Coding and Signal Processing for MIMO Communications - A Primer

Inaki Berenguer* and Xiaodong Wang
Department of Electrical Engineering
Columbia University
New York, NY 10027
{ib, wangx}@ee.columbia.edu

Abstract

Rapid growth in mobile computing and other wireless multimedia services is inspiring many research and development activities on high-speed wireless communication systems. Main challenges in this area include the development of efficient coding and modulation signal processing techniques to improve the quality and spectral efficiency of wireless systems. The recently emerged space-time coding and signal processing techniques for wireless communication systems employing multiple transmit and receive antennas offer a powerful paradigm for meeting these challenges. This paper provides an overview on the recent development in space-time coding and signal processing techniques for multiple-input multiple-output (MIMO) communication systems. We first review the information theoretic results on the capacities of wireless systems employing multiple transmit and receive antennas. We then describe two representative categories of space-time systems, namely, the BLAST systems and the space-time block coding systems. Signal processing techniques for channel estimation and decoding in space-time systems are also discussed. Finally, some other coding and signal processing techniques for wireless systems employing multiple transmit and receive antennas are also briefly touched upon.

Keywords: Multiple antennas, wireless communications, channel capacity, space-time coding.

*I. Berenguer is supported by a Fulbright fellowship.

1 Introduction

Multiple-input multiple-output (MIMO) communication technology has received significant recent attention due to the rapid development of high-speed broadband wireless communication systems employing multiple transmit and receive antennas. Information theoretic results show that MIMO systems can offer significant capacity gains over traditional single-input single-output channels [17, 45]. This increase in capacity is enabled by the fact that in rich scattering wireless environments, the signals from each individual transmitter appear highly uncorrelated at each of the receive antennas. When conveyed through uncorrelated channels between the transmitter and the receiver, the signals corresponding to each of the individual transmit antennas have attained different spatial signatures. The receiver can exploit these differences in spatial signatures to separate the signals originated from different transmit antennas.

Many MIMO techniques have been proposed targeting at different scenarios in wireless communications. The Bell-Labs Layered Space Time (BLAST) system [16, 47] is a layered space-time architecture originally proposed by Bell-Labs to achieve high data rate wireless transmissions. In this scheme, different symbol streams are simultaneously transmitted from all transmit antennas (i.e., they overlap in frequency and in time). The receive antennas receive the superposition of all symbol streams and recover them via proper signal processing. On the other hand, in Space-Time Coding (STC) systems [2, 40, 41, 43], the same information symbol stream is transmitted from different transmit antennas in appropriate manner to obtain transmit diversity. Hence, in STC systems the MIMO channel is exploited to provide more reliable communications, whereas in the BLAST system the MIMO channel is used to provide faster communications. By employing higher signal constellations the STC systems can achieve high throughput just like the BLAST system. In this paper, we give a general overview of the capacity results for MIMO systems as well as the BLAST and STC techniques.

The remainder of this paper is organized as follows. In Section 2 we summarize the capacity results for MIMO systems and discuss the impact of antenna correlation on capacity. In Section 3, we describe the BLAST system and related decoding and channel estimation techniques. In Section 4, we discuss space-time coding techniques and in particular the space-time block codes. Performance comparisons between the BLAST system and the space-time block coding system are also made. Finally, in Section 5, we briefly touch upon some other space-time coding and signal processing techniques.

2 Capacity of MIMO Systems

In this section, we summarize the information theoretic results on the capacities of MIMO channels, developed in the late 1990s [45, 17]. These results show the significant potential gains in channel capacity by employing multiple antennas at both the transmitter and receiver ends; and inspired an enormous surge of world-wide research activities to develop space-time coding and signal processing techniques that can approach the MIMO channel capacity.

2.1 Capacity Results

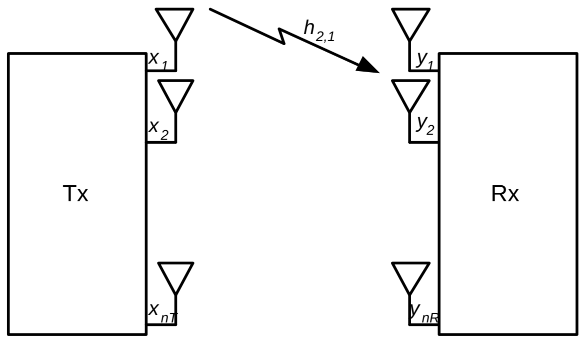

Figure 1: Schematic representation of a MIMO system.

Consider a MIMO system with n_T transmit antennas and n_R receive antennas signaling through flat fading channels, as shown in Figure 1. The input-output relationship of this system is given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{v}, \tag{1}$$

where $\mathbf{x} = \left[\ x_1, x_2, ..., x_{n_T}\ \right]^T$ is the $(n_T \times 1)$ transmitted signal vector, $\mathbf{y} = \left[\ y_1, y_2, ..., y_{n_R}\ \right]^T$ is the $(n_R \times 1)$ received signal vector, $\mathbf{v} = \left[\ v_1, v_2, ..., v_{n_R}\ \right]^T$ is the received noise vector and

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{21} & \cdots & h_{1,n_T} \\ h_{21} & h_{22} & \cdots & h_{2,n_T} \\ \vdots & & \ddots & \vdots \\ h_{n_R,1} & h_{n_R,1} & \cdots & h_{n_R,n_T} \end{bmatrix} \tag{2}$$

is the $(n_R \times n_T)$ MIMO channel matrix with h_{ij} representing the complex gain of the channel between the jth transmit antenna and the ith receive antenna.

It is assumed that the noise sample v_i, $i = 1, 2, ..., n_R$, is a circularly symmetric complex Gaussian random variable with zero mean and variance σ^2, denoted as $v_i \sim \mathcal{N}_c(0,\ \sigma^2)$. That is, $\Re\{v_i\} \sim \mathcal{N}(0,\ \frac{\sigma^2}{2})$, $\Im\{v_i\} \sim \mathcal{N}(0,\ \frac{\sigma^2}{2})$, and they are independent. It is assumed that the complex channel gains $h_{ij} \sim \mathcal{N}_c(0,\ 1)$. Note that in general, the channel gains may be correlated.

Assuming that the channel matrix $\mathbf{H}$ is known at the receiver, but not at the transmitter, the ergodic (mean) capacity of the MIMO channel with an average total transmit power P (i.e., $tr\Big(E\left\{\mathbf{x}\mathbf{x}^H\right\}\Big) \leq P$) is given by [45, 17]

$$C \ = \ E\left\{\log\det\left(\mathbf{I}_{n_R} + \frac{1}{n_T}\frac{P}{\sigma^2}\mathbf{H}\mathbf{H}^H\right)\right\}$$

$$= \quad E\left\{\log\det\left(\mathbf{I}_{n_T} + \frac{1}{n_T}\frac{P}{\sigma^2}\mathbf{H}^H\mathbf{H}\right)\right\} \quad \text{bits/s/Hz}, \tag{3}$$

where the expectation is taken with respect to the distribution of the random channel matrix $\mathbf{H}$.

To gain some insight on the capacity expression in (3), denote $\rho = P/\sigma^2$. Then the capacity can be expressed as

$$C = \sum_{k=1}^{p} E\{\log(1+\frac{\rho}{n_T}\lambda_k)\}, \tag{4}$$

where $p = \min\{n_T, n_R\}$ and $\lambda_1, ..., \lambda_p$ are the eigenvalues of the matrix $\mathbf{H}\mathbf{H}^H$ or $\mathbf{H}^H\mathbf{H}$. Note that the matrices $\mathbf{H}\mathbf{H}^H$ and $\mathbf{H}^H\mathbf{H}$ have the same eigenvalues which are all real and non-negative. If we compare (4) with the capacity of a single-input single-output (SISO) channel [11], we observe that the capacity of a MIMO system is equivalent to the sum of p parallel SISO channels, each one with an equivalent signal-to-noise ratio equal to λ_i.

Furthermore, it can be shown that when both n_T and n_R increase, the capacity increases *linearly* with respect to $\min\{n_T,\, n_R\}$. On the other hand, if n_R is fixed and n_T increases, then the capacity saturates at some fixed value; whereas if n_T is fixed and n_R increases, the capacity increases logarithmically with n_R. These asymptotic behaviors of the ergodic capacity are shown in Figure 2.

