EE E6820: Speech & Audio Processing & Recognition

Lecture 10:
ASR: Sequence Recognition

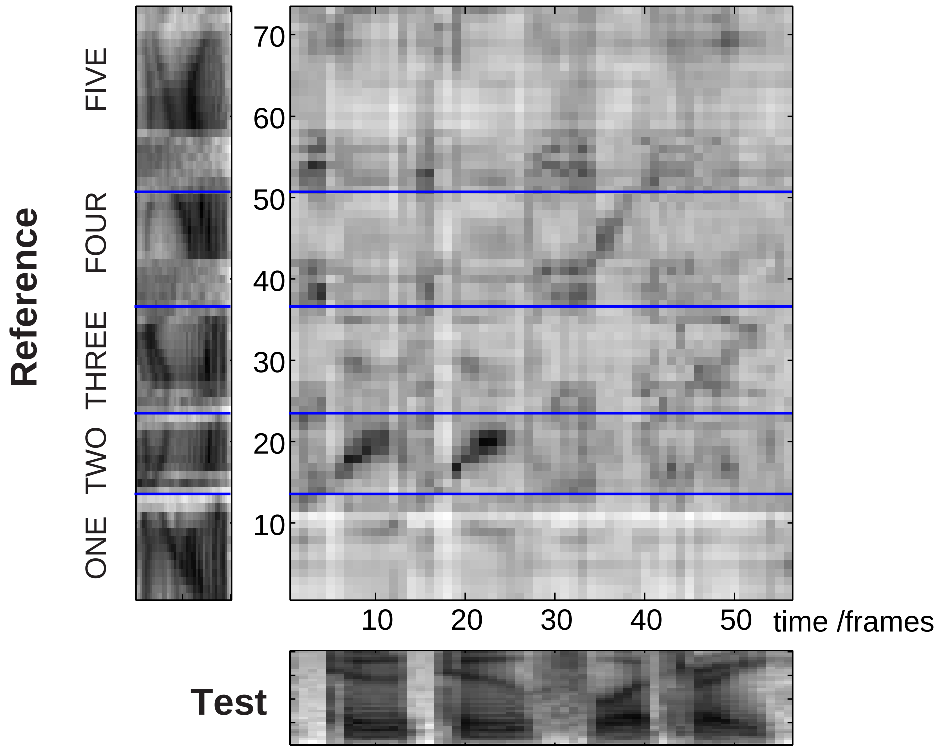
1. **Signal template matching**
2. **Statistical sequence recognition**
3. **Acoustic modeling**
4. **The Hidden Markov Model (HMM)**

Dan Ellis <dpwe@ee.columbia.edu>
http://www.ee.columbia.edu/~dpwe/e6820/

Columbia University Dept. of Electrical Engineering
Spring 2003

1 Signal template matching

- **Framewise comparison**
  of unknown word and stored templates:

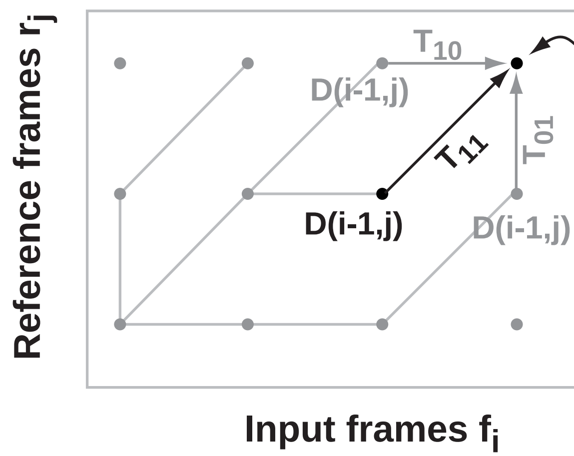


- distance metric?
- comparison across templates?
- constraints?

Dynamic Time Warp (DTW)

- **Find** lowest-cost **constrained path:**
  - matrix $d(i,j)$ of distances
    between input frame f_i and reference frame r_j
  - allowable predecessors & transition costs T_{xy}

Lowest cost to (i,j)



$$D(i,j) = d(i,j) + \min\left\{\begin{array}{l} D(i-1,j) + T_{10} \\ D(i,j-1) + T_{01} \\ D(i-1,j-1) + T_{11} \end{array}\right\}$$

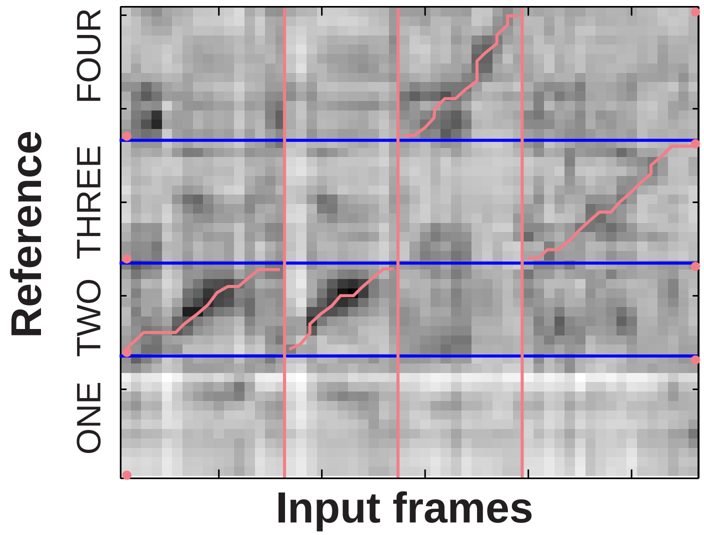
Local match cost

Best predecessor (including transition cost)

- **Best path via** traceback **from final state**
  - store predecessors for each (i,j)

DTW-based recognition

- **Reference templates for each possible word**
- **For isolated words:**
  - mark endpoints of input word
  - calculate scores through each template (+prune)
  - choose best
- **For continuous speech**
  - one matrix of template slices;
    special-case constraints at word ends



DTW-based recognition (2)

- + **Successfully handles timing variation**
- + **Able to recognize speech at reasonable cost**
- - **Distance metric?**
  - pseudo-Euclidean space?
- - **Warp penalties?**
- - **How to choose templates?**
  - several templates per word?
  - choose ‘most representative’?
  - align and average?

