Critical Band Width in Loudness Summation*

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The concept of the critical band, or *Frequenzgruppe*, is shown to apply to loudness summation. When the spacing between a group of pure tones is increased, the loudness remains constant until a critical point is reached, after which the loudness increases. The same effect occurs when the width of a band of noise of constant SPL is made larger. The critical band width at which loudness summation begins to depend on the spread of energy is approximately the same as the critical band width determined previously by methods involving thresholds, masking, and phase. The critical band as measured directly by these three methods (plus the method of loudness summation) is about two-and-a-half times as wide as the critical band derived from the assumptions made by Fletcher, but its dependence on frequency is approximately the same. The relation of the critical band to other functions is noted.

THIS paper concerns two problems: how the loudness of a group of tones depends on the spacing of the tones in the complex, and how the loudness of a band of noise of constant sound pressure level depends on the width of the band. Previous experiments by Zwicker and Feldtkeller[1] and by Bauch[2] suggest that an increase in spacing, or in band width, has little or no effect on loudness until a critical band width is reached, after which the loudness increases. In German publications this critical band has been called a "*Frequenzgruppe*." The evidence presented below confirms these findings and suggests, furthermore, that the critical band width observed in loudness summation is consistent with the critical band widths derived from observations on other parameters such as thresholds, masking, and phase relations.[3] These will be discussed in later sections, and the relation of the critical band to other functions will be noted.

Because of the sequence in which our experiments were run, a word needs to be said about the roles of the contributors to these studies. The experiments with pure tones were begun by Stevens early in 1956, with the assistance of Bertram Scharf. During the summer of 1956 Flottorp joined the staff of the laboratory and carried out the major portion of the studies with pure tones, particularly those concerned with the loudness of a four-tone complex. After Flottorp returned to Norway, Zwicker came to the laboratory and carried out the experiments involving the loudness of bands of noise. Since the final draft was prepared by Stevens, and since circumstances made it impossible for all the authors to check on the separate phases of the research, it may or may not be true that all contributors to the enterprise will find themselves agreed on all points.

GENERAL PROCEDURE

In all these experiments the signals were presented binaurally through a pair of PDR-10 earphones (calibrated on a 6-cc coupler) which were mounted in sponge Neoprene cushions MX-41/AR. The subject, seated in an anechoic chamber, adjusted the level of a comparison signal to match the loudness of a standard signal. He did this by turning the knob on a "sone potentiometer," which consisted of two 2000-ohm potentiometers ganged and cascaded. The knob was on the end of a shaft that projected from the control room through the wall into the anechoic chamber. The subjects were instructed to approach their final adjustment by "bracketing," i.e., by setting the comparison signal alternately too high and too low before settling on a final setting.

An electronic switch, keyed by a motor-driven timer, presented the two signals alternately. Each signal lasted about 1 sec and was separated from the next signal by a silent interval of about 0.5 sec.

Figure 1 shows the block diagram for both the experiments with tones and the experiments with noises. The part of the diagram on the right applies to both tones and noises, the upper left-hand section concerns the measurements with noises, the lower left-hand section concerns the measurements with tones. Note that both the standard and the comparison noises were obtained from the same noise generator, and that the only amplification used was common to both channels.

LOUDNESS *VS* TONAL SEPARATION

The major part of the experiment with pure tones involved a complex of four equally intense components supplied by four oscillators (tuned plate circuit).[4] The principal advantage of these oscillators is their stability and the fact that the oscillation of each can be stopped or started by controlling the grid bias. This permits us

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1 E. Zwicker and R. Feldtkeller, "Ueber die Lautstärke von gleichformigen Geräuschen," Acustica 5, 303–316 (1955).

2 H. Bauch, "Die Bedeutung der Frequenzgruppe für die Lautheit von Klangen," Acustica 6, 40–45 (1956).

3 R. Feldtkeller and E. Zwicker, *Das Ohr als Nachrichtenempfänger* (S. Hirzel Verlag, Stuttgart, 1956).

4 S. S. Stevens and R. Gerbrands, "A twin-oscillator for auditory research," Am. J. Psychol. 49, 113–115 (1937).

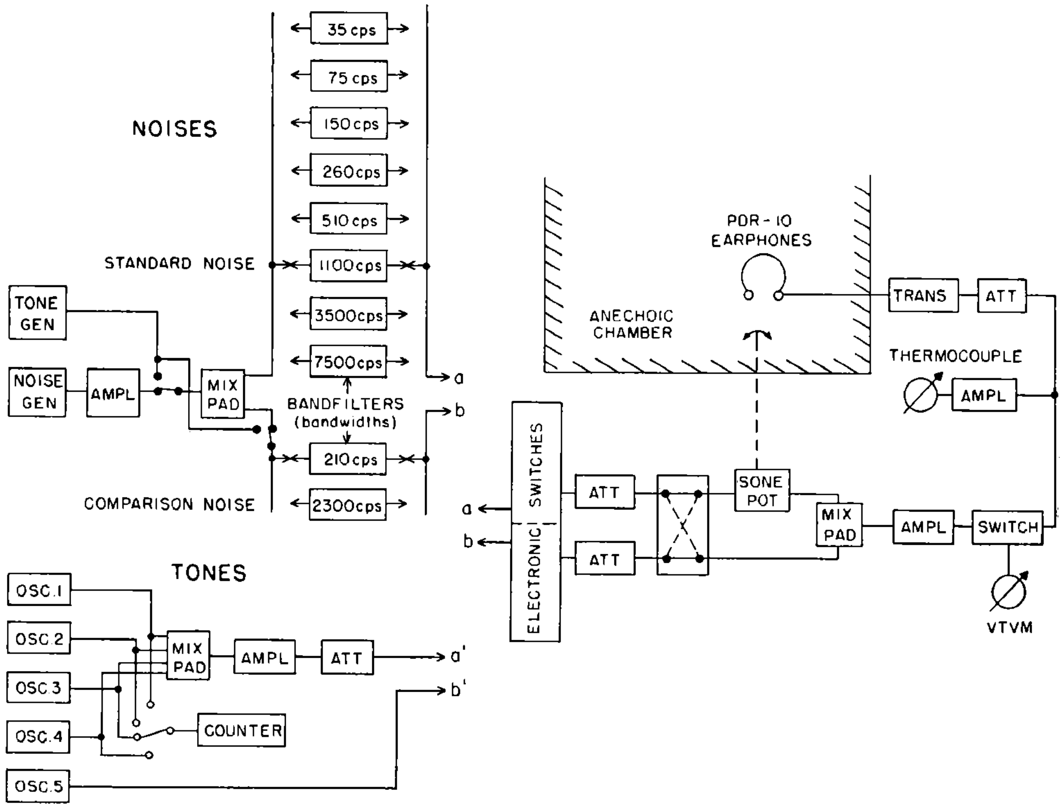

FIG. 1. Block diagram of the apparatus.

to measure the frequency (with a Hewlett-Packard Counter, 522B) as well as the amplitude of each tone without altering the external circuits of the apparatus.