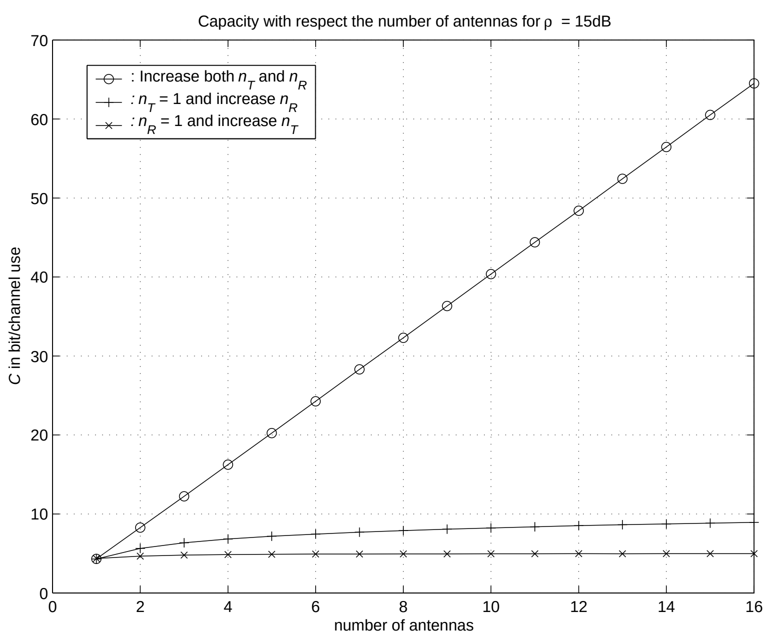

Figure 2: Ergodic capacities of uncorrelated MIMO channels. The channel is known at the receiver but not at the transmitter.

Another notion that is frequently used in practice is the outage capacity. Define the instantaneous capacity as

$$\phi\left(\mathbf{H},\rho\right)=\log\det\left(\mathbf{I}_{n_R}+\frac{\rho}{n_T}\mathbf{H}\mathbf{H}^H\right). \tag{5}$$

Obviously $\phi(\mathbf{H},\rho)$ is a random variable since $\mathbf{H}$ is random. Given a certain outage probability P_{out}, the corresponding outage capacity C_{out} is defined through the following equation,

$$P\left\{\phi(\mathbf{H},\rho)\le C_{out}\right\}=P_{out}. \tag{6}$$

So far we have assumed that the channel matrix $\mathbf{H}$ is known at the receiver but not at the transmitter. Another scenario is that the channel is known at both the transmitter and receiver. This is the case, for example, when the system employs time-division duplex (TDD) so that the uplink and downlink channels are reciprocal to each other. In this case, the instantaneous capacity is given by the following "water-filling" equation [38]

$$\psi(\mathbf{H},\rho)=\sum_{i=1}^{n_T}\left[\log\left(\upsilon\lambda_i\right)\right]^+\quad\text{bits/s/Hz}, \tag{7}$$

where $\lambda_1,...,\lambda_{n_T}$ are the eigenvalues of the matrix $\mathbf{H}^H\mathbf{H}$, υ is chosen such that $\rho=\sum_{i=1}^{n_T}\left[\upsilon-\frac{1}{\lambda_i}\right]^+$ and the operator $(\cdot)^+$ is specified as

$$(x)^+=\begin{cases}x & \text{if}\quad x>0,\\ 0 & \text{if}\quad x\le 0.\end{cases} \tag{8}$$

The ergodic capacity is then given by $C=E\{\psi(\mathbf{H},\rho)\}$. Moreover, the outage capacity in this case is specified by

$$P\left\{\psi(\mathbf{H},\rho)\le C_{out}\right\}=P_{out}. \tag{9}$$

Figure 3 shows the 10% outage capacity of uncorrelated MIMO channels with and without water-filling. It is seen that by knowing the channel at the transmitter, some capacity gain can be obtained at low signal-to-noise ratio.

2.2 Effects of Antenna Correlations

It has been observed that antennas placed with large enough separations will receive essentially uncorrelated signals [30]. However, in handsets or terminals, large separations among the antennas may not be feasible. On the other hand, when the transmitter or receiver is not surrounded by scatterers, no local scattering or diversity occurs, and the spatial fading at the antennas is correlated. Hence, insufficient antenna spacing and lack of scattering cause the individual antennas to be correlated.

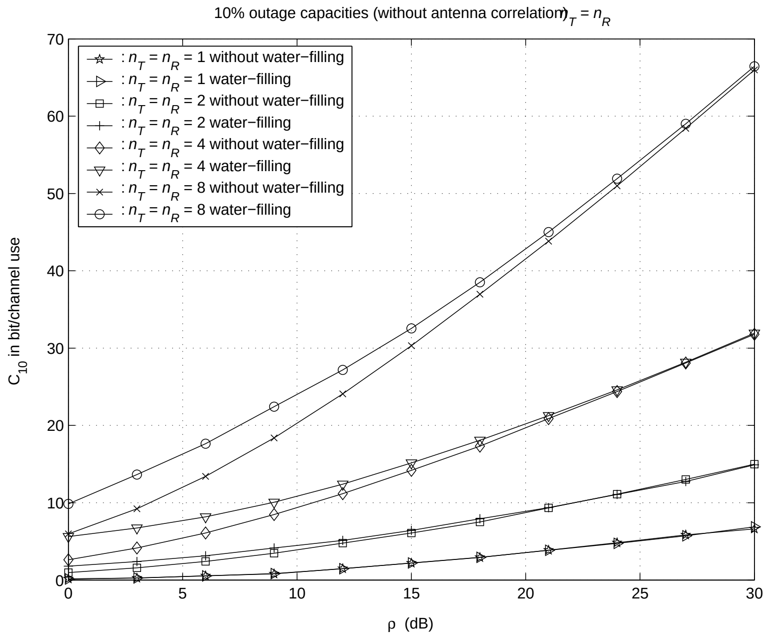

Figure 3: 10% outage capacities of uncorrelated MIMO channels with and without employing water-filling.

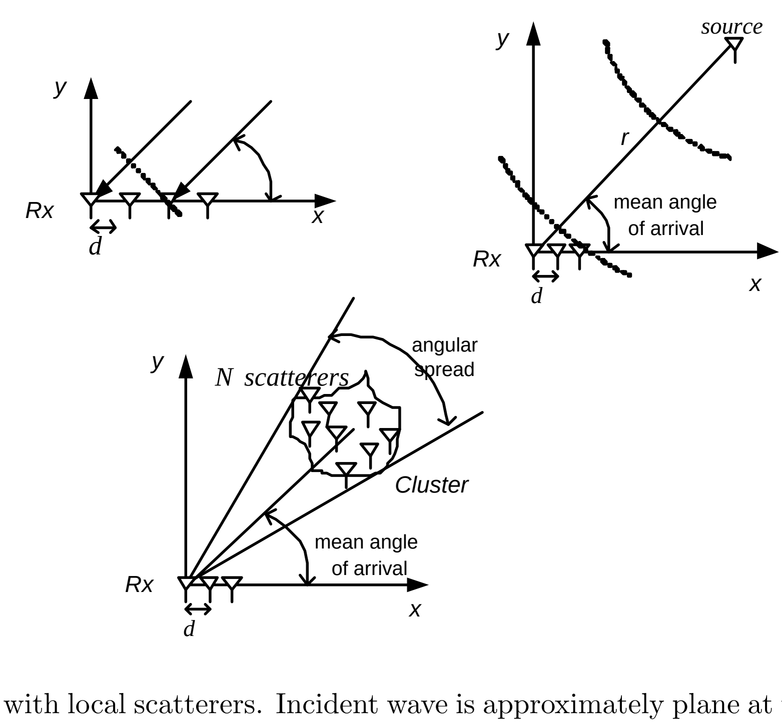

Figure 4: Model with local scatterers. Incident wave is approximately plane at the receiving array.

We next discuss the correlation model and the effect of antenna correlation on capacity. Following [8], assuming correlations at both the transmitter and receiver, the $(n_R \times n_T)$ channel response matrix can be modeled as

$$\mathbf{H} = \mathbf{R}_r^{1/2}\mathbf{H}_w\mathbf{R}_t^{1/2} \tag{10}$$

with $\mathbf{H}_w$ being an uncorrelated $(n_R \times n_T)$ matrix with i.i.d. $\mathcal{N}_c(0,1)$ entries and $\mathbf{R}_t$ and $\mathbf{R}_r$, of size $(n_T \times n_T)$ and $(n_R \times n_R)$, representing the covariance matrices inducing transmit and receive correlations respectively. Note that for the case of uncorrelated transmitter (receiver), we have $\mathbf{R}_t = \mathbf{I}$ ($\mathbf{R}_r = \mathbf{I}$).