$\rightarrow$ **need a rigorous foundation...**

Outline

1. Signal template matching
2. **Statistical sequence recognition**
   - state-based modeling
3. Acoustic modeling
4. The Hidden Markov Model (HMM)

2 Statistical sequence recognition

- **DTW limited because it's hard to optimize**
  - interpretation of distance, transition costs?
- **Need a theoretical foundation: Probability**
- **Formulate recognition as MAP choice among models:**

$$M^* = \underset{M_j}{\operatorname{argmax}}\ p(M_j | X, \Theta)$$

  - X = observed features
  - M_j = word-sequence models
  - Θ = all current parameters

Statistical formulation (2)

- **Can rearrange via Bayes' rule (& drop $p(X)$):**

$$M^{*} = \underset{M_j}{\operatorname{argmax}}\ p(M_j|X, \Theta)$$

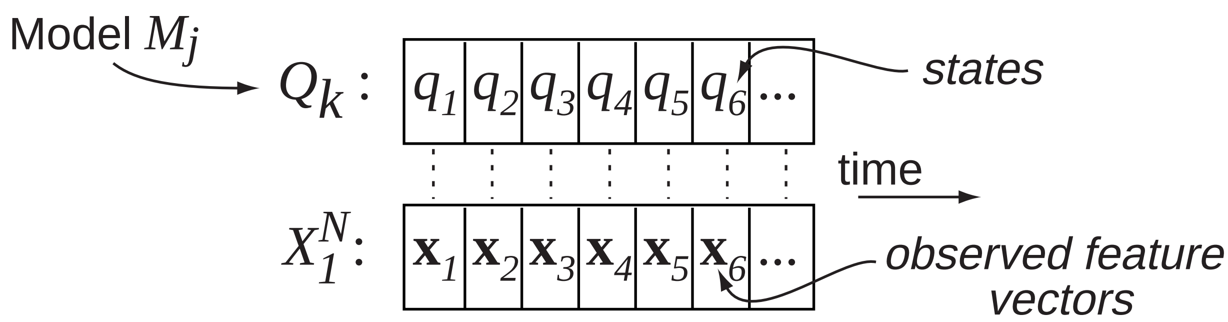
$$= \underset{M_j}{\operatorname{argmax}}\ p(X|M_j, \Theta_A)\,p(M_j|\Theta_L)$$

  - $p(X \mid M_j)$ =
    likelihood of observations under model
  - $p(M_j)$ = prior probability of model
  - Θ_A = acoustics-related model parameters
  - Θ_L = language-related model parameters

- **Questions:**
  - what form of model to use for $p(X|M_j, \Theta_A)$?
  - how to find Θ_A (training)?
  - how to solve for M_j (decoding)?

State-based modeling

- **Assume discrete-state model for the speech:**
  - observations are divided up into time frames
  - model → states → observations:



- **Probability of observations given model is:**

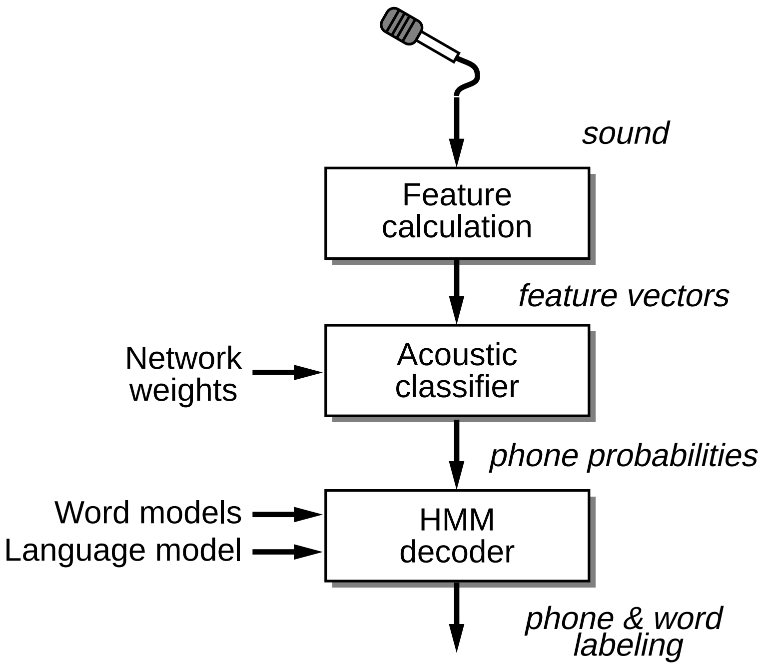
$$p(X|M_j) = \sum_{\text{all } Q_k} p(X_1^N \,|\, Q_k, M_j) \cdot p(Q_k \,|\, M_j)$$

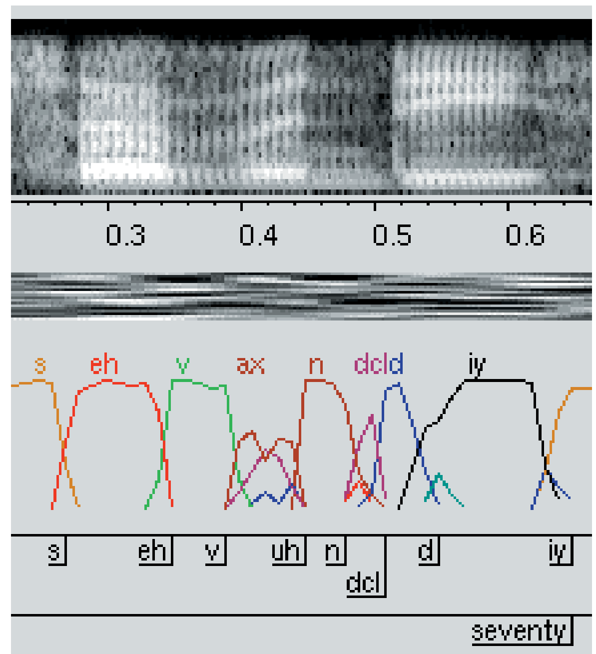
  - sum over all possible state sequences Q_k

- **How do observations depend on states?**
  How do state sequences depend on model?

The speech recognition chain

- **After classification, still have problem of classifying the sequences of frames:**





- **Questions**
  - what to use for the acoustic classifier?
  - how to represent ‘model’ sequences?
  - how to score matches?