When a subject had adjusted the level of the complex to match the loudness of a single pure tone, the adjusted level was determined by measuring one component in the complex. This gave a steady reading on the voltmeter. Then the median sound-pressure level of the multitone complex was later determined with the aid of a thermocouple. The loudness match made by adjusting the complex was usually complemented by the reverse procedure in which the complex was set at a fixed level and the subjects adjusted the level of a single pure tone to match the loudness of the complex.

The number of subjects who made each match varied from 6 to 22. In general, fewer subjects were used for the easier matches, for which the variability was small, and larger numbers of subjects were used when the variability was large or when an attempt was being made to test effects of spacings that produce only small differences. In all cases the median of the loudness matches was taken as the final measure.

Figure 2 shows the effect on loudness produced by changing the over-all spacing, ΔF, of four tones centered around the frequencies, 500, 1000, and 2000 cps. By over-all spacing is meant the frequency difference between the highest and the lowest tone. The symbols T and C represent the medians obtained under the two procedures: T=single tone adjusted (either 500, 1000, or 2000 cps) and C=complex adjusted. As is usual in loudness balances, the final match depends on which stimulus is adjusted. At the level employed here (57.5 db SPL) the stimulus that is adjusted is set high relative to the match determined when the other stimulus is adjusted. It has been shown elsewhere that at high stimulus levels this constant "error" may reverse its direction.[5] The lines through the data in Fig. 2 are drawn in order to show how well the results agree with the hypothesis that within a critical band the loudness is independent of the spacing of the tones, and that, when the over-all spacing ΔF exceeds a critical value, the loudness increases. The position of the discontinuity, where the two segments of the lines form an angle, was determined independently of these particular data. The values of ΔF predicted for the discontinuities in Fig. 2, and in similar graphs, were derived from measurements of the critical band width in other types of experiments discussed below (see Fig. 12). In this sense the lines in Fig. 2 are predicted values. Although the points do not fit perfectly, it seems clear that their general behavior is in line with the hypothesis.

The points in Fig. 2, represented by the inverted T and C (1000 cps), represent the results of a check experiment run on 12 observers. It was noted that the other points in the vicinity of ΔF values from 50 to 100 cps fall below the horizontal line representing a constant sound pressure level. Since most of the other results (see Fig. 3) suggest that for these values of ΔF the loudness is determined by the sound pressure level, the problem is whether the departure of some of these

[5] S. S. Stevens, "Calculation of the loudness of complex noise," J. Acoust. Soc. Am. 28, 807–832 (1956).

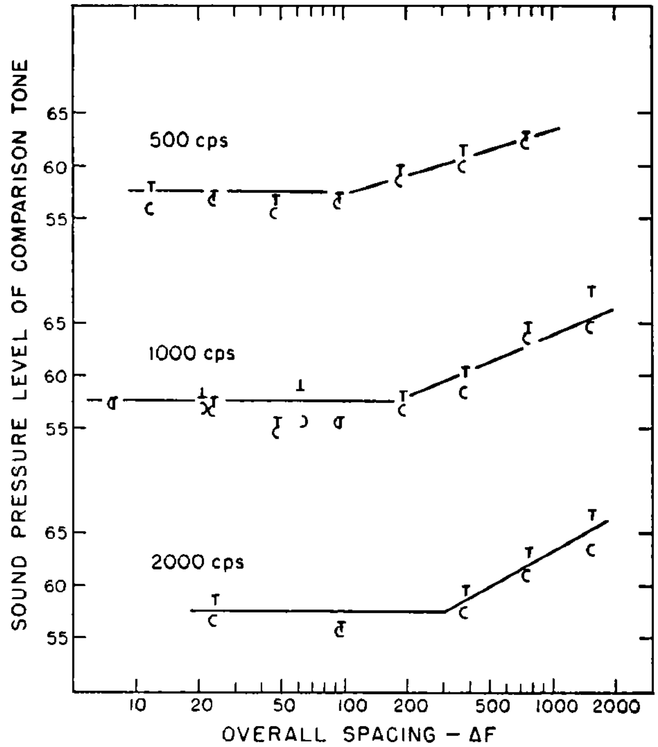

FIG. 2. Dependence of loudness on the spacing of the components in a four-tone complex. The four tones were spaced approximately uniformly in frequency about the center frequency indicated. Loudness balances between the center frequency and the complex were made by groups of from 16 to 22 subjects. T means the tone was adjusted and C means the complex was adjusted. The lines through the data have a break at the point predicted by the critical-band hypothesis.

points represents a real effect or whether it may be due to experimental variability. Since the inverted points lie more nearly where we would expect them to lie, it seems reasonable to conclude that the other points are probably less representative. In other words, we are not forced to reject the hypothesis that for components lying within a critical band the loudness is determined by the effective sound pressure level.

Figure 3 shows how the loudness of a group of four tones varies with spacing and with sound pressure level. The center frequency was 1000 cps. Here we see that the discontinuity representing the critical band width occurs at the same value of ΔF regardless of the sound pressure level. Another feature that seems evident in Fig. 3 is that the increase in loudness level, as ΔF is made greater than a critical band, is greatest for the medium levels. At the lowest level tested (SPL=17.5 db) there is no apparent increase in loudness level as ΔF increases.

The apparent *decrease* in loudness level at the largest value of ΔF (1536 cps) may have been due to the fact that at this wide spacing some of the individual tones fell below the threshold of some of the subjects. But there is also evidence that some subjects, according to their own statements, feel that they change their criterion for judging loudness when the tones are faint and widely separated. Instead of "integrating" the loudnesses of the components, these subjects feel that they tend to judge the total loudness to be equal to that of a single component in the complex. In other words, it may be that for low levels and wide spacings the "loudness integration," whatever that is, tends to break down. A similar effect was observed in an experiment[5] in which the observer tried to match the loudness of a pair of octave bands of noise widely separated in frequency—a low rumble and a high hiss. It was apparently impossible for at least some subjects to "integrate" the loudnesses of these two disparate sounds.