The form of cross-correlation between the waves impinging on antenna elements (i.e., $\mathbf{R}_r$ or $\mathbf{R}_t$) has been studied and modeled in several references [3, 8, 10, 14, 38]. These models use similar parameters to characterize the correlation. Specifically, assuming that no line of sight exists between the transmit and the receive antennas, the signal reaching the receive antennas can be modeled as arriving from a number of equivalent point sources or scatterers in the vicinity of the receiver as shown in Figure 4. Assuming that the antennas are omnidirectional (i.e. they radiate and receive from all directions in space), there are three main parameters that characterize the correlation between antennas (see Figure 4):

- Distance d between antennas in terms of wavelengths,
- Angular spread of the arrival incident waves δ_o^R,
- Mean angle of arrival of incident waves ϕ_o^R.

Large values of the angular spread δ_o^R result in uncorrelated signals at each of the antennas. The angular spread is a function of the distance of the cluster to the antenna array and radius of the cluster. For example, in an outdoor environment, a cluster could be a building located far away from the antenna array deriving in a small angular spread δ_o^R. In an indoor environment, the cluster of scatterers will be the walls surrounding the array. In this case, there will be signals impinging the antenna array from all directions resulting in a large angular spread value; therefore, uncorrelated fading among the antennas can be expected. Figure 5 depicts different scattering scenarios similar to those defined for COST-259 models [38]. In this representation, the circle represents a cluster of scatterers. The five different scenarios correspond to:

- Uplink: This scenario corresponds to a base station operating as a receiver from some high point without any scatterer nearby. The receiver, usually a handset or terminal, will be surrounded by scatterers. The angular spread corresponding to the receiver (i.e., base station) is very low resulting in correlation among the receive antennas.

- Downlink: This scenario is similar to the uplink but with the base station acting as a transmitter.
- Urban area: Medium size angular spread for both the transmitter and the receiver. Scatterer clusters represent buildings.
- Rural area: Low angular spread for both the transmitter and the receiver. Scatterer clusters represent mountains and hills.
- Indoor: Large angular spread for both the transmitter and the receiver. Impinging waves arrive from all directions in the space.

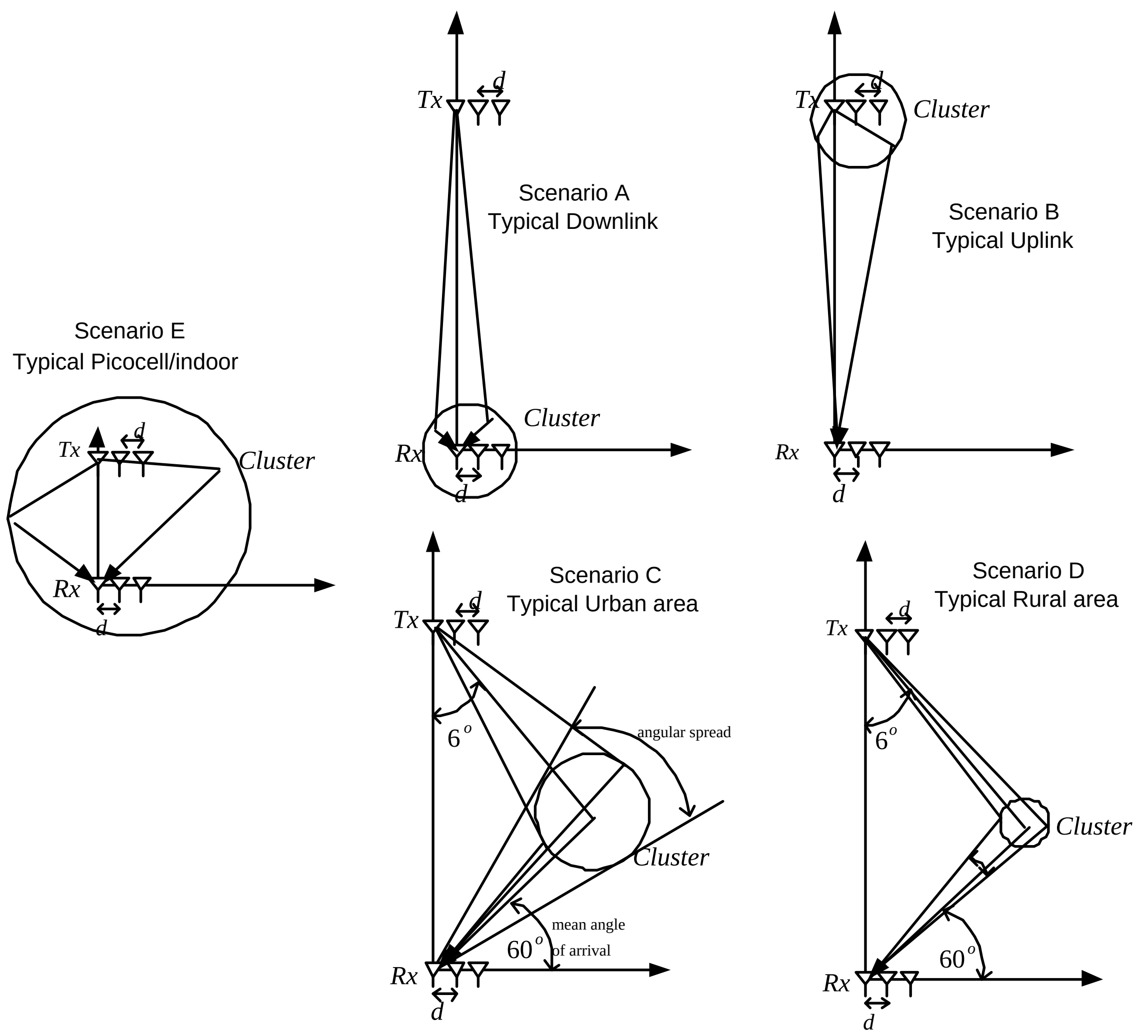

Figure 5: MIMO scattering scenarios.

Figure 6 shows the 10% outage capacities for the different scenarios defined in Figure 4 with $n_T = n_R = 4$ and an antenna spacing of $d = 0.5\lambda$. We assume that the channel is known at the receiver but not at the transmitter. We have used the correlation model described in [3]. We also show the SISO capacity for comparison. It is seen that urban and indoor scenarios with rich scattering offer much higher MIMO capacities than rural environments.

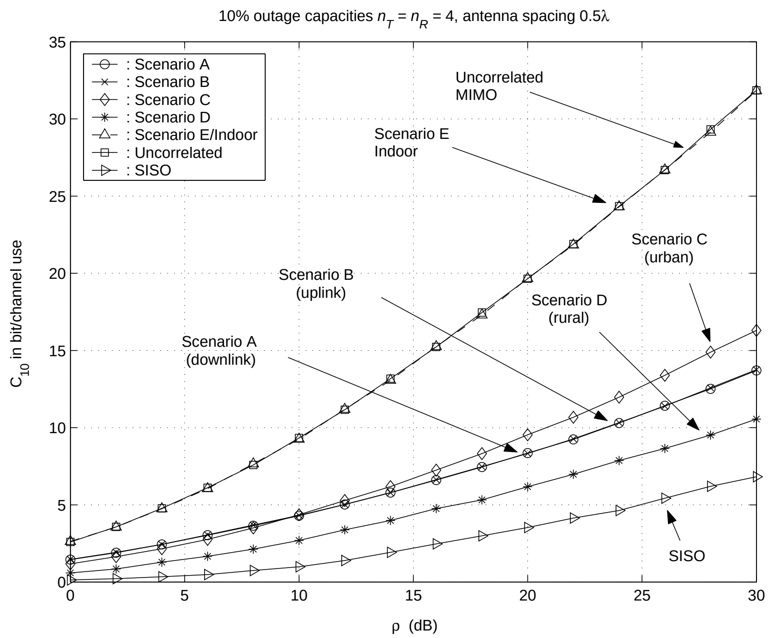

Figure 6: MIMO outage capacities for different channel scenarios described in Figure 5.

Figure 7 shows the 10% outage capacities of a correlated MIMO channel with and without water-filling. The correlation scenario corresponds to an urban area depicted in Figure 5 with an antenna spacing of $d = 0.5\lambda$. Comparing with Figure 3, it is seen that significant capacity gain can be achieved when there are antenna correlations and the channel is known at both the transmitter and the receiver.

3 The BLAST System

The information theoretical results from the preceding section indicates the enormous capacity gain by employing multiple antennas at both ends of the communication systems. Realizing such a potential gain, researchers at Bell-Labs developed the first MIMO architecture for high-speed wireless communications – the BLAST systems.

