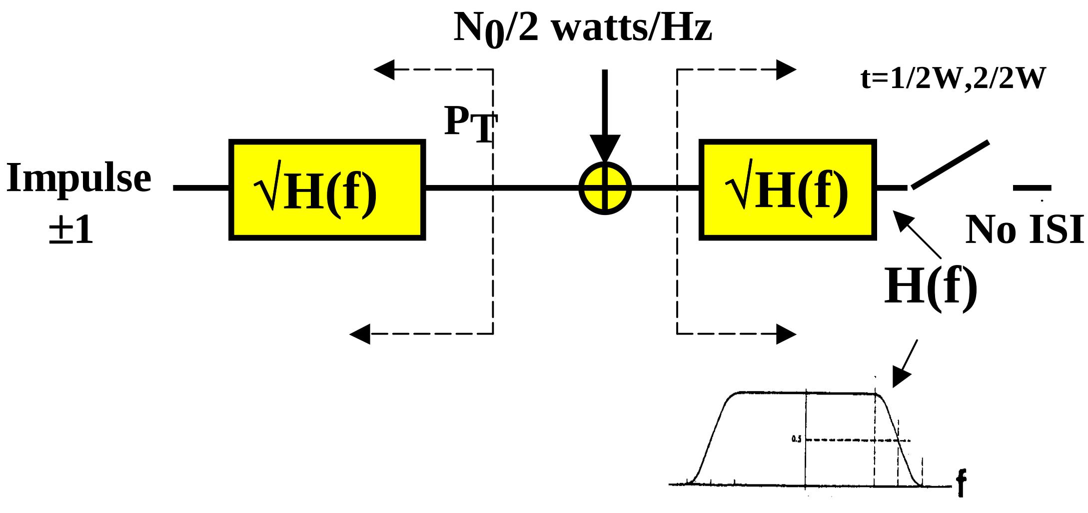
*35% rolloff



Optimum Filtering

To achieve the maximum signal-to-noise ratio at receiver output

• Optimum Filtering to achieve the maximum signal-to-noise ratio at receiver output



OPTIMUM SOLUTION:

Square-Root Nyquist Filter at transmitter and receiver

$$H_1(f) = H_2(f) = \sqrt{H(f)}$$

- **Transmitted Power remains equal to P_T**
- **Output S/N is maximized**

$$S/N,_{max} = 2E/N_0$$

Square-Root Raised Cosine Pulse, g(t)

$$g(t)=\frac{\sin[\pi(1-r)t']+4rt'\cos[\pi(1+r)t']}{\pi t'[1-(4rt')^2]}$$

where t'=t/T and $0 \le r \le 1$.

The spectrum G(f), is

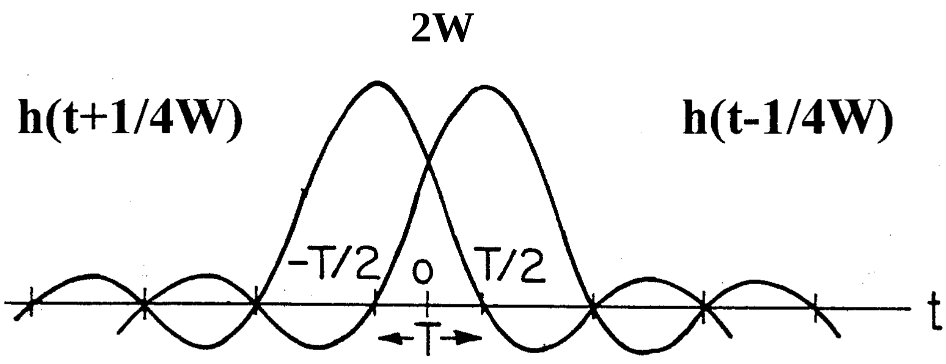
$$G(f)=\begin{cases} T, & 0 \le |f| \le (1-r)(1/2T) \\ \frac{T}{2}\sqrt{1-\sin\left[(\pi T/r)\{|f|-(1/2T)\}\right]}, & \text{for } (1-r)(1/2T) \le |f| \le (1+r)(1/2T) \end{cases}$$