The data for the lowest level in Fig. 3 suggest an interesting possibility. If the tones in a complex are sufficiently faint, it may be that when their spacing is made greater than a critical band the loudness tends to decrease instead of increase. A corollary of this would be the possibility that at some level (in the vicinity of 20 phons?) the loudness of a complex is independent of the spacing of the components. This problem of the summation of loudness at very low levels needs further exploration.

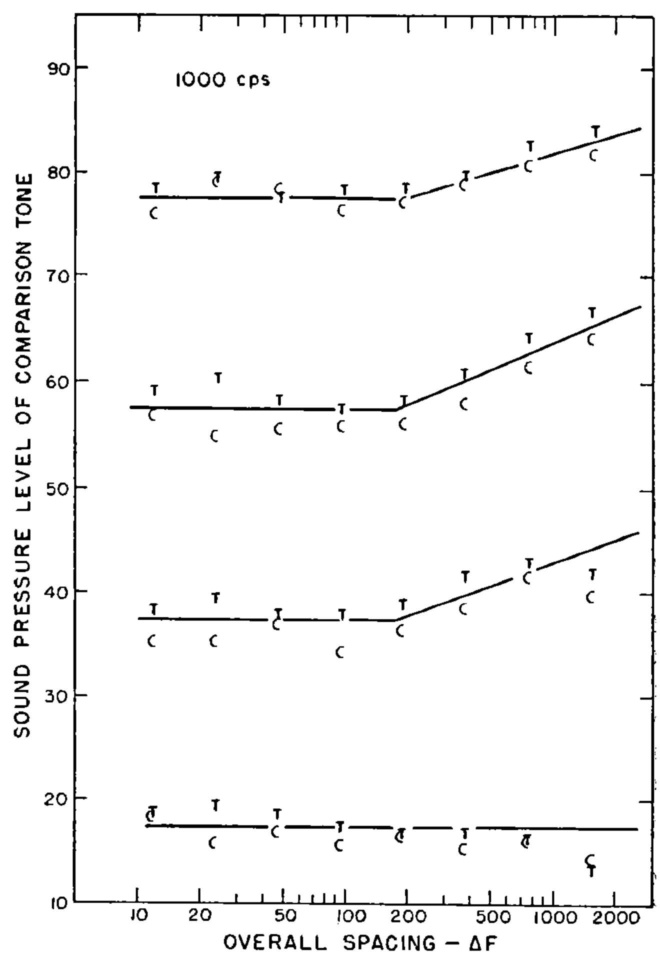

FIG. 3. Dependence of loudness on spacing and level. The critical band width is approximately invariant with level, although little or no loudness summation occurs at levels near threshold. Each point represents the median of two judgments by each of ten listeners. In each experimental session the spacing was kept constant and the levels were tested in irregular order.

Another point should be noted. When the tonal components are so close together that slow beats occur, we would expect the loudness to be judged more in terms of the maximum amplitude of the beat envelope. We did not try to pursue this question, but an effect of this sort is evident in Bauch's results.

EFFECT OF LEVEL

The dependence of loudness summation on level can be seen more clearly in Fig. 4, where some of the data of Fig. 3 are plotted in a different manner. The points represent the average of the T and C values from Fig. 3. The solid points show that when the components lie within a critical band the loudness of the complex is the same as the loudness of a 1000-cy tone having the same sound pressure level as the complex. The unfilled points are for values of ΔF greater than a critical band. The fact that the curves through these points pass through a maximum suggests that loudness summation is greatest for loudness levels in the vicinity of 50 to 60 phons. It will be noted that two of the points do not lie near the curve for the widest spacing (1536 cps). As indicated above, we are not certain of the reason for this discrepancy, although it may well be a real effect rather than an error of measurement.

The crosses in Fig. 4 are from another study[5] and are reproduced for purposes of comparison. These values represent the relative loudness level of a 1000-cy tone interrupted at the rate of 130 per sec. Modulation of this sort turns the single pure tone into a complex consisting of a central frequency (1000 cps) and a group of side bands, and we see that as a function of level the loudness of this complex behaves much like the loudness of a complex of four equally intense tones.

Another set of data, obtained in some of the tests run by Scharf, is shown in Fig. 5, where the loudness level of the complex is plotted as a function of the loudness level of one of the single components of the complex. These results are consistent with those shown in Fig. 4, in that

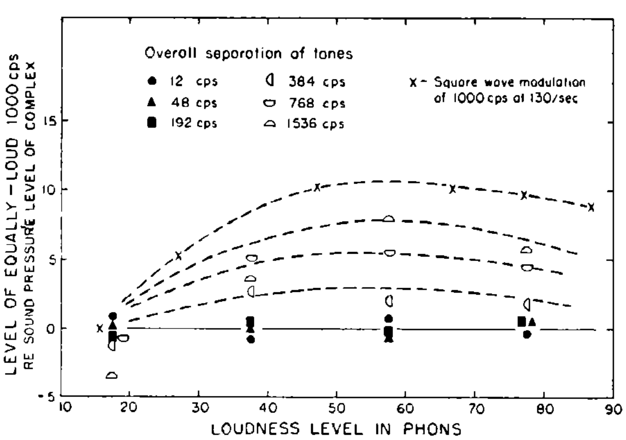

FIG. 4. Relative loudness levels of different complexes as a function of level and spacing. All but the crosses represent data from Fig. 3. The crosses show the relative loudness level of a 1000-cy tone interrupted at a rate of 130 per sec.

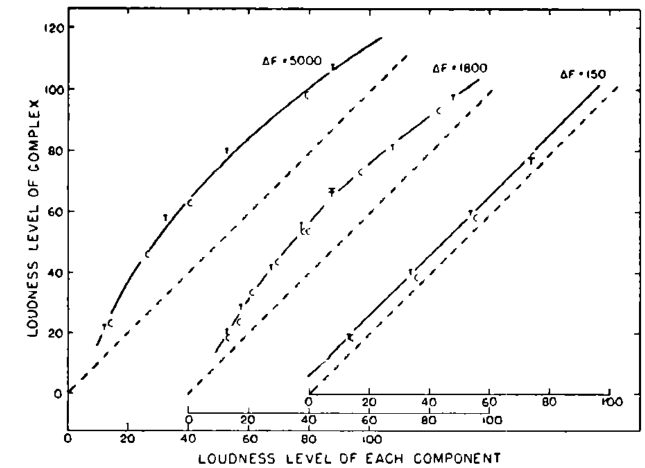

FIG. 5. Loudness levels of various complexes as a function of the level of a single component. The two curves on the left represent complexes of five equally intense tones. The curve on the right is for four tones. The center frequencies of the complexes were 1800, 1250, and 1000 cps for the three curves (from left to right).

loudness summation is greatest at medium levels. The curve at the right is for a group of four tones lying within a critical band ($\Delta F = 150$ cps). Here the loudness level of the complex is proportional to the loudness level of the single component. The other two curves represent complexes of five tones at spacings greater than the critical band width. As we should expect, the loudness level of the complex exceeds that of a single component by a greater amount when the spacing of the components is greater.