Outline

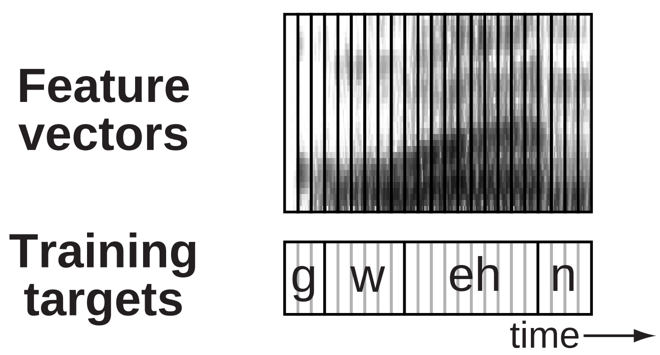
1. Signal template matching
2. Statistical sequence recognition
3. **Acoustic modeling**
   - defining targets
   - neural networks & Gaussian models
4. The Hidden Markov Model (HMM)

3 Acoustic Modeling

- **Goal: Convert features into probabilities of particular labels:**
  i.e find $p(q_n^i \mid X_n)$ **over some state set {q^i}**
    - conventional statistical classification problem
- **Classifier construction is data-driven**
    - assume we can get examples of known good Xs for each of the q^is
    - calculate model parameters by standard training scheme
- **Various classifiers can be used**
    - GMMs model distribution under each state
    - Neural Nets directly estimate posteriors
- **Different classifiers have different properties**
    - features, labels limit ultimate performance

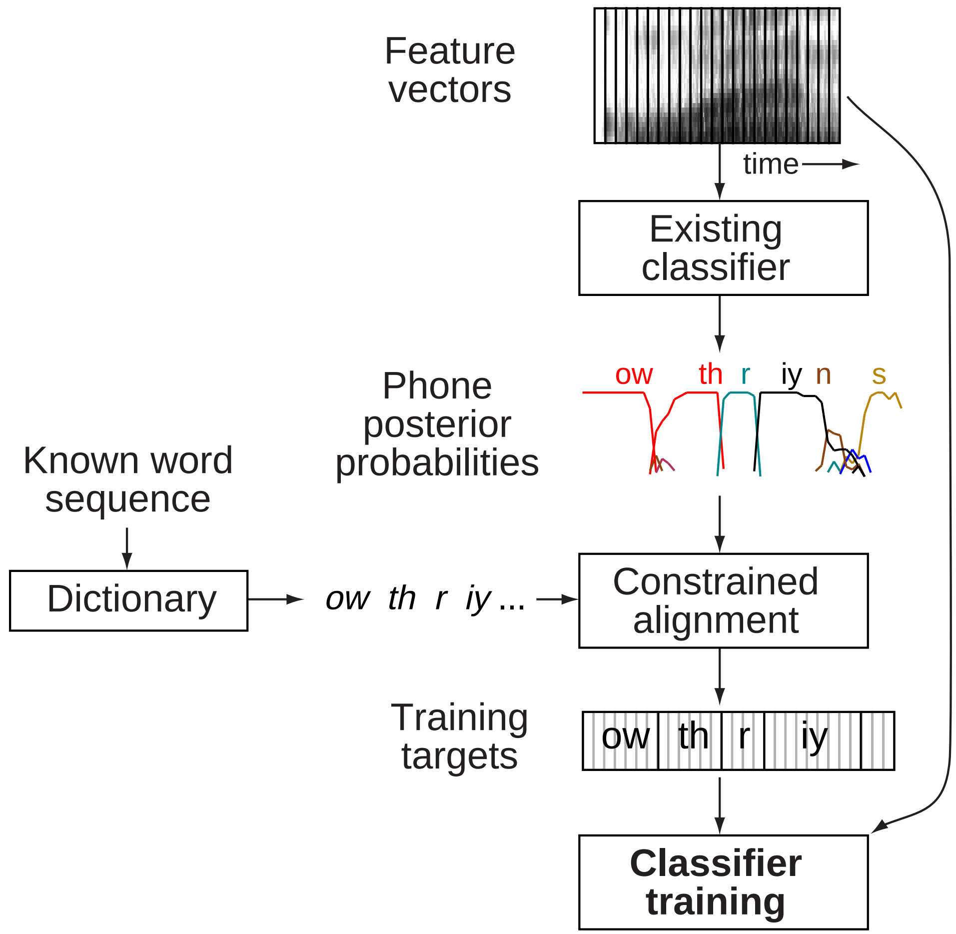
Defining classifier targets

- **Choice of {q^i} can make a big difference**
  - must support recognition task
  - must be a practical classification task
- **Hand-labeling is one source...**
  - 'experts' mark spectrogram boundaries
- **...Forced alignment is another**
  - 'best guess' with existing classifiers, given words
- **Result is targets for each training frame:**



Forced alignment

- **Best labeling given existing classifier constrained by known word sequence**



Gaussian Mixture Models vs. Neural Nets

- **GMMs fit distribution of features under states:**
  - separate 'likelihood' model for each state q^i

$$p(\mathbf{x}|q^k) = \frac{1}{(\sqrt{2\pi})^d |\Sigma_k|^{1/2}} \cdot \exp\left[-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu}_k)^T \Sigma_k^{-1} (\mathbf{x}-\mu_k)\right]$$

  - match any distribution given enough data

- **Neural nets estimate posteriors directly**

$$p(q^k|\mathbf{x}) = F[\textstyle\sum_j w_{jk} \cdot F[\sum_j w_{ij} x_i]]$$

  - parameters set to discriminate classes

- **Posteriors & likelihoods related by Bayes' rule:**

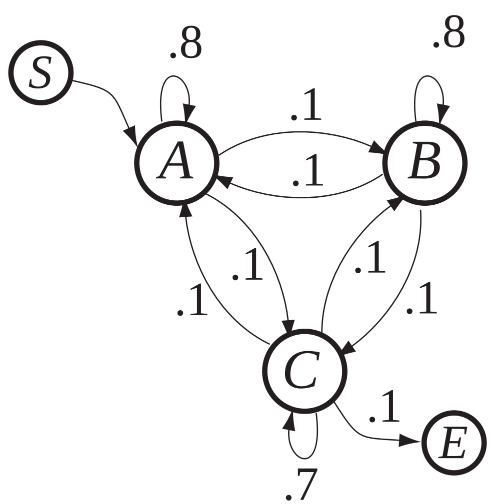
$$p(q^k|\mathbf{x}) = \frac{p(\mathbf{x}|q^k) \cdot Pr(q^k)}{\sum_j p(\mathbf{x}|q^j) \cdot Pr(q^j)}$$

Outline

1. Signal template matching
2. Statistical sequence recognition
3. Acoustic classification
4. **The Hidden Markov Model (HMM)**
   - generative Markov models
   - hidden Markov models
   - model fit likelihood
   - HMM examples

Markov models

- **A (first order) Markov model is a finite-state system whose behavior depends only on the current state**
- **E.g. generative Markov model:**