***G(f) is the Square-Root Nyquist Spectrum,**

i.e., $G(f)=\sqrt{H(f)}$

PARTIAL RESPONSE SIGNALS

Duobinary Signal



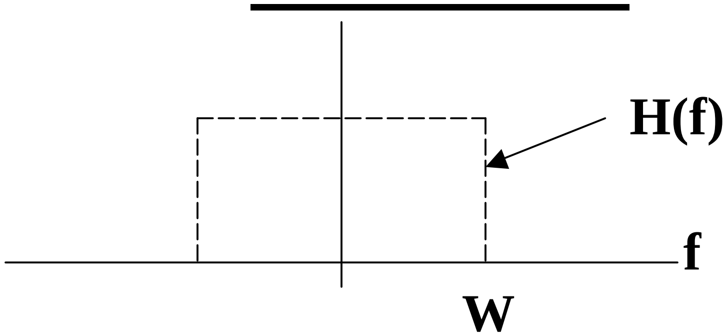
T=1/2W

$$p(t)= h(t-1/4W)+h(t+1/4W)$$

$$P(f)=H(f)\, e^{-j2\pi f/4W}+H(f)\, e^{j2\pi f/4W}$$

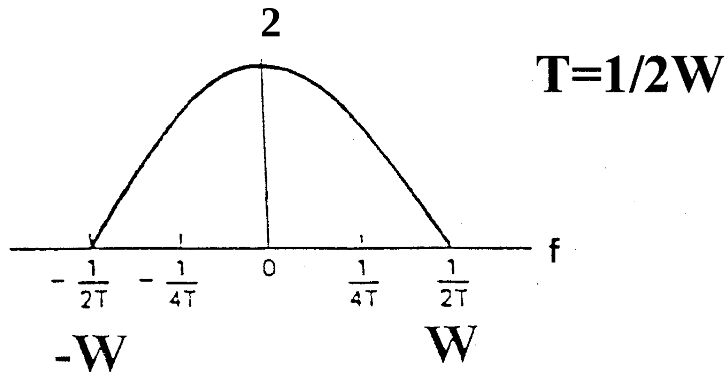
$$P(f)=2\, H(f) \cos 2\pi f/4W$$

- **Introduces controlled ISI**

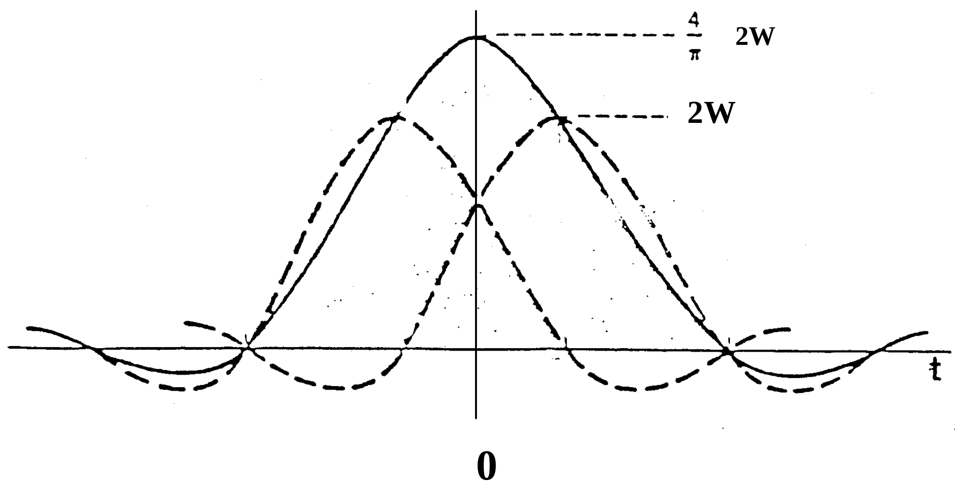


DUOBINARY SIGNAL

P(f)



p(t)