EFFECT OF IRREGULAR SPACING

The next question concerns the dependence of loudness on the relative spacing of the tones when the overall spacing ΔF is kept constant. For a given ΔF, does it make any difference how the intermediate tones are spaced in frequency? The answer appears to be that it does, and that uniform spacing produces greater loudness than nonuniform spacing.

Three different experiments were run in which loudness balances were made between a single tone (500, 1000, and 2000 cps) and four-tone complexes in which the relative spacings were as shown in Fig. 6. Groups of 18 to 22 subjects made one balance by adjusting the single tone and one by adjusting the complex. The combined results, giving the sound pressure level of the single tone that matches the loudness of the complex, are listed in Fig. 6.

We see that in each instance the loudness is greatest when the spacing of the tones in the complex is uniform in frequency. Although we did not test the question, there is reason to believe that if the spacing were made proportional to critical band width, which would make the separations approximately equal in mels, the loudness would be slightly greater still. Figure 6 shows also that the more uneven the spacing, the lower the loudness. There is some tendency for the loudness to be

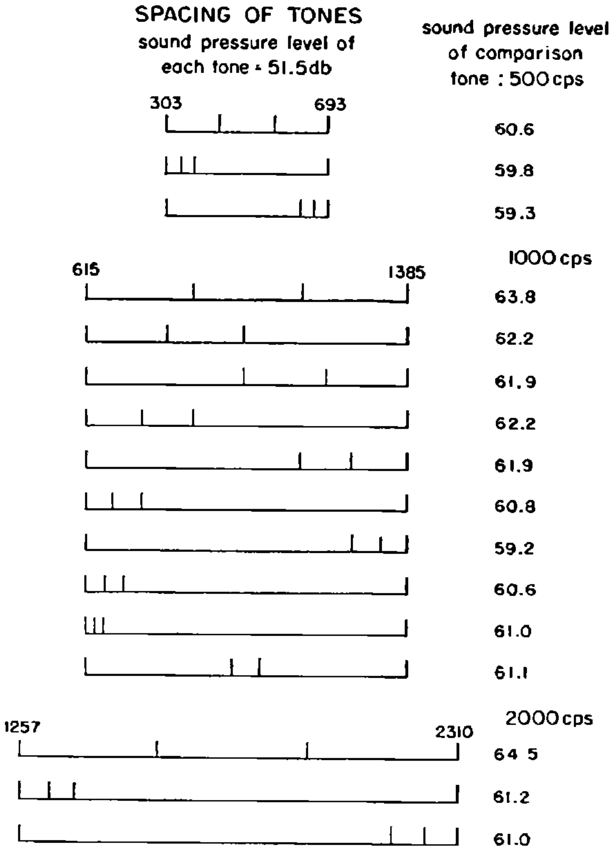

FIG. 6. Loudness as a function of the relative spacing among the four tones in a complex. Three different over-all spacings ΔF were used as indicated. Within a given ΔF the tones were spaced evenly or were bunched in one frequency region or another. The sound pressure level of the tone (center frequency) that was matched in loudness to the complex is listed beside each diagram showing the relative spacings.

slightly greater when the tones are bunched at the low end of the interval than when they are bunched at the high end.

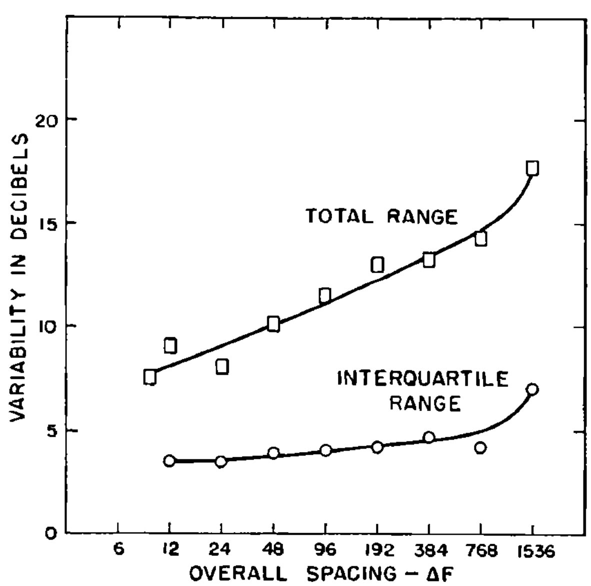

FIG. 7. Showing how the total range and the interquartile range of the loudness balance varies with the over-all spacing of the tones in a complex.

VARIABILITY

When two sounds differ in quality, the matching of their loudnesses is difficult and subject to considerable variability. In general, the variability increases as the difference between the characters of the two sounds increases. Thus when a single tone is matched to a complex, variability increases as the spacing of the tones in the complex is made wider. Evidence for this fact is shown in Fig. 7.

Variability also changes with level. Although there were no significant differences among the three highest levels tested (Fig. 3), the variability was less at the lowest level (17.5 db). On the other hand, for the results shown in Fig. 6 there seemed to be no significant difference in variability for the different spacings of the tones within a constant ΔF.

When the subjects adjusted the complex to match the single tone, the variability was less than when they adjusted the tone to match the complex. The median of the interquartile ranges for all sessions was 4 db when the complex was adjusted, and 4.5 db when the tone was adjusted. This effect has also been noted elsewhere.[5] In addition, there was the systematic tendency already described for the adjusted signal to be set relatively too high.

Although the final results have been reported in terms of medians, the means of the loudness balances were also calculated. The agreement between the two was always within ± 1.9 db, but there was a slight residual skewness in the data, as shown by the fact that the mean values averaged about 0.1 db lower than the median values. The direction of this skewness is what we should expect from the relation between loudness (in sones) and decibels.

One of the persistent difficulties in experiments on loudness is the tendency for different subjects to use different criteria in judging the loudness of a complex. This difference in criterion is especially marked when the tones are faint and widely spaced. Apparently two extreme attitudes may be taken toward the complex: the subject may take an "analytic" attitude and judge in terms of the loudness of a single component, or he may take an "integrative" attitude and judge in terms of a summation of the loudnesses of the components. Different subjects seemed to take these attitudes in different degrees and to hold rather consistently to them. Of course, those who took a more integrative attitude judged the complex to be louder than did those who took a more analytic attitude.