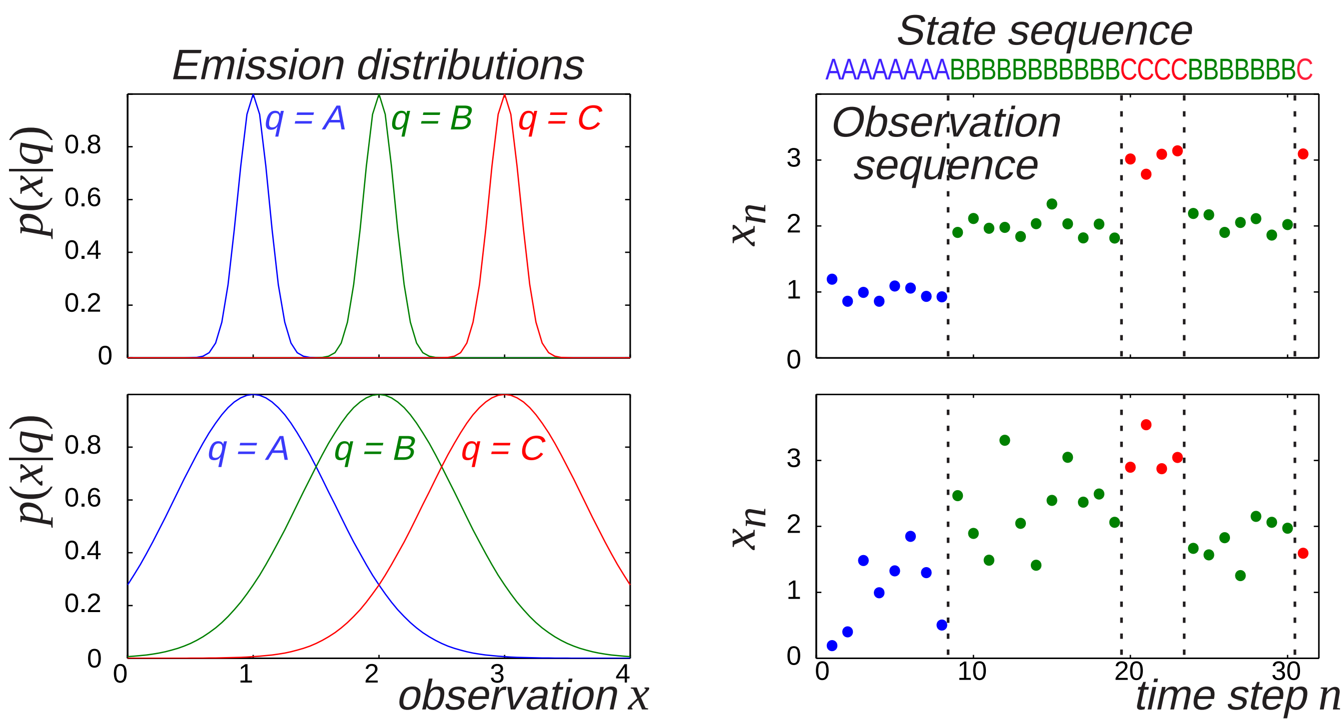


$p(q_{n+1} \mid q_n)$		q_{n+1}: S	A	B	C	E
q_n	S	0	1	0	0	0
	A	0	.8	.1	.1	0
	B	0	.1	.8	.1	0
	C	0	.1	.1	.7	.1
	E	0	0	0	0	1

S A A A A A A A A B B B B B B B B B C C C C B B B B B B C E

Hidden Markov models

- **= Markov model where state sequence $Q = \{q_n\}$ is not directly observable (= 'hidden')**
- **But, observations X *do* depend on Q:**
  - x_n is rv that depends on current state: $p(x|q)$



  - can still tell *something* about state seq...

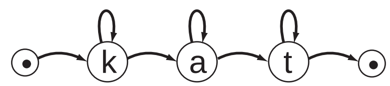
(Generative) Markov models (2)

- **HMM is specified by:**

- states q^i

- transition probabilities a_{ij}

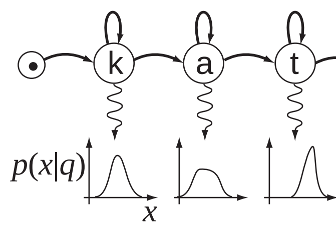
$$p(q_n^j | q_{n-1}^i) \equiv a_{ij}$$



	k	a	t	•
•	1.0	0.0	0.0	0.0
k	0.9	0.1	0.0	0.0
a	0.0	0.9	0.1	0.0
t	0.0	0.0	0.9	0.1

- emission distributions $b_i(x)$

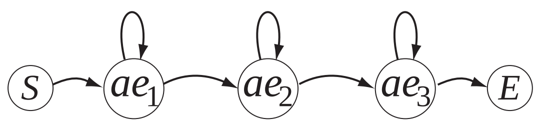
$$p(x | q^i) \equiv b_i(x)$$

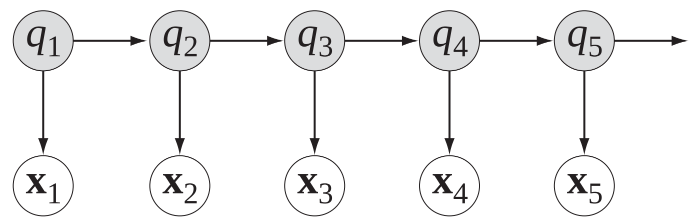


+ (initial state probabilities $p(q_1^i) \equiv \pi_i$)

Markov models for speech

- **Speech models** M_j
  - usually left-to-right HMMs (sequence constraint)
  - observation & evolution
    are conditionally independent of rest
    given (hidden) state q_n





  - self-loops for time dilation

Markov models for sequence recognition

- **Independence of observations:**
  - observation x_n depends only current state q_n

$$
\begin{aligned}
p(X|Q) &= p(x_1, x_2, \ldots x_N | q_1, q_2, \ldots q_N) \\
&= p(x_1|q_1) \cdot p(x_2|q_2) \cdot \ldots p(x_N|q_N) \\
&= \prod_{n=1}^{N} p(x_n|q_n) = \prod_{n=1}^{N} b_{q_n}(x_n)
\end{aligned}
$$

- **Markov transitions:**
  - transition to next state q_{i+1} depends only on q_i

$$
\begin{aligned}
p(Q|M) &= p(q_1, q_2, \ldots q_N | M) \\
&= p(q_N|q_1 \ldots q_{N-1}) p(q_{N-1}|q_1 \ldots q_{N-2}) \ldots p(q_2|q_1) p(q_1) \\
&= p(q_N|q_{N-1}) p(q_{N-1}|q_{N-2}) \ldots p(q_2|q_1) p(q_1) \\
&= p(q_1) \prod_{n=2}^{N} p(q_n|q_{n-1}) = \pi_{q_1} \prod_{n=2}^{N} a_{q_{n-1} q_n}
\end{aligned}
$$

