ELEN E4810 Digital Signal Processing **Final Exam**

Monday 2012-12-17 09:00-12:00 (180 min) **Corrected version** Dan Ellis <dpwe@ee.columbia.edu>

This test consists of 4 questions, each with equal weight.
You have three hours (180 minutes) to complete the test.
This test is open-book: you are permitted to refer to your notes and textbooks during the test.
You may use a calculator for numerical work, but not for graphing.
You must show all your workings to get credit for an answer.

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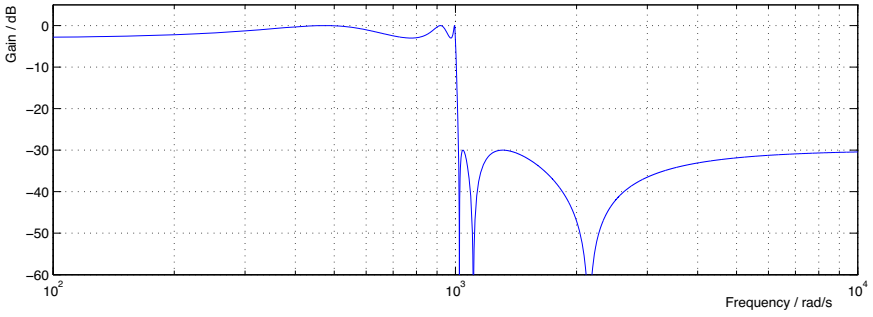
1. A system H_1 has impulse response $h_1[n] = \{\underline{1}, 0, -1\}$, where the underline indicates the $n = 0$ point.
   (a) What kind of system is this?
   (b) Evaluate and sketch the magnitude and phase response of H_1.
   (c) A system H_2 is constructed as two instances of H_1 in sequence. Evaluate and sketch its magnitude and phase response.
   (d) System H_3 is built as H_2 followed by a factor-of-two decimation (i.e., every odd-indexed sample is discarded). Evaluate and sketch its magnitude and phase response.

2. The figure below shows the magnitude response of an optimal minimax continuous-time filter. All features (ripples and notches) are visible.
   (a) What filter is this? Be as specific as possible about how it was designed.
   (b) Sketch its pole-zero diagram as well as you can. Explain how you chose where to put poles and zeros.
   (c) A discrete-time filter with cutoff frequency 0.25π rad/samp is derived from this filter. Sketch its magnitude response and pole-zero diagram.
   (d) An input signal $x[n] = 1 + \cos(\pi n)$ is fed to this system. What is $|Y(e^{j\omega})|$, the magnitude of the DTFT of the output?



3. (a) Express the 4-point DFT as a matrix multiplication, evaluating all constants.

   (b) Write expressions for the real and imaginary parts of the outputs, $\mathrm{Re}\{X[k]\}$ and $\mathrm{Im}\{X[k]\}$, $k = 0 \ldots 3$, in terms of the real and imaginary parts of the inputs, $\mathrm{Re}\{x[n]\}$ and $\mathrm{Im}\{x[n]\}$.

   (c) Draw the structure for an 8-point FFT that includes a first stage of 4-point DFTs. What is the minimum number of real multiplies that this structure requires? How does it compare to a pure radix-2 implementation?

4. The *Discrete Cosine Transform* (DCT) is a transform widely used in data compression (including JPEG image compression) for its ability to compact energy from a signal with local continuity into a small number of coefficients. The $(N+1)$-point type-I DCT of a sequence $x[n]$, $n = 0 \ldots N$ is given by:

$$X[k] = \frac{1}{2}(x[0] + (-1)^k x[N]) + \sum_{n=1}^{N-1} x[n] \cos \frac{\pi n k}{N} \quad \text{for } k = 0 \ldots N \tag{1}$$

   Note: *In the original version of this question, as described in the solutions, the above equation omitted the plus sign, i.e., it read:*

$$X[k] = \frac{1}{2}(x[0] + (-1)^k x[N]) \sum_{n=1}^{N-1} x[n] \cos \frac{\pi n k}{N} \quad \text{for } k = 0 \ldots N \tag{2}$$

   (a) A $2N$-point real sequence $y[n]$, $n = -(N-1) \ldots N$ has a $2N$-point DFT $Y[k]$, $k = 0 \ldots 2N-1$ that is equal to the DCT $X[k]$ for $k = 0 \ldots N$. Find $y[n]$ in terms of $x[n]$.

   (b) What is the DCT of $x[n] = \cos \frac{\pi r n}{N}$ for integer r? (**Note:** *in the original version, it read* $x[n] = \cos \pi r n$).

   (c) Discuss any advantages and disadvantages you see for the DCT as a replacement for the DFT.