BLAST (Bell-Labs Layered Space Time) [16, 47] is a high speed wireless communication scheme employing multiple antennas at both the transmitter and the receiver. In a BLAST system, the transmitted data is split equally into n_T transmit antennas and then simultaneously sent to the channel overlapping in time and frequency. The signals are received by n_R receive antennas as shown in Figure 8 and signal processing at the receiver attempts to separate the received signals and recover the transmitted data. The input-output relationship of a BLAST

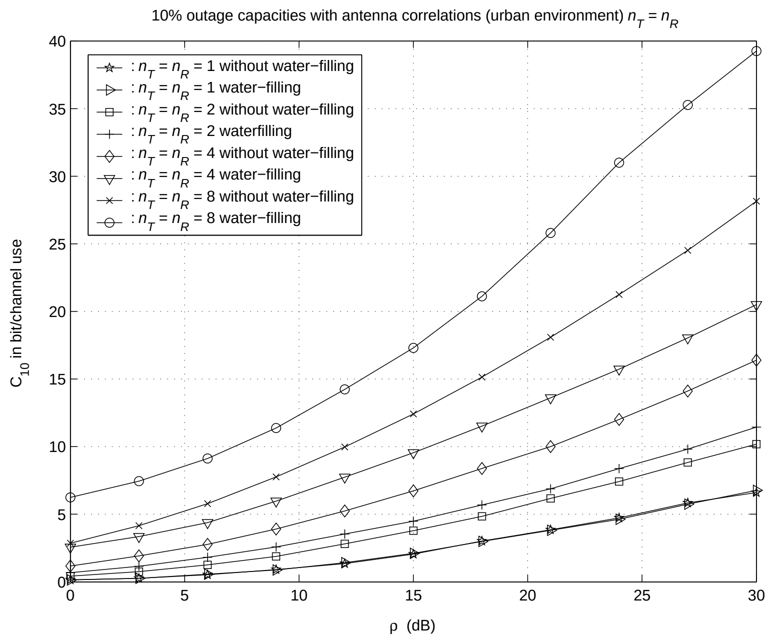

Figure 7: 10% outage capacities of a correlated MIMO channel corresponding to an urban scenario, with and without employing water-filling.

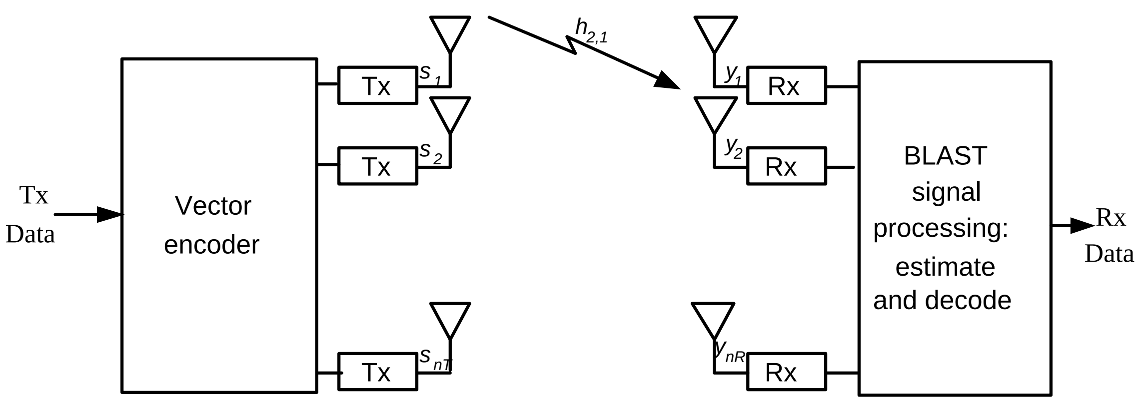

Figure 8: Schematic representation of a BLAST system.

system can be expressed as

$$\mathbf{y} = \sqrt{\frac{\rho}{n_T}}\mathbf{H}\mathbf{s} + \mathbf{v} \tag{11}$$

where $\mathbf{s} = \begin{bmatrix} s_1, s_2, ..., s_{n_T} \end{bmatrix}^T$ is the $(n_T \times 1)$ transmit signal vector with s_i belonging to a finite constellation $\mathcal{A}$, $\mathbf{v} = \begin{bmatrix} v_1, v_2, ..., v_{n_R} \end{bmatrix}^T$ is the $(n_R \times 1)$ receive noise vector with $v_i \sim \mathcal{N}_c(0,\, 1)$, $\mathbf{H}$ is defined in (2) and ρ is the total signal-to-noise ratio independent of the number of transmit antennas. It is assumed unitary power for the transmitted symbols, $E\left\{|s_i|^2\right\} = 1$.

3.1 BLAST Detection Algorithms

It is seen from (11) that the receive antennas see the superposition of all the transmitted signals. The task of a BLAST detector is to recover the transmitted data $\mathbf{s}$ from the received signal $\mathbf{y}$. In what follows, we describe several BLAST detection algorithms [18, 20]. Here we assume the channel matrix $\mathbf{H}$ is known at the receiver. We will discuss channel estimation algorithms in Section 3.2.

Maximum Likelihood (ML) Receiver

The ML detector is the optimal receiver in terms of bit error rate. Let $\mathcal{A}$ be the symbol constellation set (e.g., QPSK or M-QAM) whose size is M. Then, the ML detection rule is given by

$$\hat{\mathbf{s}} = \arg \min_{\mathbf{s} \in \mathcal{A}^{n_T}} \left\| \mathbf{y} - \sqrt{\frac{\rho}{n_T}}\mathbf{H}\mathbf{s} \right\|^2 . \tag{12}$$

Note that the minimization problem is performed over all possible transmitted signal vectors $\mathbf{s}$ in the set $\mathcal{A}^{n_T}$. The computational complexity of an exhaustive search is then $\mathcal{O}(M^{n_T})$. Hence, although the ML receiver is optimal, its complexity grows exponentially with the number of transmit antennas. A low complexity local search method called "sphere decoding" whose complexity is $\mathcal{O}(M^3)$ is developed in [12, 15].

Zero Forcing and Cancellation Receiver

A simpler receiver is the zero forcing (ZF) receiver. The ZF receiver considers the signal from each transmit antenna as the desired signal and the remainder as interferers. Nulling is performed by linearly weighting the received signals to satisfy the ZF criterion, i.e., inverting the channel response. Furthermore, a superior performance can be obtained by using nonlinear techniques by means of symbol cancellation. Using symbol cancellation, the already detected and sliced symbol from each transmit antenna is subtracted out from the received signal vector, similarly to decision feedback equalization or multiuser detection with successive interference cancellation. Therefore, the next signal to be decoded will see one interferer less.

For simplicity, assume $n = n_T = n_R$. Denote the QR factorization of $\mathbf{H}$ as $\mathbf{H} = \mathbf{QR}$ where $\mathbf{Q}$ is unitary, i.e., $\mathbf{QQ}^H = \mathbf{I}$ and $\mathbf{R}$ is upper triangular. The nulling operation of the received vector $\mathbf{y}$ is performed by

$$\mathbf{z} = \mathbf{Q}^H\mathbf{y} = \sqrt{\frac{\rho}{n_T}}\mathbf{RS} + \mathbf{Q}^H\mathbf{v}; \tag{13}$$

that is

$$\begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix} = \sqrt{\frac{\rho}{n}} \begin{bmatrix} r_{1,1} & r_{1,2} & \dots & r_{1,n} \\ 0 & r_{2,2} & \dots & r_{2,n} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & r_{n,n} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}. \tag{14}$$

Note that since $\mathbf{Q}$ is unitary, there is no noise amplification, i.e., $\mathbf{w} = \mathbf{Q}^H\mathbf{v}$ is also $\mathcal{N}_c(0, \mathbf{I})$. In (14), the decision statistic z_n is just a noisy scaled version of s_n which can be directly estimated and then subtracted from z_{n-1}. Repeating the estimating and subtracting operations until all transmitted signals are decoded, the algorithm can be summarized as follows

$$\boxed{\begin{aligned} \hat{s}_n &= \text{Quantize}\left\{\frac{1}{r_{n,n}}\sqrt{\frac{n}{\rho}}z_n\right\} \\ \hat{s}_{n-1} &= \text{Quantize}\left\{\frac{1}{r_{n-1,n-1}}\left(\sqrt{\frac{n}{\rho}}z_{n-1} - r_{n-1,n}\hat{s}_n\right)\right\} \\ &\vdots \\ \hat{s}_i &= \text{Quantize}\left\{\frac{1}{r_{i,i}}\left(\sqrt{\frac{n}{\rho}}z_i - \sum_{k=i+1}^{n} r_{i,k}\hat{s}_k\right)\right\} \\ &\vdots \\ \hat{s}_1 &= \text{Quantize}\left\{\frac{1}{r_{1,1}}\left(\sqrt{\frac{n}{\rho}}z_{n-1} - \sum_{k=2}^{n} r_{1,k}\hat{s}_k\right)\right\} \end{aligned}}$$

where the quantizer takes values from the constellation $\mathcal{A}$.