Model-fit calculation

- **From ‘state-based modeling’:**

$$p(X|M_j) = \sum_{\text{all } Q_k} p(X_1^N \big| Q_k, M_j) \cdot p(Q_k \big| M_j)$$

- **For HMMs:**

$$p(X|Q) = \prod_{n=1}^{N} b_{q_n}(x_n)$$

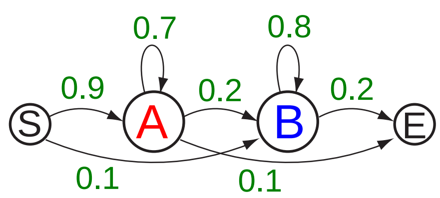
$$p(Q|M) = \pi_{q_1} \cdot \prod_{n=2}^{N} a_{q_{n-1}q_n}$$

- **Hence, solve for M^* :**
  - calculate $p(X|M_j)$ for each available model, scale by prior $p(M_j) \rightarrow p(M_j|X)$

- **Sum over *all* Q_k ???**

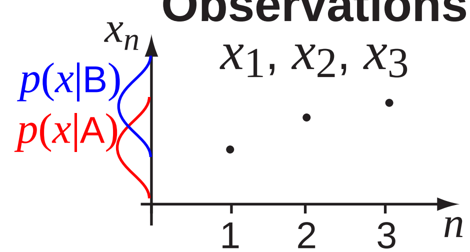
Summing over all paths

Model M_1



	S	A	B	E
S	•	0.9	0.1	•
A	•	0.7	0.2	0.1
B	•	•	0.8	0.2
E	•	•	•	1

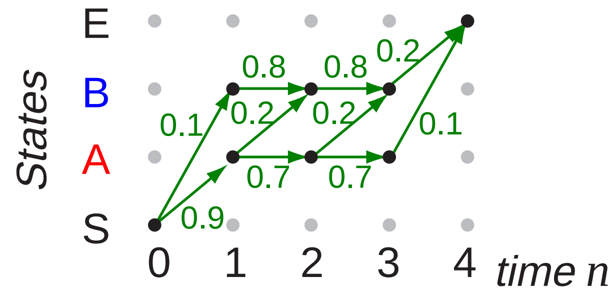
Observations



Observation likelihoods

$p(x\|q)$		x_1	x_2	x_3
q	A	2.5	0.2	0.1
	B	0.1	2.2	2.3

Paths



All possible 3-emission paths Q_k from S to E

q_0	q_1	q_2	q_3	q_4	$p(Q\mid M) = \prod_n p(q_n\|q_{n\text{-}1})$	$p(X\mid Q,M) = \prod_n p(x_n\|q_n)$	$p(X,Q\mid M)$
S	A	A	A	E	.9 x .7 x .7 x .1 = **0.0441**	2.5 x 0.2 x 0.1 = 0.05	0.0022
S	A	A	B	E	.9 x .7 x .2 x .2 = 0.0252	2.5 x 0.2 x 2.3 = 1.15	0.0290
S	A	B	B	E	.9 x .2 x .8 x .2 = 0.0288	2.5 x 2.2 x 2.3 = 12.65	**0.3643**
S	B	B	B	E	.1 x .8 x .8 x .2 = 0.0128	0.1 x 2.2 x 2.3 = 0.506	0.0065
					Σ = 0.1109		$\boxed{\Sigma = p(X\mid M) = \mathbf{0.4020}}$

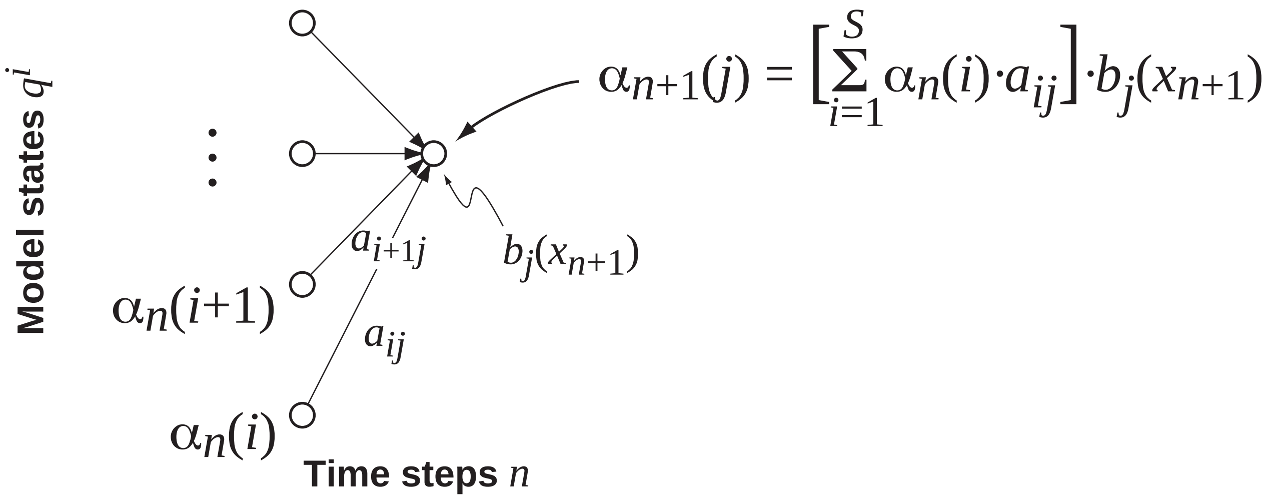
(length 3 paths only)

The 'forward recursion'

- **Dynamic-programming-like technique to calculate sum over all Q_k**
- **Define $\alpha_n(i)$ as the probability of getting to state q^i at time step n (by any path):**

$$\alpha_n(i) = p(x_1, x_2, \ldots x_n, q_n = q^i) \equiv p(X_1^n, q_n^i)$$

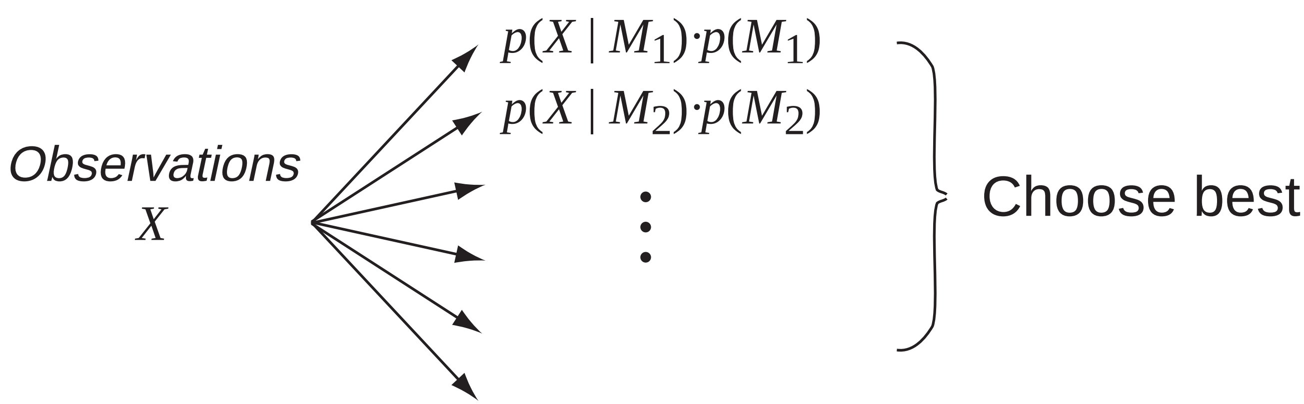
- **Then $\alpha_{n+1}(j)$ can be calculated recursively:**