LOUDNESS *VS* BAND WIDTH OF NOISE

The foregoing results concern the loudness of line spectra, and the next question is, how does the loudness of a continuous flat spectrum (white noise) change with band width when the effective sound pressure level is held constant. (In the experiment by Zwicker and Feldtkeller[1] the spectrum studied was uniform masking noise.) In order to test this question it was necessary to design sets of filters having different pass bands centered (geometrically) about a given frequency. Three such sets of filters were used to provide bands of various widths centered about the frequencies 440, 1420, and 5200 cps.

Groups of 12 subjects adjusted a comparison signal (either a band of noise or a tone) to match the loudness of each of the filtered bands. For a given experiment the levels of the filtered bands were held constant in terms of the reading on a thermocouple voltmeter, and after the loudness balance had been made the level of the adjusted signal was read on the same meter. The median results are shown in Figs. 8, 9, and 10.

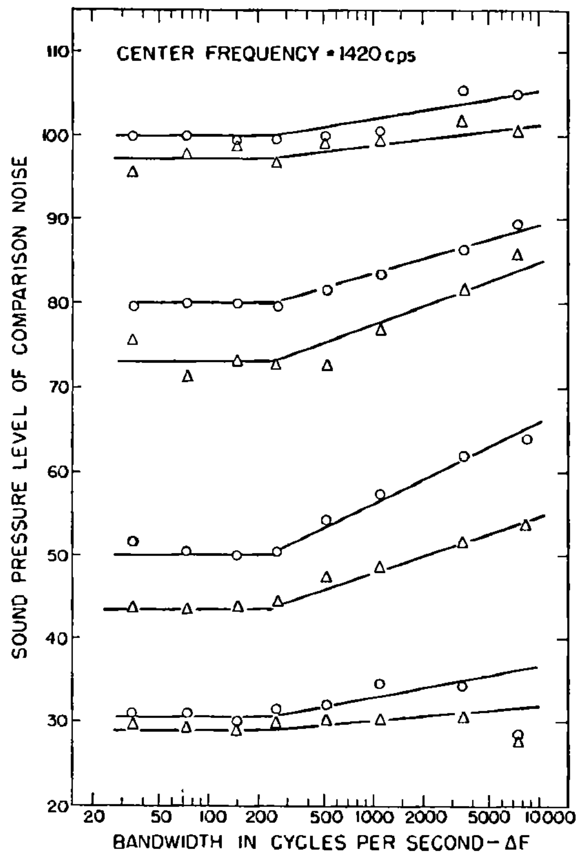

FIG. 8. Dependence of loudness on the band width of a noise of constant SPL having a center frequency of 1420 cps. Comparison noises of two band widths, 210 cps (circles) and 2300 cps (triangles), were matched in loudness to each band width ΔF at a constant SPL.

As in Figs. 2 and 3, the lines through the data were drawn to show the agreement between the data and the critical-band hypothesis. The point at which the two segments of each line form an angle was determined by the critical-band measurements discussed below. It seems clear that the results, taken all together, support the hypothesis that the loudness of a band of noise of constant SPL is invariant with band width provided the band width is smaller than the critical band. When the band width is increased beyond the critical value, the loudness increases. The point at which the increase begins is relatively independent of level.

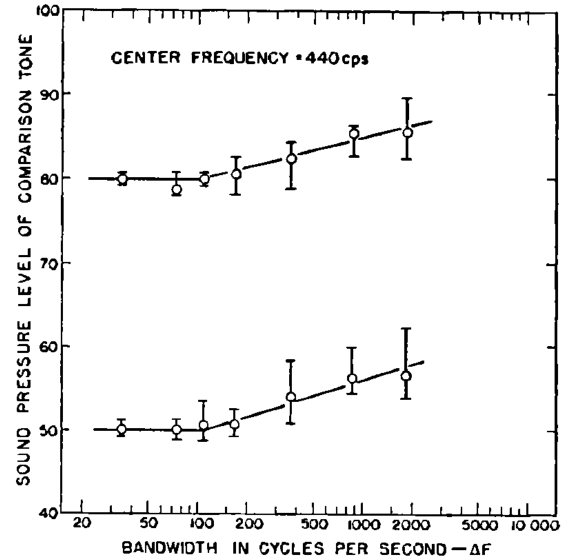

FIG. 9. Dependence of loudness on the band width of a noise of constant SPL having a center frequency of 440 cps. The subjects adjusted a 440-cy tone to match the loudness of each band width ΔF. The vertical bars show the interquartile ranges of the adjustments.

Certain other features of these results also deserve comment. In Fig. 8 four different levels were tested (30, 50, 80, and 100 db) and two different comparison noises were adjusted to match the loudness of the filtered band at each value of ΔF. These comparison noises had band widths of 210 and 2300 cps. Since for the same SPL the wider of these noises is louder, the levels to which the subjects adjusted it (triangles) are lower than the levels to which they adjusted the 210-cy band (circles). The variabilities of the loudness

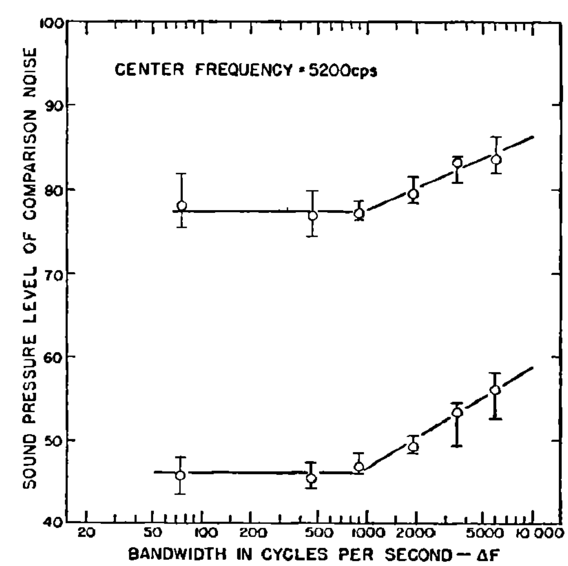

FIG. 10. Dependence of loudness on the band width of a noise of constant SPL having a center frequency of 5200 cps. The subjects adjusted a comparison noise (band width 2500 cps) to match the loudness of each band width ΔF. The vertical bars show the interquartile ranges of the adjustments.

matches showed the expected relations: variability was less when the narrow band was matched to other narrow bands and when the wide band was matched to other wide bands. Consequently, the lower segments of the functions are better determined by the circles (210 cps) and the upper segments by the triangles (2300 cps).