Nulling and Cancellation Receivers with Ordering

In the decoding algorithm discussed above, a wrong decision in the detection of a symbol adds interference to the next symbols to be detected. It is shown in [18, 20] that it is advantageous to first find and detect the symbol s_k with the highest signal to-noise ratio, i.e., with the highest reliability. The detected symbol is then subtracted from the rest of the received signals. Therefore, after cancelling s_k, we have a system with $n_T - 1$ transmit antennas and n_R receive antennas, i.e., the corresponding channel matrix is obtained by removing column k from $\mathbf{H}$. The same process is then applied on this $(n_T - 1, n_R)$ system and the algorithm continues until all transmitted symbols have been decoded. That is, the nulling and cancellation operation is performed from the more reliable symbols to the less reliable ones.

The nulling operation can be performed by means of ZF or minimum mean-square error (MMSE). Similarly to the ZF equalization in single antenna systems, the ZF criterion yields the following two problems: (1) The algorithm can encounter singular matrices that are not invertible; and (2) ZF focuses on cancelling the interference (i.e., overlapping signals) completely at the expense of enhancing the noise, possibly significantly. On the other hand, the MMSE criterion minimizes the error due to the noise and the interference combined. In the ordering operation, the MMSE method nulls the component with the smallest MSE. Following [20], the BLAST decoding algorithm based on the MMSE nulling and cancellation with ordering is as follows:

$\mathbf{G}$	$=$	$\mathbf{H}$	
$\mathbf{r}$	$=$	$\mathbf{y}$	
FOR	$i = 1 : n_T$	DO	
$\mathbf{P}$	$=$	$(\frac{\rho}{n}\mathbf{G}^H\mathbf{G} + \mathbf{I})^{-1}$	
k_i	$=$	$\text{argmin}\{P_{j,j}\}, \quad j \notin \{k_1, k_2, ..., k_{i-1}\}$	(ordering: find min MSE)
$\mathbf{w}$	$=$	$(\mathbf{GP})(:, k_i)$	(nulling vector)
z	$=$	$\mathbf{w}^H\mathbf{r}$	
$\hat{s}_{k_i}$	$=$	$\text{Quantize}(z)$	
$\mathbf{r}$	$=$	$\mathbf{r} - \sqrt{\frac{\rho}{n}}\mathbf{H}(:, k_i)\hat{s}_{k_i}$	(cancellation)
$\mathbf{G}$	$=$	$\mathbf{G} \setminus \mathbf{H}(:, k_i)$	(remove column of that transmit antenna)
END			

Figure 9 compares the BER performance of the four detection methods discussed above in a BLAST system with $n_T = n_R = 4$ antennas and QPSK modulation. It is seen that the ML decoder has the best BER performance although for every transmitted code vector, the receiver needs to evaluate (12) over $4^4 = 256$ possibilities. On the other hand, the MMSE nulling and cancellation algorithm with ordering exhibits the best performance among the suboptimal algorithms.

3.2 MIMO Channel Estimation Algorithms

So far, we have assumed that the MIMO channel matrix $\mathbf{H}$ is known at the receiver. In practice, the receiver needs to estimate this matrix prior to the start of the decoding process. We next discuss the channel estimation methods based on a training preamble [34].

Suppose $T \geq n_T$ MIMO training symbols $\mathbf{s}(1)$, $\mathbf{s}(2)$, ..., $\mathbf{s}(T)$ are used to probe the channel. The received signals corresponding to these training symbols are

$$\mathbf{y}(i) = \sqrt{\frac{\rho}{n_T}}\mathbf{H}\mathbf{s}(i) + \mathbf{v}(i), \quad i = 1, 2, ..., T. \tag{15}$$

Denote $\mathbf{Y} = [\ \mathbf{y}(1), \mathbf{y}(2), ..., \mathbf{y}(T)\ ]$, $\mathbf{S} = [\ \mathbf{s}(1), \mathbf{s}(2), ..., \mathbf{s}(T)\ ]$ and $\mathbf{V} = [\ \mathbf{v}(1), \mathbf{v}(2), ..., \mathbf{v}(T)\ ]$. Then (15) can be written as

$$\mathbf{Y} = \sqrt{\frac{\rho}{n_T}}\mathbf{H}\mathbf{S} + \mathbf{V}. \tag{16}$$

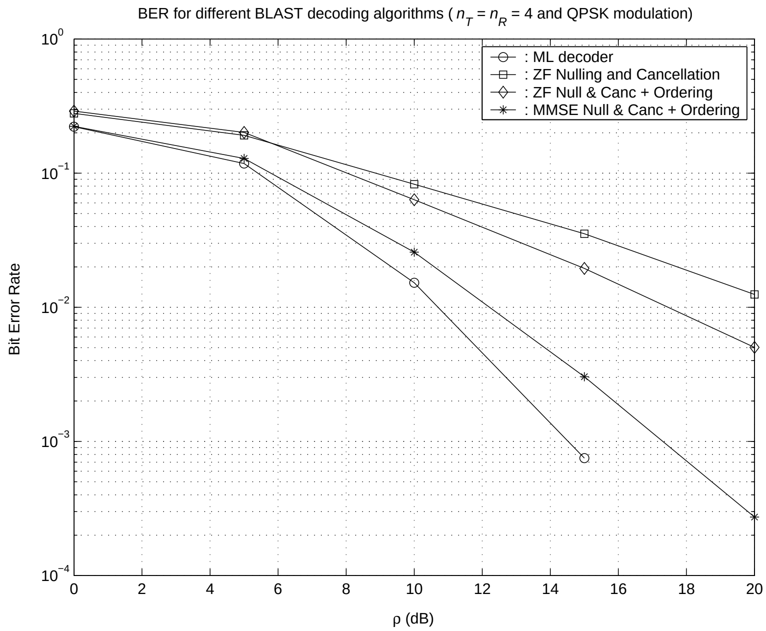

Figure 9: BER performance of different BLAST decoding algorithms with $n_T = n_R = 4$ and QPSK. Uncorrelated MIMO channels and perfect channel knowledge at the receiver are assumed.

The maximum likelihood estimate of the channel matrix $\mathbf{H}$ is given by

$$\begin{aligned} \hat{\mathbf{H}}_{ML} &= \arg\min_{\mathbf{H}} \left\| \mathbf{Y} - \sqrt{\frac{\rho}{n_T}} \mathbf{H}\mathbf{S} \right\|^2 \\ &= \sqrt{\frac{n_T}{\rho}} \mathbf{Y}\mathbf{S}^H (\mathbf{S}\mathbf{S}^H)^{-1}. \end{aligned} \tag{17}$$

According to [34], the optimal training symbol sequence $\mathbf{S}$ that minimizes the channel estimation error should satisfy

$$\mathbf{S}\mathbf{S}^H = T \cdot \mathbf{I}_{n_T}. \tag{18}$$

One way to generate such optimal training sequences is to use the Hadamard matrices [26] (when they exist for n_T). As an example, consider a system with $n_T = 4$ and a training sequence length $T = 16$ symbol intervals. We first generate a (4×4) Hadamard matrix as

$$\mathbf{A} = \frac{1+i}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}. \tag{19}$$

Then the optimal training sequence can be constructed by concatenating four $\mathbf{A}$ matrices as

$$\mathbf{S} = \begin{bmatrix} \mathbf{A} & \mathbf{A} & \mathbf{A} & \mathbf{A} \end{bmatrix}. \tag{20}$$

As an alternative to the ML channel estimator, the linear MMSE channel estimator is obtained as a linear transformation of the received signals $\mathbf{Y}$ that minimizes the estimation error and it is given by

$$\hat{\mathbf{H}}_{MMSE} = \sqrt{\frac{\rho}{n_T}}\mathbf{Y}\mathbf{S}^H\left(\frac{\rho}{n_T}\mathbf{S}\mathbf{S}^H + \mathbf{I}\right)^{-1}. \tag{21}$$

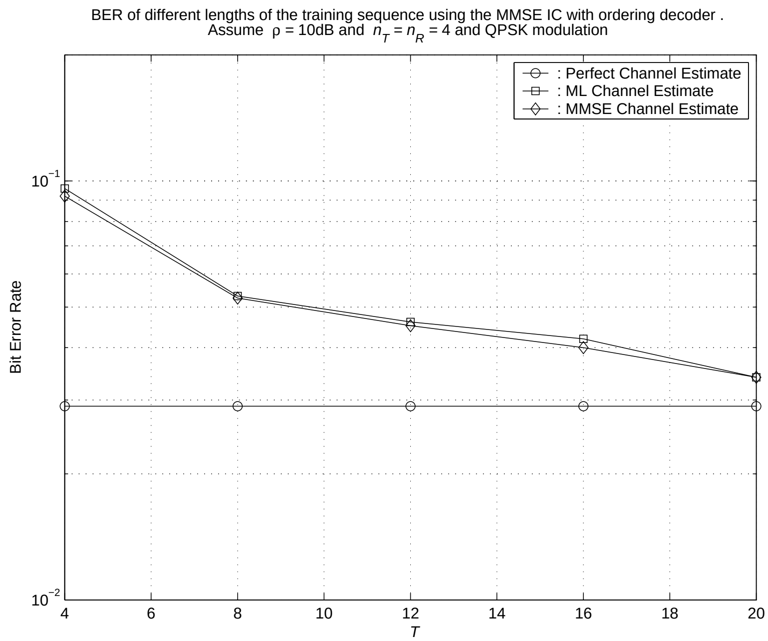

Figure 10: Effect of the training length T on the BER performance.