Forward recursion (2)

- **Initialize** $\alpha_1(i) = \pi_i \cdot b_i(x_1)$
- **Then total probability** $p(X_1^N | M) = \sum_{i=1}^{S} \alpha_N(i)$

$\rightarrow$ **Practical way to solve for** $p(X \mid M_j)$
and hence perform recognition



Optimal path

- **May be interested in actual q_n assignments**
  - which state was 'active' at each time frame
  - e.g. phone labelling (for training?)
- **Total probability is over *all* paths...**
- **... but can also solve for single *best* path = "Viterbi" state sequence**
- **Probability along best path to state q^j_{n+1}:**

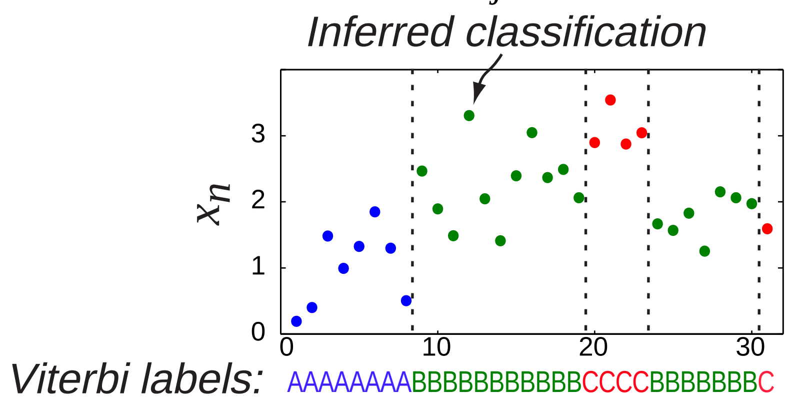
$$\alpha^*_{n+1}(j) = \left[\max_i \{\alpha^*_n(i) a_{ij}\}\right] \cdot b_j(x_{n+1})$$

  - backtrack from final state to get best path
  - final probability is product only (no sum) → log-domain calculation is just summation
- **Total probability often dominated by best path:**

$$p(X, Q^* | M) \approx p(X | M)$$

Interpreting the Viterbi path

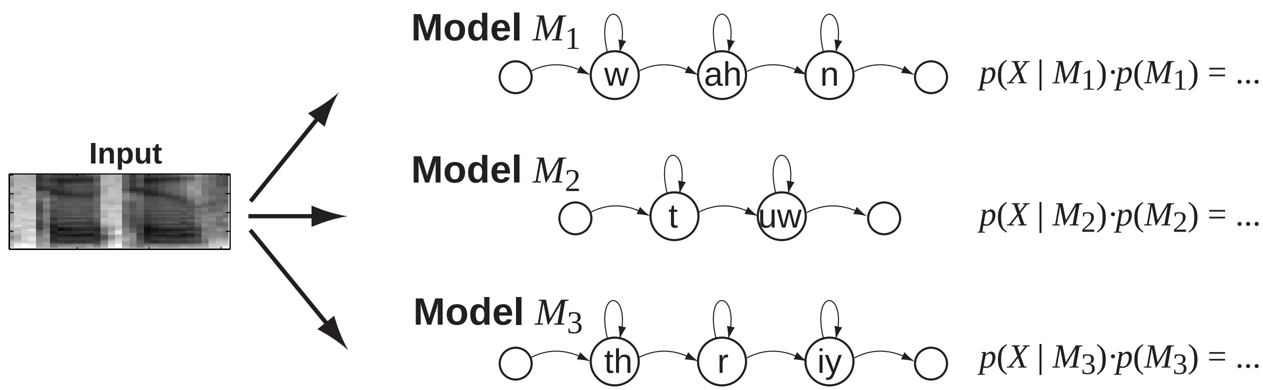
- **Viterbi path assigns each x_n to a state q^i**
  - performing classification based on $b_i(x)$
  - ... at the same time as applying transition constraints a_{ij}



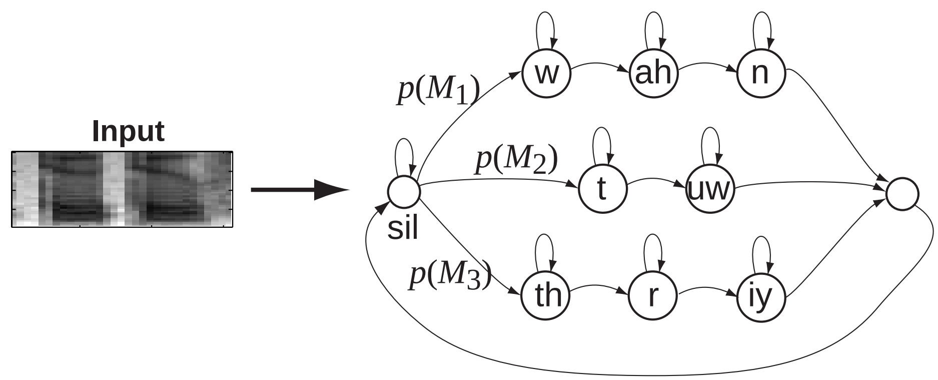
- **Can be used for segmentation**
  - train an HMM with ‘garbage’ and ‘target’ states
  - decode on new data to find ‘targets’, boundaries
- **Can use for (heuristic) training**
  - e.g. train classifiers based on labels...