The dependence of loudness summation on level is seen to be rather similar in Fig. 3 (tones) and Fig. 8 (noise). At the highest level (100 db) the growth of loudness, as ΔF is increased beyond the critical band width, is less rapid than at medium levels. And again at very low levels the growth of loudness with ΔF beomes smaller. In fact, at the lowest level tested, the largest value of ΔF (about 7500 cps) seems to be less loud than the narrower bands (see also reference 1). This is reminiscent of the behavior of the lowest curve in Fig. 3, but it is not yet certain that these effects are based on the same causes. The behavior of loudness summation at very low levels still presents many puzzles.

Figure 9 shows the results for bands of noise centered about 440 cps at two levels: 50 and 80 db. In this case the subjects adjusted a pure tone of 440 cps to match the loudness of the bands. The vertical bars through the points indicate the interquartile ranges of the adjustments. In Fig. 10, similar results are shown for bands centered about a frequency of 5200 cps. To match the loudness of these bands the subjects adjusted the level of a comparison noise whose band width was 2500 cps.

CONCEPT OF CRITICAL BANDS

A word is in order regarding the relation between the *Frequenzgruppe*, which we are translating "critical band," and the concept originated by Fletcher. Although a measure called κ was used earlier (1937), it was in his 1940 paper that Fletcher outlined the concept of the critical band as an aid to determining "position coordinates" on the basilar membrane.[6] In order to know how to divide the frequency spectrum into bands of equal effectiveness he wanted to know where on the membrane various frequencies produce excitation. One approach to this problem was via the assumption that successive difference limens for frequency mark off equal steps along the membrane. The other was to assume that masking is proportional to excitation and then to see how effectively different bands of the spectrum produce excitation by determining how pure tones are masked by a broad-band noise. Fletcher assumed that the part of the noise that is effective in masking a tone is the part of the spectrum lying near the tone and containing the same amount of power as the tone. Those parts of the spectrum that are far from the tone contribute no masking. His critical band, then, becomes that width of the spectrum which contains the same acoustic power as the tone that is just masked.

This concept gives us, in effect, a critical band by definition, or by assumption. The masking measurements used to calculate its value have been confirmed in other laboratories,[7] but the direct experimental evidence for the validity of its underlying assumption is not very substantial. Thus far, it seems that a sharp discontinuity has not been found in plots relating the masked threshold of a tone to the band width of a masking noise.[8] Nevertheless, the concept has been a powerful aid in the solution of the problem Fletcher posed, for it turns out that the width of the critical band thus defined varies with frequency in about the same manner as the difference limen. Hence, Fletcher was able to combine information from these two sources and produce a useful "excitation map" of the basilar membrane—one that also agrees with other evidence. A more recent development of this notion is contained in his 1953 book.[9]

Since attempts to determine empirically the width of the critical band by the procedure of masking a tone by bands of noise of different widths have not been definitive, in actual practice the measure of the critical band has usually been taken to be the ratio between the power in the masked tone and the power in a 1-cy band of the masking noise. In other words, the practice rests on a defined quantity or ratio, which might perhaps be called the "critical ratio" or the "critical masking ratio."

Now it happens that the critical band (or ratio) defined by Fletcher leads to a band width that is approximately proportional to the "*Frequenzgruppe*," which is one of the reasons for our suggesting that the *Frequenzgruppe* be called a critical band. This latter critical band has the advantage, however, that it does not rest on assumptions or definitions, but is empirically determined by at least four kinds of independent experiments. The evidence for a critical band in loudness summation has already been presented above, and we will turn now to a brief review of the other types of evidence for it. The four types of experiments that provide direct measures of a critical band width have to do with thresholds, masking, phase, and, as we have seen, loudness summation.

THRESHOLD MEASURES OF THE CRITICAL BAND

In a study by Gässler[10] a critical band width emerges as a discontinuity in the function relating two variables:

[6] H. Fletcher and W. A. Munson, "Relation between loudness and masking," J. Acoust. Soc. Am. 9, 1–10 (1937); H. Fletcher, "Auditory patterns," Revs. Modern Phys. 12, 47–65 (1940).

[7] J. E. Hawkins, Jr., and S. S. Stevens, "The masking of pure tones and of speech by white noise," J. Acoust. Soc. Am. 22, 6–13 (1950).

[8] Schafer, Gales, Shewmaker, and Thompson, "The frequency selectivity of the ear as determined by masking experiments," J. Acoust. Soc. Am. 22, 490–496 (1950).

[9] H. Fletcher, *Speech and Hearing in Communication* (D. Van Nostrand, Company, Inc., New York, 1953).

[10] G. Gässler, "Ueber die Hörschwelle für Schallereignisse mit verschieden breitem Frequenzspektrum," Acustica 4, Akust. Beih. 1, 408–414 (1954).

(1) the level at which a single component in a complex of uniformly spaced tones reaches threshold and (2) the number of tones in the complex. It is assumed that all the components have the same amplitude.

In principle, the procedure is this. We begin with a single pure tone and find its threshold (using the "tracking" procedure of the Békésy audiometer). We then add another tone of the same amplitude but spaced 10 cps lower than the first tone, and again we measure the threshold. We find, of course, that in order to reach threshold the amplitude of each tone can be less for two components than it is for one. We proceed in this fashion to add tones 10 cps apart and to measure the resulting threshold. Up to a certain point, the amplitude of the individual tones decreases as more of them are added, but beyond this point no further decrease occurs. This transition point is a measure of the critical band.

Actually, since the threshold of the typical ear is not flat over a wide range of frequencies, Gässler found it necessary to make the audiogram flat by introducing a uniform masking noise. He was able to show, however, that the behavior at the absolute threshold is similar to the behavior at the masked threshold, regardless of the level of the masked thresholds. Using up to 40 components at various spacings and in various frequency regions, he mapped the width of the critical band in the ears of two observers. The results are shown by two curves in Fig. 11, one for each observer. For purposes of clarity, the curves in Fig. 11 have been separated vertically by one-half of a logarithmic unit.

The crosses represent results with tones under the method just described, and the circles show the results of using an analogous procedure with bands of noise. When the threshold for different band widths of noise is measured, the level per cycle of the noise decreases with bandwidth, up to the critical band. Beyond the critical point, the level per cycle remains constant as the band width is increased. The points in Fig. 11 were determined from the intersections of two line segments on plots that look superficially like those shown in Figs. 2, 3, 8, 9, and 10. In the threshold measurements the slope of the slanting line is such that it represents a constant sound pressure level of the complex.