We next give a simulation example. Consider a BLAST system with $n_T = n_R = 4$ antennas and QPSK modulation. We assume uncorrelated fading and a signal-to-noise ratio $\rho = 10$dB. Figure 10 shows the BER of different channel estimation algorithms for different lengths of the optimal training sequence. As a decoder we use the MMSE nulling and cancellation with ordering algorithm. It is seen that the MMSE and ML channel estimators have similar performance. Figure 11 compares the BER performance of the MMSE nulling and cancellation with ordering decoder using the ML channel estimator with different lengths of the optimal training sequence.

4 Space-Time Coding

In the previous section, we discussed the BLAST system which increases the data rate by simultaneously transmitting symbols from multiple transmit antennas. However, the BLAST approach suffers from two major drawbacks: (1) it requires $n_R \geq n_T$ that is not always feasible when the receiver is a small or battery operated device; and (2) the performance of the suboptimal BLAST decoding algorithms is limited by error propagation. In this section, we discuss the space-time coding approach that exploits the concept of diversity.

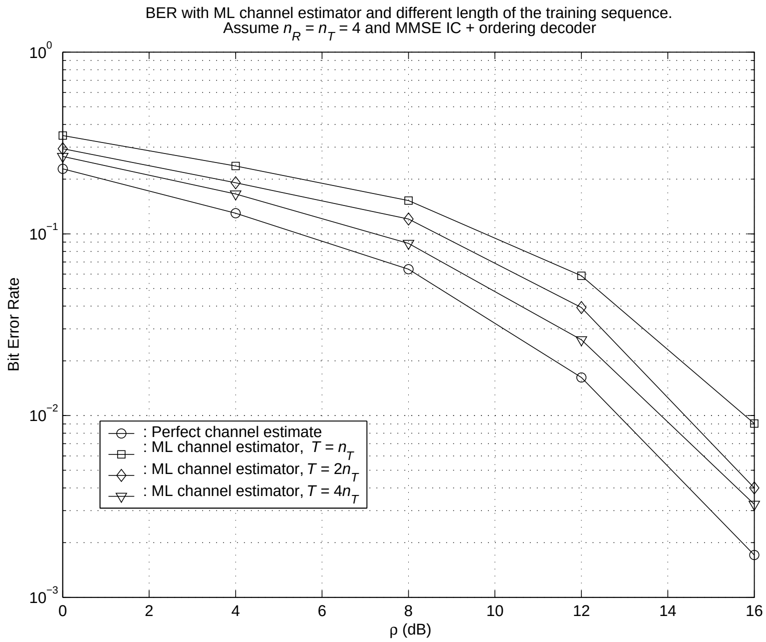

Figure 11: BER performance of the ML channel estimator with different lengths of the optimal training sequence.

4.1 The Concept of Diversity

With space-time codes (STC) [2, 40, 41, 43], instead of transmitting independent data streams as in BLAST, the same information is transmitted in appropriate manner simultaneously from different transmit antennas to obtain transmit diversity. The main idea of transmit diversity is that if a message is lost in a channel with probability p and we can transmit replicas of the message over n independent such channels, the loss probability becomes p^n. Using diversity, more reliability is given to the symbols which allows employing higher order constellation resulting in higher throughput just like the BLAST system. The main difference between BLAST and STC can be summarized as: (1) BLAST transmits more symbols, i.e., n_T symbols/channel use; and (2) STC transmits only (at most) 1 *reliable* symbol/channel use by means of diversity.

As an example, consider a systems willing to transmit 4 bit/s/Hz with 2 transmit antennas. BLAST would use QPSK symbols per antenna, i.e., 4 bit/s/Hz. STC can only send 1 symbol/channel use, therefore 16-QAM symbols would need to be employed. In the end, the same data is transmitted through higher order constellations. There are two main types of STCs, namely space time trellis codes (STTC) [43] and space time block codes (STBC) [41].

The STTC is an extension of trellis coded modulation [6] to the case of multiple transmit and receive antennas. It provides both full diversity and coding gain. However, it has the disadvantage of high decoding complexity which grows exponentially with the number of antennas. Specific space-time trellis codes designed for two or four antennas perform very well in slow fading environments and come within 2-3 dB of the outage capacity. STTC's are designed to achieve full diversity and then, among the codes that achieve full diversity, maximize the coding gain. For further references on STTC refer to [4, 43].

In hope of reducing the exponential decoding complexity of STTC, Alamouti proposed a simple space-time coding scheme using two transmit antennas [2]. Later, the STBC introduced in [40], generalized the Alamouti transmission scheme to an arbitrary number of transmit antennas. STBC achieve full diversity as the STTC although they do not provide any coding gain. This is not a problem since they can be concatenated with an outer channel code [5]. Besides achieving full diversity, the main property of STBC is that there is a very simple ML decoding algorithm based only on linear processing. These codes are based on some specific linear matrices and the reduced complexity receiver is due to the orthogonal properties of these matrices.

4.2 Space-Time Block Codes

We assume a wireless communication system where the transmitter is equipped with n_T and the receiver with n_R antennas. A space time block code matrix is represented as

$$\mathbf{C}_{p,n_T} = \overset{\leftarrow \textbf{space} \rightarrow}{\begin{bmatrix} c_{1,1} & c_{1,2} & \cdots & c_{1,n_T} \\ c_{2,1} & c_{2,2} & \cdots & c_{2,n_T} \\ \vdots & & \ddots & \vdots \\ c_{p,1} & c_{p,2} & \cdots & c_{p,n_T} \end{bmatrix}} \quad \begin{matrix} \uparrow \\ \textbf{time} \\ \downarrow \end{matrix} \tag{22}$$

At each time slot t, signals $c_{t,i}$, i=1,2,...,n_T, are transmitted simultaneously from the n_T transmit antennas as shown in Figure 12. Therefore, at time t, transmitter antenna i will transmit $c_{t,i}$ in the matrix ($1 \leq t \leq p$ and $1 \leq i \leq n_T$, with p being the length of the block code). Next, we describe the encoding and decoding operations of the STBC for two transmit antennas, namely the Alamouti code.

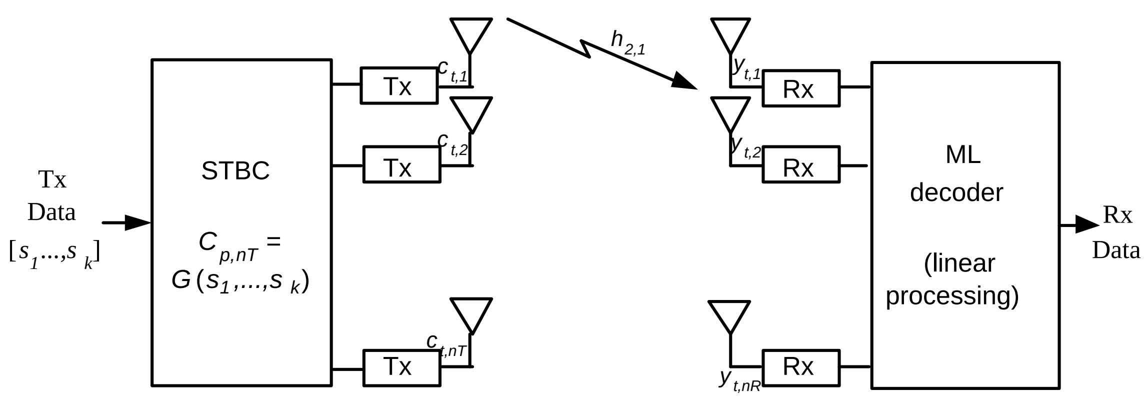

Figure 12: Schematic representation of an STBC system.