$$p(t) = \frac{4}{\pi}(2W)\,\frac{\cos 2\pi Wt}{1-(4Wt)^2}$$

- There is actually a 2.1 dB detectability loss

NYQUIST BASEBAND SIGNALING

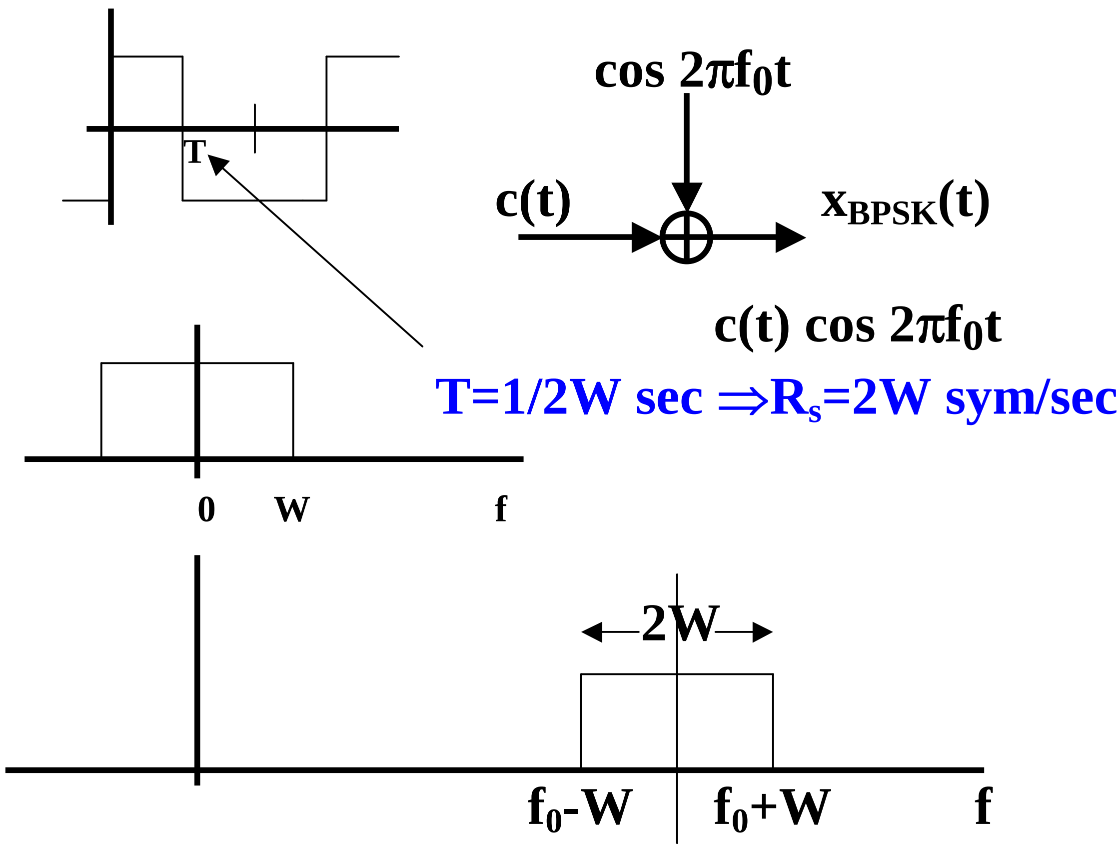
FOR BPSK, QPSK, SQPSK, MPSK, QAM

- **The perfectly time-limited rectangular pulses are replaced by perfectly bandlimited Nyquist signals.Theoretically the rolloff is 0%.**

- **NOT constant envelope**

These signals are used when we are looking for *narrow bandwidth signals* -However we *lose* the *constant envelope* properties of the original modulations.

BPSK
Nyquist I Signals



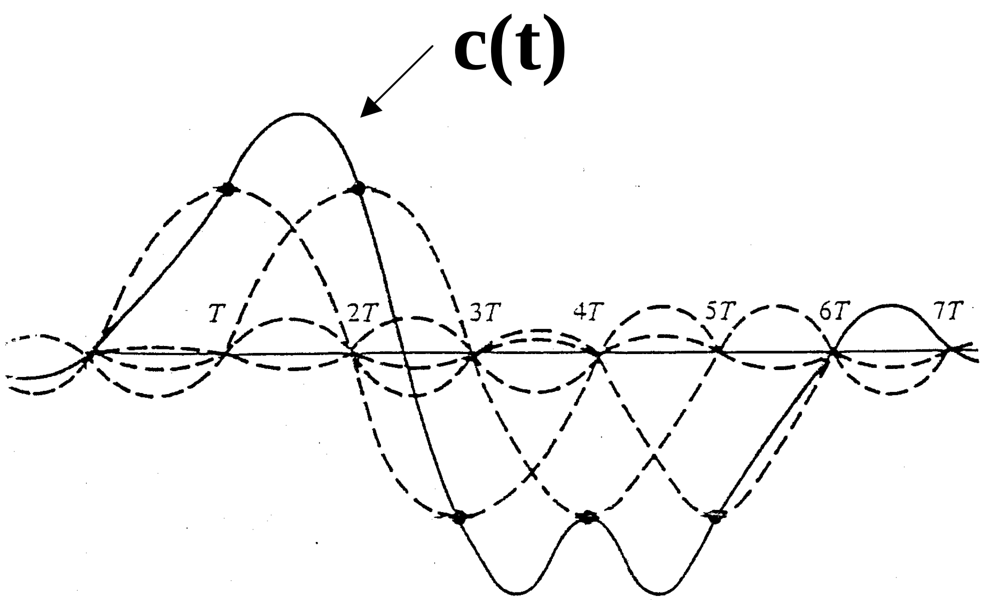
R_S=2W sym/sec

Notice that using the <u>ideal 0% Nyquist</u> signal generates a transmitted signal with a <u>bandwidth exactly equal to the symbol rate, R_S.</u>

We replace the perfectly time-limited rectangular pulses with perfectly bandlimited Nyquist signals.

The signal which now multiples the cosine carrier is c(t). A typical c(t) is shown below.

A typical baseband signal, c(t), generated by a series of Nyquist I signals with no ISI.



BPSK

-The output signal, $c(t) \cos 2\pi f_0 t$, is no longer constant envelope