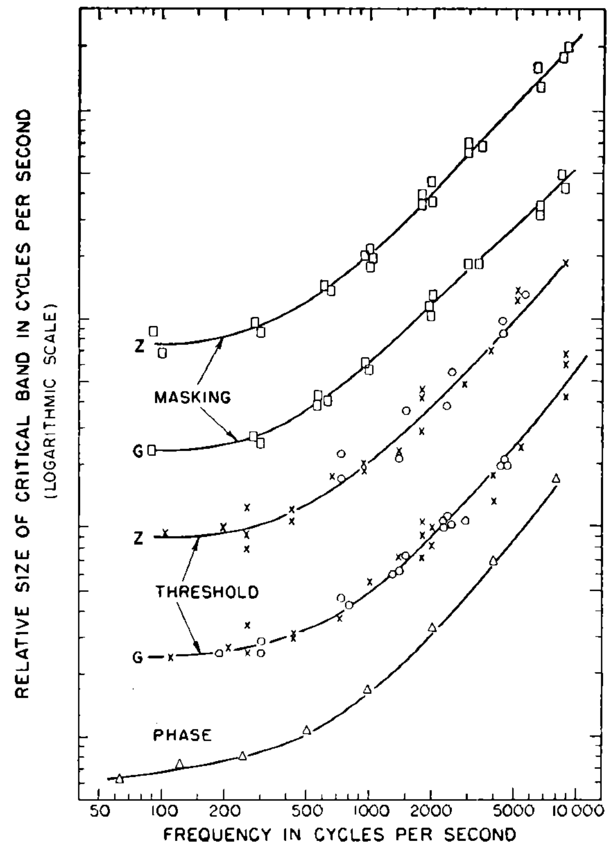

FIG. 11. Functions relating the width of the critical band to frequency, as determined by three experimental methods: masking, threshold, and phase. The letters Z and G refer to individual observers. On the curves derived from threshold measurements, the crosses represent studies with pure tones and the circles represent studies with bands of noise. The points on the lowest curve represent average values for four observers. For purposes of clarity the curves have been separated by one-half a logarithmic unit.

MASKING

The procedure used to measure the critical band by means of masking is in a sense the inverse of Fletcher's procedure. He used a noise to mask a tone; the inverse experiment[11] uses two tones to mask a noise. A narrow band of noise is placed midway between two pure tones of equal amplitude, and the threshold of the noise is measured. Then the two tones are moved farther apart in frequency, and the threshold of the noise is measured again. It is found that the masked threshold of the noise remains constant until the separation between the two tones reaches a critical value, after which the masked threshold decreases rather abruptly. The point at which this decrease begins is taken as the measure of the critical band.

Critical band widths determined by these transition points for two observers are plotted in Fig. 11 (top two curves). The location of the critical transition points in the masking curves is apparently invariant with level, at least over a range of about 80 db. At high levels a basic asymmetry enters the masking audiogram, which may obscure the effect we are looking for.

PHASE

The manner in which experiments on phase permit us to measure the critical band is through the effect of phase on modulation. To see how this happens we need to note that the difference between a small frequency modulation (FM) and a small amplitude modulation (AM) is essentially a matter of the phase relations among the side bands. When the *frequency* of a tone is modulated sinusoidally through a small range, there results a complex of three principal components: the

[11] E. Zwicker, "Die Verdeckung von Schmalbandgeräuschen durch Sinustöne, Acustica 4, Akust. Beih. 1, 415–420 (1954).

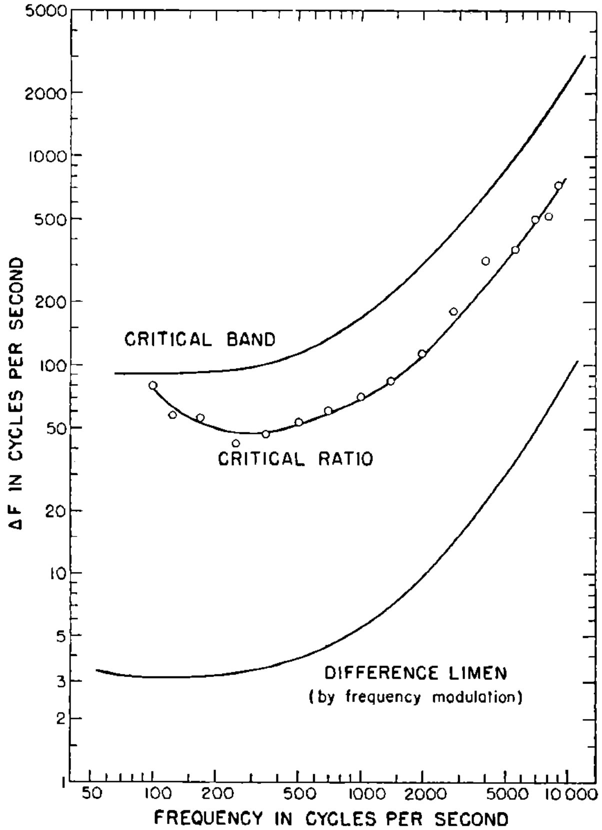

FIG. 12. The top curve, derived from four kinds of experiments, shows the width ΔF of the critical band as a function of the frequency of the center of the band. The middle curve shows the width of the band derived from the "critical ratio," which is defined as the ratio between the intensity per cycle of a noise and the intensity of a pure tone that is just masked by the noise (points are from Hawkins and Stevens). The bottom curve shows the just noticeable range of frequency modulation.

original tone (carrier) and a side band on either side. The spacing between carrier and side band corresponds to the rate (frequency) of the modulation. If the ratio of the range to the rate of modulation is sufficiently small, the side bands beyond the first are negligible. When the *amplitude* of a tone is modulated sinusoidally the same thing happens. There is produced a carrier and two side bands. The difference between AM and FM then is merely one of phase: relative to the phasing of the components produced by AM, under FM one of the side bands is 180° out of phase. In other words, to a first approximation, AM becomes FM if the phase of one of the side bands is reversed.

Now we ask the question, how sensitive is the ear to AM and to FM? When this sensitivity is measured by finding the just detectable amount of modulation—degree of AM and range of FM—an interesting thing is observed.[12] In terms of the amplitudes of the side bands, at low rates of modulation the just detectable AM is less than the just detectable FM. In other words, in order to be heard as a modulation, the amplitude of the side bands must be greater under FM than under AM. However, as the rate of modulation increases, and the side bands are spread wider apart, a point is reached beyond which the just detectable modulation is the same for both FM and AM. At and beyond this point the *phase* of the side bands no longer makes any difference to the ear.