STBC with $n_T = 2$: Alamouti Code

The Alamouti code is an STBC using $n_T = 2$ transmit antennas and any number of receive antennas. The Alamouti code matrix $\mathbf{O}_{c,2}$ is defined as [2]

$$\mathbf{O}_{c,2} = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}. \tag{23}$$

Consider transmitting symbols of a signal constellation $\mathcal{A}$ of size 2^b. Every two time slots, $2b$ bits arrive at the encoder and select constellation signals s_1 and s_2. Setting $x_1 = s_1$ and $x_2 = s_2$ in $\mathbf{O}_{c,2}$, we arrive at the following transmission matrix

$$\mathbf{C}_{2,2} = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix}. \tag{24}$$

Then, in the first time slot, antenna 1 transmits s_1 and antenna 2 transmits s_2. In the next time slot, antenna 1 transmits $-s_2^*$ and antenna 2 transmits s_1^*. Since two time slots are needed to transmit two symbols (s_1, s_2), the rate of the code is $R = 1$ symbol/channel use.

At the receiver, the received signal by antenna i during two consecutive time slots (t=1,2) is

$$\begin{aligned} \begin{bmatrix} y_{1,i} \\ y_{2,i} \end{bmatrix} &= \sqrt{\frac{\rho}{2}}\mathbf{C}_{2,2}\mathbf{h}_i + \mathbf{v}_i \\ &= \sqrt{\frac{\rho}{2}}\begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix}\begin{bmatrix} h_{i,1} \\ h_{i,2} \end{bmatrix} + \begin{bmatrix} v_{1,i} \\ v_{2,i} \end{bmatrix}, \quad i = 1, 2, \end{aligned} \tag{25}$$

which can be rewritten as

$$\underbrace{\begin{bmatrix} y_{1,i} \\ y_{2,i}^* \end{bmatrix}}_{\mathbf{y}_i} = \sqrt{\frac{\rho}{2}}\underbrace{\begin{bmatrix} h_{i,1} & h_{i,2} \\ h_{i,2}^* & -h_{i,1}^* \end{bmatrix}}_{\mathbf{H}_i}\underbrace{\begin{bmatrix} s_1 \\ s_2 \end{bmatrix}}_{\mathbf{s}} + \underbrace{\begin{bmatrix} v_{1,i} \\ \tilde{v}_{2,i} \end{bmatrix}}_{\tilde{\mathbf{v}}_i}, \quad i = 1, 2. \tag{26}$$

We note that the orthogonality of the code $\mathbf{O}_{c,2}$ implies the orthogonality of $\mathbf{H}_i$, i.e., $\mathbf{H}_i^H\mathbf{H}_i = \left(|h_{i,1}|^2 + |h_{i,2}|^2\right)\mathbf{I}_2$. Assuming that the receiver has knowledge of the channel coefficients $h_{i,j}$, we form a decision statistic at each receive antenna by left multiplying the received vector in (26) by $\mathbf{H}_i^H$ which results in

$$\mathbf{z}_i = \begin{bmatrix} z_{1,i} \\ z_{2,i} \end{bmatrix} = \mathbf{H}_i^H\mathbf{y}_i = \sqrt{\frac{\rho}{2}}\mathbf{H}_i^H\mathbf{H}_i\mathbf{s} + \mathbf{H}_i^H\tilde{\mathbf{v}}_i. \tag{27}$$

Hence, using the orthogonality property of $\mathbf{H}_i$ it yields

$$\mathbf{z}_i = \begin{bmatrix} z_{1,i} \\ z_{2,i} \end{bmatrix} = \sqrt{\frac{\rho}{2}}\left(|h_{i,1}|^2 + |h_{i,2}|^2\right)\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} w_{1,i} \\ w_{2,i} \end{bmatrix}. \tag{28}$$

Adding all the statistics from all n_R receive antennas we obtain

$$\mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \sum_{i=1}^{n_R}\begin{bmatrix} z_{1,i} \\ z_{2,i} \end{bmatrix} = \sqrt{\frac{\rho}{2}}\sum_{i=1}^{n_R}\left(|h_{i,1}|^2 + |h_{i,2}|^2\right)\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \sum_{i=1}^{n_R}\begin{bmatrix} w_{1,i} \\ w_{2,i} \end{bmatrix}. \tag{29}$$

In (29), in the absence of noise, z_1 will be just an scaled version of s_1 and z_2 will be an scale version of s_2 without any cross dependency. To estimate the symbols that were sent, we just scale and quantize the decisions statistics in (29) as

$$\begin{aligned} \hat{s}_1 &= Q(z_1), \\ \text{and} \quad \hat{s}_2 &= Q(z_2). \end{aligned} \tag{30}$$

We recall that the decoupling has been possible because of the orthogonality of the Alamouti code matrix.

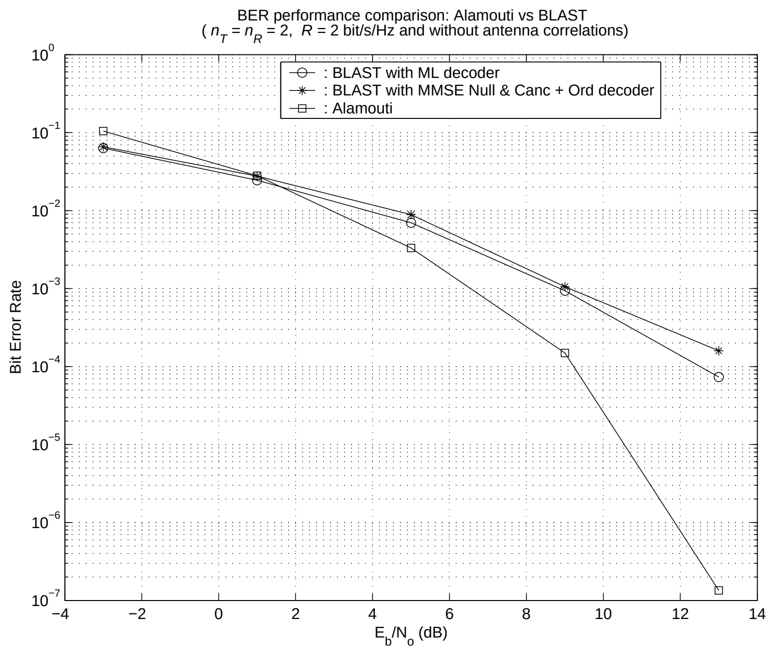

Figure 13: BER performance comparison between BLAST (BPSK modulation) and Alamouti (QPSK modulation) with $n_T = n_R = 2$ (transmission rate $R = 2$ bit/s/Hz). Uncorrelated MIMO channel and perfect channel knowledge at the receiver are assumed.

We now compare the performance of the Alamouti scheme with that of the BLAST system discussed in the previous section. For both systems, we consider $n_T = n_R = 2$. We assume that both schemes have a transmission rate $R = 2$ bit/s/Hz. This rate can be achieved using BLAST with BPSK or using the Alamouti code with QPSK modulation. For a fair comparison, we compare the two systems in terms of signal-to-noise ratio per bit, i.e., E_b/N_o. Assuming perfect channel estimation at the receiver and no antenna correlations, Figure 13 shows that Alamouti performs better than BLAST and this improvement is greater at higher signal-to-noise ratio. We next compare their performance in correlated MIMO channels. We consider a medium level of correlation typical of urban environments as described in Figure 5. It is seen from Figure 14 that Alamouti performs much better than BLAST in such a scenario.

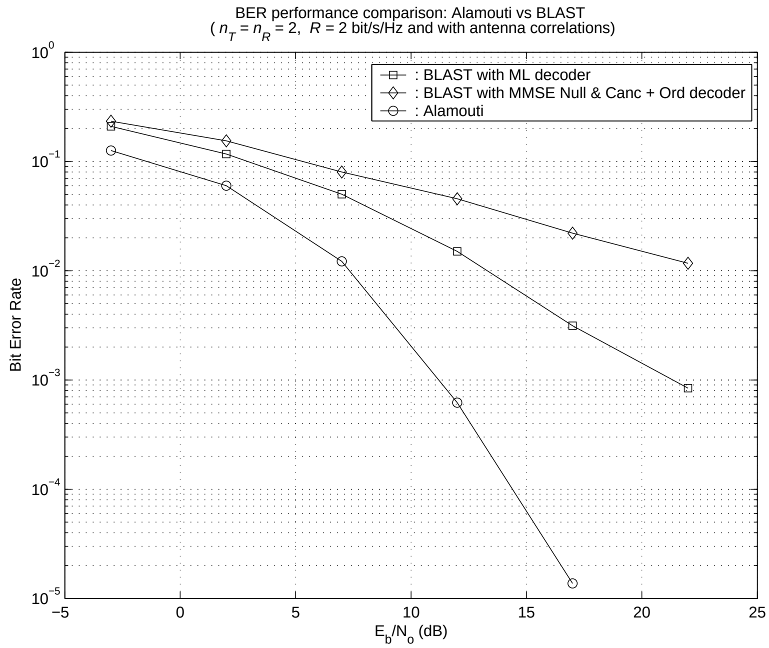

Figure 14: BER Performance comparison between BLAST (BPSK modulation) and Alamouti (QPSK modulation) with $n_T = n_R = 2$ (transmission rate $R = 2$ bit/s/Hz). Correlated MIMO channel (urban environment in Figure 5) and perfect channel knowledge at the receiver are assumed.