Recognition with HMMs

- **Isolated word**
  - choose best $p(M|X) \propto p(X|M)p(M)$

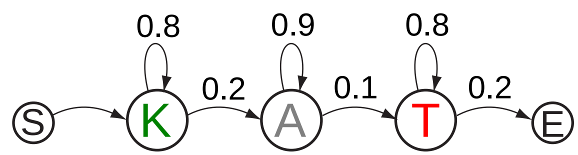


- **Continuous speech**
  - Viterbi decoding of one large HMM gives words

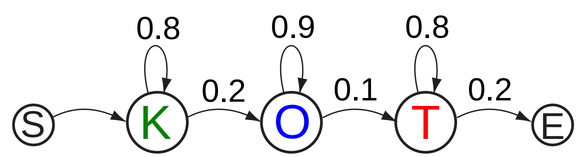


HMM examples: Different state sequences

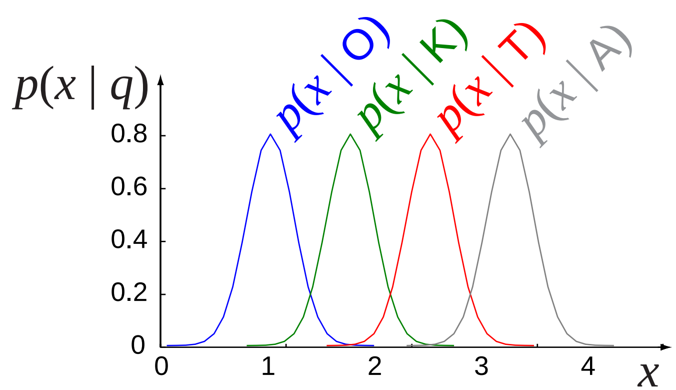
Model M_1



Model M_2

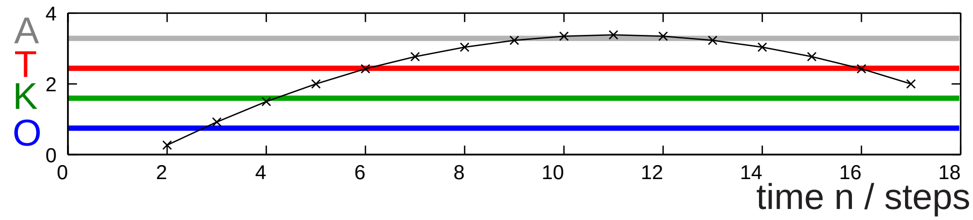


Emission distributions



Model matching: Emission probabilities

Observation sequence
x_n

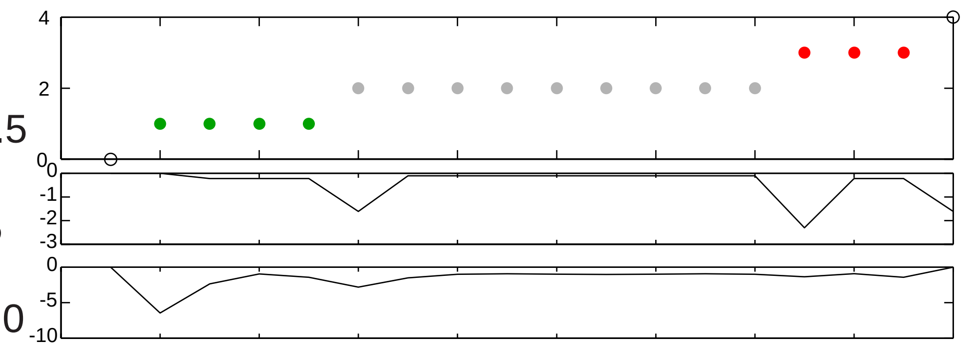


Model M_1
$\log p(X \mid M) = -32.1$

state alignment
$\log p(X,Q^* \mid M) = -33.5$

log trans.prob
$\log p(Q^* \mid M) = -7.5$

log obs.l'hood
$\log p(X \mid Q^*,M) = -26.0$

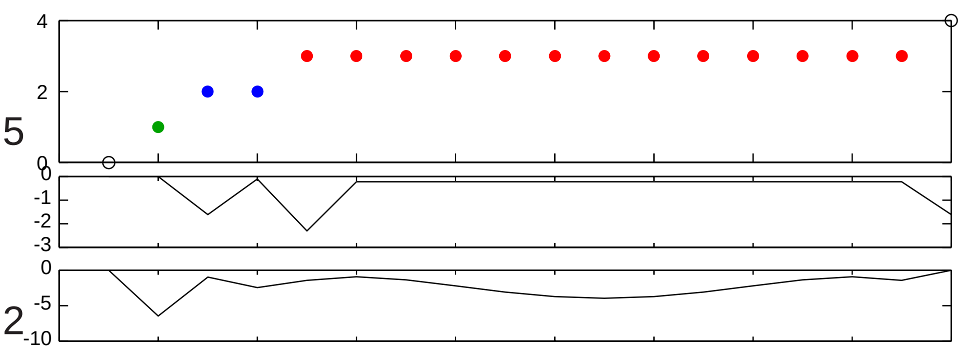


Model M_2
$\log p(X \mid M) = -47.0$

state alignment
$\log p(X,Q^* \mid M) = -47.5$

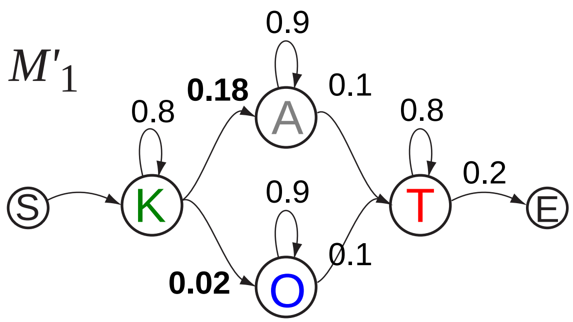
log trans.prob
$\log p(Q^* \mid M) = -8.3$