If we take the over-all frequency difference between the two side bands at this critical rate of modulation to be the measure of the critical band, we can use the results of such experiments to measure this band. The bottom curve in Fig. 11 shows the results of such measurements on four observers. The just detectable amounts of AM and FM were measured at various carrier frequencies, at various rates of modulation (from 1 to 6000 per sec), and at loudness levels ranging from 30 to 80 phons. As was true with the other measures of the critical band width (threshold, masking, and loudness summation) the critical band determined by phase sensitivity turns out to be independent of level over the ranges tested.

As is evident in Fig. 11, the width of the critical band determined by these various methods is approximately the same, and its dependence on frequency follows a similar course. Over the low-frequency range the critical band tends to remain constant at a width of about 90 cps, but with increasing frequency it grows rapidly until in the vicinity of 10 000 cps its width is about 2000 cps.

RELATIONS TO OTHER FUNCTIONS

On the basis of the evidence available from the direct methods for measuring the critical band, an attempt has been made to construct a function relating its width ΔF to frequency. This function, shown as the top curve in Fig. 12, represents our present best estimate of the size of the critical band (ordinate), as a function of the frequency of the center of the band (abscissa). As we have seen, this curve is derived from four types of experiments. At very low frequencies the width of the critical band becomes indeterminate, for its lower limit falls, so to speak, beyond the bottom end of the scale.

Approximate values for two sets of critical band widths are listed in Table I. Since we are free to choose either the lower or the upper cut-off frequency of a critical band, other sets of values could be tabled with the aid of Fig. 12.

We have already mentioned the fact that the critical band measured by direct methods is approximately proportional to the size of the band derived by Fletcher from the "critical ratio." As can be seen from the middle curve in Fig. 12, the directly measured band is about two-and-a-half times as wide as Fletcher's band. Since over most of the frequency range the forms of these two functions are similar, it seems reasonable to suppose that they may reflect aspects of the same underlying process. The two functions differ mainly at

[12] E. Zwicker, "Die Grenzen der Hörbarkeit der Amplitudenmodulation und der Frequenzmodulation eines Tones," Acustica 2, Akust. Beih. 3, 125–133 (1952).

the lower end, where the "critical ratio" curve turns up. A possible explanation of this upturn is that at low frequencies, where the critical band is narrow, the masked threshold tends to rise because the "envelope" of a narrow (critical) band of noise is not as steady, relative to the time constant of the ear, as the envelope of a wide band of noise.[13] With narrow bands the ear begins to hear the amplitude irregularity of the envelope and this irregularity may contribute to the masking of a tone.

The critical band seems also to bear a relation to certain other auditory phenomena such as the difference limen for frequency, the function relating frequency to subjective pitch in mels, and the function relating frequency to the position of stimulation on the basilar membrane.

That the difference limen for frequency approximates a constant fraction of a critical band is shown by the bottom curve in Fig. 12. This bottom curve has about the same form as the top curve. The bottom curve represents the just detectable change in frequency as measured by the detectable range of frequency modulation when the rate of modulation is about 4 per sec.[13] This curve, for an SPL of 80 db, is based on measurements on four observers who used the method of "tracking" to determine the difference limen, defined as the total range of the just detectable frequency modulation. These values agree reasonably well with those of Shower and Biddulph.[14]

An interesting relation between the critical band and the mel scale[15] of pitch is suggested by the fact that over most of the frequency range the width of the critical band approximates a constant number of mels. The average width of the critical band is about 137 mels. It varies from about 100 mels at low frequencies to about 180 mels at high frequencies. In view of the difficulty of determining the pitch scale with great precision, this degree of agreement between critical bands and intervals of subjective pitch makes it reasonable to entertain the hypothesis that the two may be closely related. If we were to plot in Fig. 12 a curve showing the ΔF corresponding to a constant interval of pitch, we would obtain a curve not greatly unlike the others shown there (see, for example, Licklider's Fig. 18[16]).

Finally, as has been pointed out elsewhere,[17] Békésy's determinations of the positions at which various frequencies produce a maximum vibration of the basilar membrane result in a cochlear map that suggests an additional interesting hypothesis. Critical bands, equal mel intervals, and difference limens may correspond to equal distances along the basilar membrane. The precision with which some of these things can be measured does not yet permit a precise test of this possibility, but the general similarity of the several functions justifies our using it as a working hypothesis.

TABLE I. Examples of critical band widths ΔF. The left-hand center column gives the cut-off frequencies of bands whose center frequencies are listed in the right-hand center column, and vice versa. Example: The band 900 to 1060 has a width ΔF of 160 and a center frequency of 980 cps. The band 830 to 980 has a width ΔF of 150 and a center frequency of 900 cps.

ΔF	Center and cut-off frequencies		ΔF
	20		
90		65	
	110		90
90		155	
	200		95
95		250	
	295		95
100		345	
	395		105
108		450	
	503		110
120		560	
	625		130
130		690	
	755		140
145		830	
	900		150
160		980	
	1060		175
190		1155	
	1250		200
210		1355	
	1460		225
240		1580	
	1700		255
270		1835	
	1970		295
320		2130	
	2290		350
380		2480	
	2670		420
450		2900	
	3120		500
560		3400	
	3680		620
680		4020	
	4360		760
840		4780	
	5200		920
1000		5700	
	6200		1150
1300		6850	
	7500		1550
1800		8400	
	9300		2100
2400		10500	
	11700		2800
3300		13300	
	15000		4000
		17300	

[13] E. Zwicker, "Die elementaren Grundlagen zur Bestimmung der Informationskapazität des Gehörs," Acustica **6**, 365–381 (1956).

[14] E. G. Shower and R. Biddulph, "Differential pitch sensitivity of the ear," J. Acoust. Soc. Am. **3**, 275–287 (1931).

[15] S. S. Stevens and J. Volkmann, "The relation of pitch to frequency: a revised scale," Am. J. Psychol. **53**, 329–353 (1940).

[16] J. C. R. Licklider, "Basic correlates of the auditory stimulus," S. S. Stevens, editor, *Handbook of Experimental Psychology* (John Wiley and Sons, Inc., New York, 1951).

[17] G. von Békésy and W. A. Rosenblith, "The mechanical properties of the ear," S. S. Stevens, editor, *Handbook of Experimental Psychology* (John Wiley and Sons, Inc., New York, 1951).