General STBC Based on Orthogonal Designs ($n_T \geq 2$)

The Alamouti scheme presented above works only with two transmit antennas. This scheme was later generalized in [40, 41] to an arbitrary number of transmit antennas. Similarly to the Alamouti code in (23), the general STBC is defined by a code matrix with orthogonal columns. Just like in the Alamouti scheme, a simple linear receiver is also obtained due to the orthogonality of the columns of the code matrix. In general, an STBC is defined by a $(p \times n_T)$ matrix $\mathbf{G}$. The entries of the matrix $\mathbf{G}$ are linear (possibly complex) combinations of the variables $x_1, x_2, ..., x_k$ (representing symbols). The columns of the matrix represent antennas and the rows time slots. Therefore, p time slots are needed to transmit k symbols, resulting in a code rate $R = k/p$ symbols/channel use. It is of special interest code matrices achieving the maximum transmission rate permitted by the STC theory, i.e, $R = 1$ symbol/channel use. For a fixed n_T, among the code matrices that achieve the maximum rate, we will be interested in those with minimum values of p or equivalently, minimum number of time slots needed to transmit a block. These code matrices are referred as delay optimal and they are interesting because they minimize the memory requirements at the transmitter and at the receiver (i.e., encoding and decoding delay). We recall that $p \geqslant n_T$.

STBC for real constellations

For real signal constellations such as PAM, the entries of the code matrices are only real linear combinations of $x_1, x_2, ..., x_k$. General STBC based on real orthogonal designs achieving full diversity and full rate, can be found for any number of transmit antennas n_T [43]. Using $n_T = 2$, 4 and 8 antennas, STBC code matrices can be found with $p = n_T$ (i.e., minimum possible delay in STBC). As an example, an STBC suitable for real constellations with $n_T = 4$ is

$$\mathbf{G}_4 = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & -x_2 \\ -x_4 & -x_3 & x_2 & x_1 \end{bmatrix} \tag{31}$$

for which it can be verified that $\mathbf{G}_4^T \mathbf{G}_4 = \left(\sum_{i=1}^{4} x_i^2 \right) \cdot \mathbf{I}_4$. The encoding process at the transmitter is similar to that for the Alamouti code, as follows. Consider a real constellation of size 2^b. At time 1, $4b$ bits arrive at the encoder and select symbols s_1, s_2, s_3, s_4. Let $x_i = s_i$ in matrix $\mathbf{G}_4$ in (31) to obtain the code matrix $\mathbf{C}_4$. At time $t = $ 1,2,3 and 4, the t-th row of $\mathbf{C}_4$ is transmitted from the four transmit antennas simultaneously. Therefore, with $n_T = 4$ transmit antennas and employing the code matrix $\mathbf{C}_4$, four symbols are transmitted during four symbol intervals achieving $R =$ 1 symbol/channel use, i.e., the maximum rate allowed by the STC theory. At the receiver, the orthogonality of the matrix $\mathbf{C}_4$ simplifies the ML decoder decoupling the detection of each of the transmitted symbols.

STBC for complex constellations

Complex STBC are analogous to the real ones except that the code matrices contain entries $\pm x_1, \pm x_2, ..., \pm x_k$, their conjugates, and multiples of them by $\sqrt{-1}$, making them useful for complex constellations such as M-PSK or M-QAM. As an example, an STBC with $n_T = 4$ for complex constellations can be constructed using the real orthogonal design in (31) as

$$\mathbf{G}_{c,4} = \begin{bmatrix} \mathbf{G}_4 \\ \mathbf{G}_4^* \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & -x_2 \\ -x_4 & -x_3 & x_2 & x_1 \\ x_1^* & x_2^* & x_3^* & x_4^* \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & x_4^* & x_1^* & -x_2^* \\ -x_4^* & -x_3^* & x_2^* & x_1^* \end{bmatrix}. \tag{32}$$

As before, the code $\mathbf{C}_{c,4}$ can be obtained substituting x_i by the data symbols s_i in $\mathbf{G}_{c,4}$. In this code, transmitting each row at a time, 8 symbols intervals are needed to transmit 4 symbols, therefore having a rate $R = 1/2$ symbol/channel use, i.e., half of the maximum rate permitted by the STC theory. Complex STBC of $R = 1/2$ achieving full diversity can be built for any number of transmit antennas n_T from real STBC using $\mathbf{G}_{c,n_T} = \begin{bmatrix} \mathbf{G}_{n_T} \\ \mathbf{G}_{n_T}^* \end{bmatrix}$.

It has been shown that complex STBC with full rate (i.e., $R = 1$) exists only for $n_T = 2$, i.e., the Alamouti code. In this sense, the Alamouti codes is quite unique. Codes that achieve a rate $R = 3/4$ with complex constellations have been found with $n_T = 3$ and $n_T = 4$ [40].

5 Further Topics and Conclusions

In this paper, we have discussed the huge increase in capacity that can be obtained in rich scattering environments by using multiple antennas at the transmitter and the receiver; and we have given an overview of the main classes of space-time techniques recently developed in the literature. In conclusion, the area of space-time coding and signal processing is new, active and full of challenges. The following is a list of some other important topics related to MIMO systems and space-time coding and signal processing:

- Space-time trellis codes (STTC): An STTC is basically a trellis-coded modulation (TCM) code, which can be defined in terms of a trellis tree. Rather than transmitting the output code symbols serially from a single transmitter antenna as in the traditional TCM scheme, in STTC all the output code symbols at each time are transmitted simultaneously from multiple transmitter antennas. The first STTC communication system was proposed in [43]. Some design criteria and performance analysis for STTC in the presence of channel estimation error are given in [44]. Some improved STTC codes found by exhaustive computer search are given in [4].

- Differential space-time codes: Previous sections assumed that the receiver had knowledge of the channel matrix before starting the detection algorithms. In some situations, this is not possible since no training symbols are available. In some other situations, the channel changes so rapidly that channel estimation is difficult or requires to send training symbols very often. That is the reason why it is interesting to consider differential techniques that do not require estimation of the channel response neither at the receiver nor at the transmitter. Differential STBC based on orthogonal designs are proposed in [29, 39] and that based on unitary group codes were proposed in [28]. Similarly to the SISO case, differential decoding incurs a performance penalty of about 3dB compared with coherent detection.

- Space-time precoding: The space-time coding schemes presented in this paper only require channel knowledge at the receiver. In some cases, channel status can be fedback to the transmitter or directly estimated by the transmitter such as in a TDD system. In such scenarios, the performance can be improved if the transmitter uses this channel information. Different precoding schemes have been proposed in [36].

- MIMO antenna selection: Usually, the RF chain (amplifier, digital-to-analog converters, etc.) in wireless devices is one of the most expensive blocks. A promising approach for reducing the cost and complexity while retaining a reasonably large fraction of the high potential data rate of a MIMO system is to employ a reduced number of RF chains at the receiver (or transmitter) and attempt to optimally allocate each chain to one of a larger number of receive (transmit) antennas. In this way, only the best set of antennas is used, while the remaining antennas are not employed, thus reducing the number of required RF chains. Different approaches to selecting those antennas are recently proposed in the literature [21, 22, 25].

- MIMO applications in OFDM and CDMA systems: Recently, the use of MIMO systems in frequency-selective fading channels in combination with orthogonal frequency division multiplexing (OFDM) and coherent detection has been considered [1, 7]. Code design criteria for the MIMO OFDM systems are given in [32, 33], and specific code designs are given in [9]. Moreover, MIMO coding and signal processing techniques for code-division multiple-access (CDMA) systems are developed in [27, 35].

- Turbo processing for MIMO systems: Iterative or turbo demodulation and decoding for coded BLAST or coded STC systems have been investigated in [13, 23, 31, 33, 37, 46].

- Other space-time coding schemes: Other classes of codes are being developed for MIMO systems. As an example, linear dispersion (LD) codes [24] can be used with any configuration of transmit and receive antennas and they are designed to optimize the mutual information between the transmitted and received signals. The LD codes can be decoded using any BLAST detection algorithm. Moreover, layered space-time coding schemes are developed in [19, 42].

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