log obs.l'hood
$\log p(X \mid Q^*,M) = -39.2$

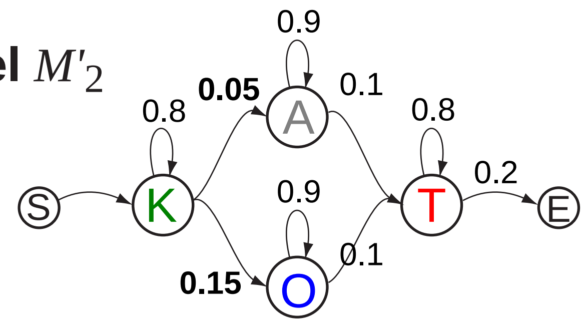


Model matching: Transition probabilities

Model M'_1



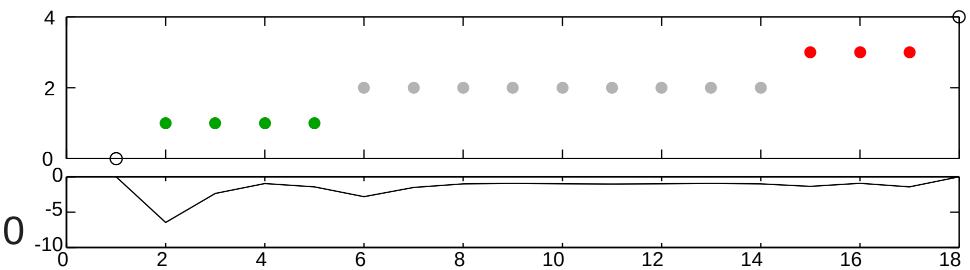
Model M'_2



state alignment

log obs.l'hood

$\log p(X \mid Q^*,M) = -26.0$

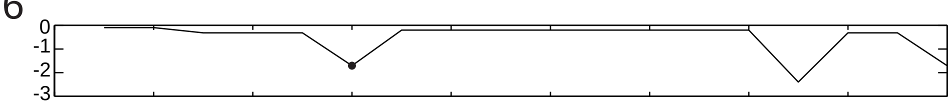


Model M'_1 $\log p(X \mid M) = -32.2$

$\log p(X,Q^* \mid M) = -33.6$

log trans.prob

$\log p(Q^* \mid M) = -7.6$

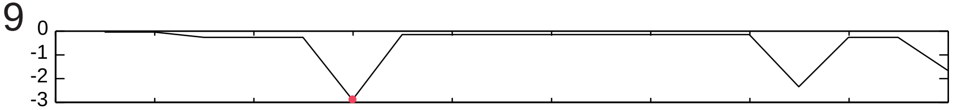


Model M'_2 $\log p(X \mid M) = -33.5$

$\log p(X,Q^* \mid M) = -34.9$

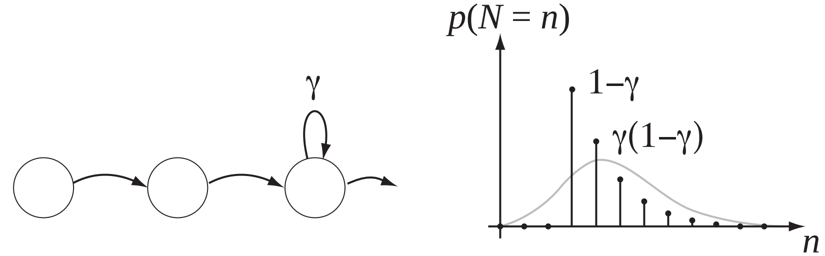
log trans.prob

$\log p(Q^* \mid M) = -8.9$

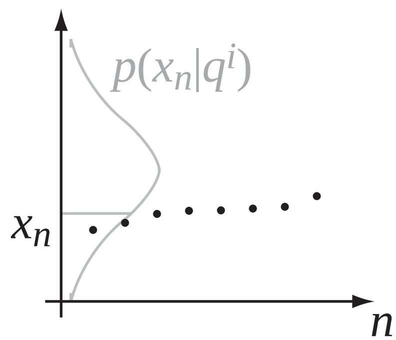


Validity of HMM assumptions

- **Key assumption is conditional independence:**
  Given q^i, future evolution & obs. distribution are independent of previous events
  - duration behavior:
    self-loops imply exponential distribution

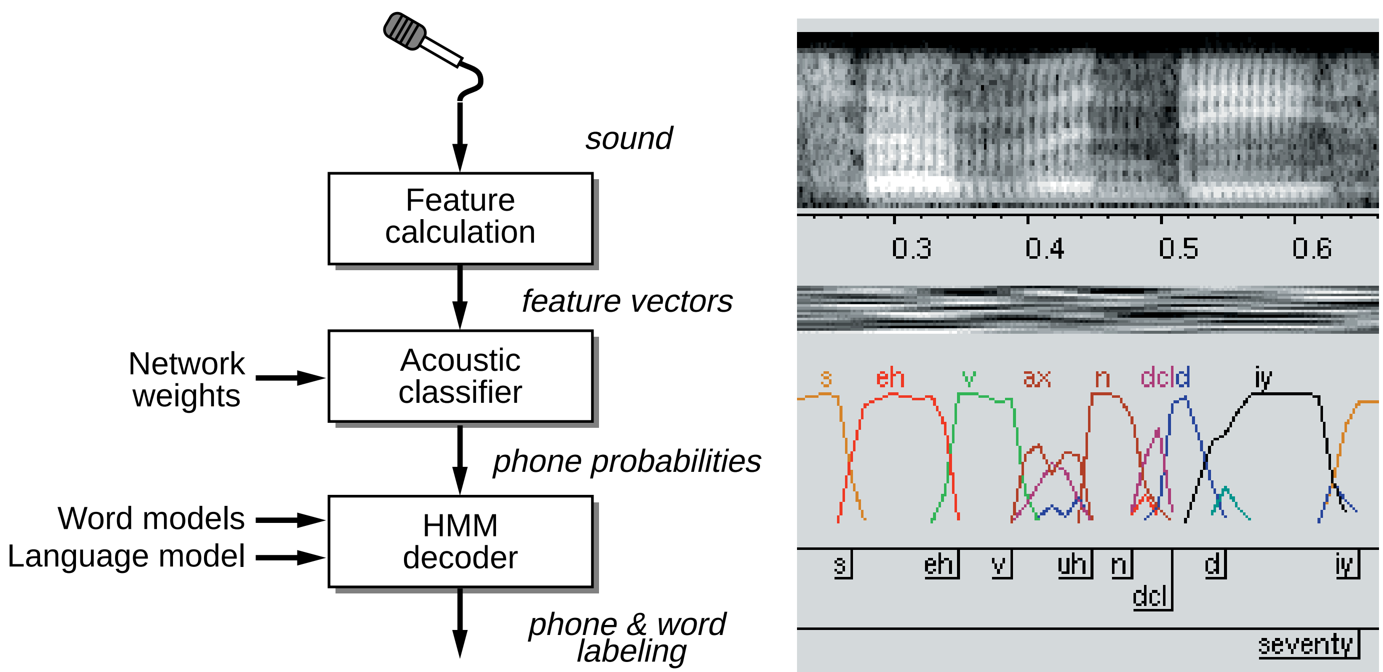


  - independence of successive x_ns



$$p(X) = \prod p(x_n|q^i) \text{ ?}$$

Recap: Recognizer Structure



- **We now know how to execute each stage**
- **.. but how to train HMMs?**
- **.. where to get word/language models?**

Summary

- **Speech is modeled as a sequence of features**
  - need temporal aspect to recognition
  - best time-alignment of templates = DTW
- **Hidden Markov models are rigorous solution**
  - self-loops allow temporal dilation
  - exact, efficient likelihood calculations

Parting thought:
How to set the HMM parameters? (training)