EE E6820: Speech & Audio Processing & Recognition

Lecture 3: Acoustics

1. **The wave equation**
2. **Acoustic tubes: reflections & resonance**
3. **Oscillations & musical acoustics**
4. **Spherical waves & room acoustics**

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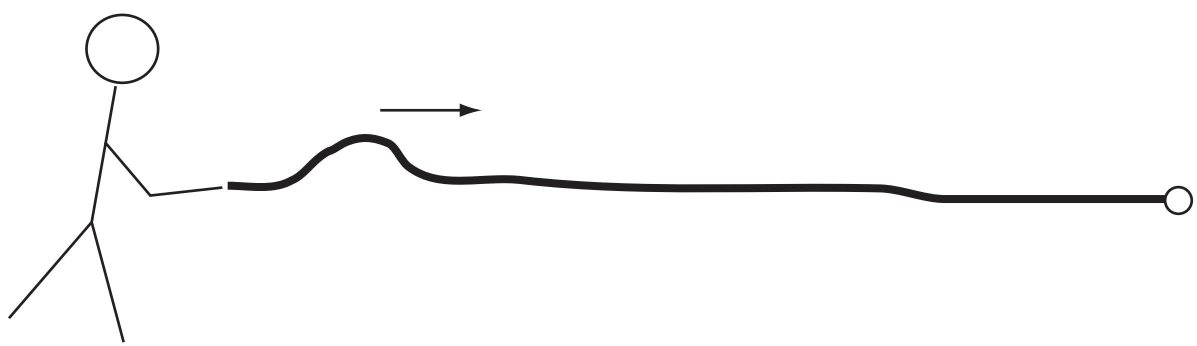
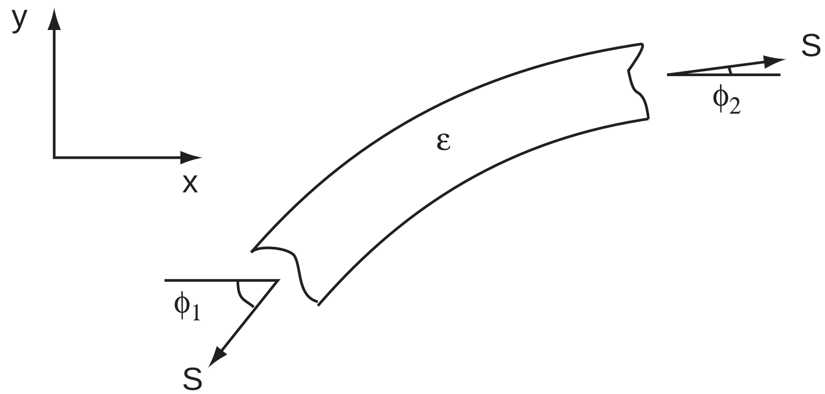
Acoustics & sound

- Acoustics is the study of physical waves
- (Acoustic) waves transmit energy without permanently displacing matter
(e.g. ocean waves)
- Same math recurs in many domains
- Intuition: pulse going down a rope



The wave equation

- **Consider a small section of the rope:**



- **displacement is y(x), tension S, mass ε·dx**

→ **lateral force is** $F_y = S \cdot \sin(\phi_2) - S \cdot \sin(\phi_1)$

$$= S \cdot \frac{\partial^2 y}{\partial x^2} \cdot dx$$

Wave equation (2)

- **Newton's law:** $F = ma$

$$S \cdot \frac{\partial^2 y}{\partial x^2} \cdot dx = \varepsilon dx \cdot \frac{\partial^2 y}{\partial t^2}$$

- **Call** $c^2 = S/\varepsilon$ **(tension to mass-per-length)**

  hence the *wave equation*:

$$c^2 \cdot \frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}$$

.. partial DE relating curvature and acceleration

Solution to the wave equation

- **If** $y(x, t) = f(x - ct)$

  then

  $$\frac{\partial y}{\partial x} = f'(x - ct) \qquad \frac{\partial y}{\partial t} = -c \cdot f'(x - ct)$$

  $$\frac{\partial^2 y}{\partial x^2} = f''(x - ct) \qquad \frac{\partial^2 y}{\partial t^2} = c^2 \cdot f''(x - ct)$$

  also works for $y(x, t) = f(x + ct)$

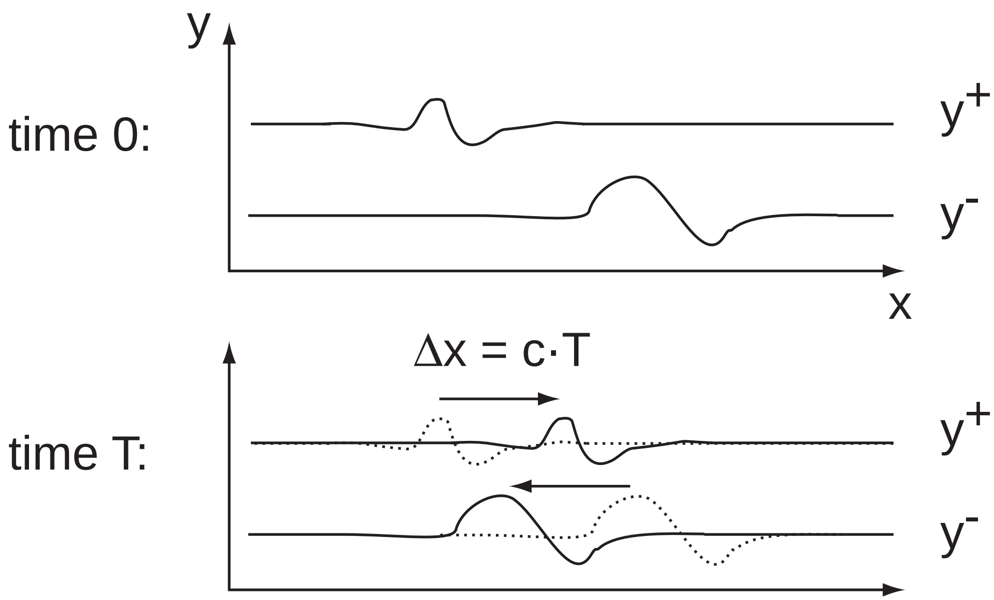
  Hence, general solution:

  $$c^2 \cdot \frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}$$

  $$\Rightarrow \quad y(x, t) = y^+(x - ct) + y^-(x + ct)$$

Solution to the wave equation (2)

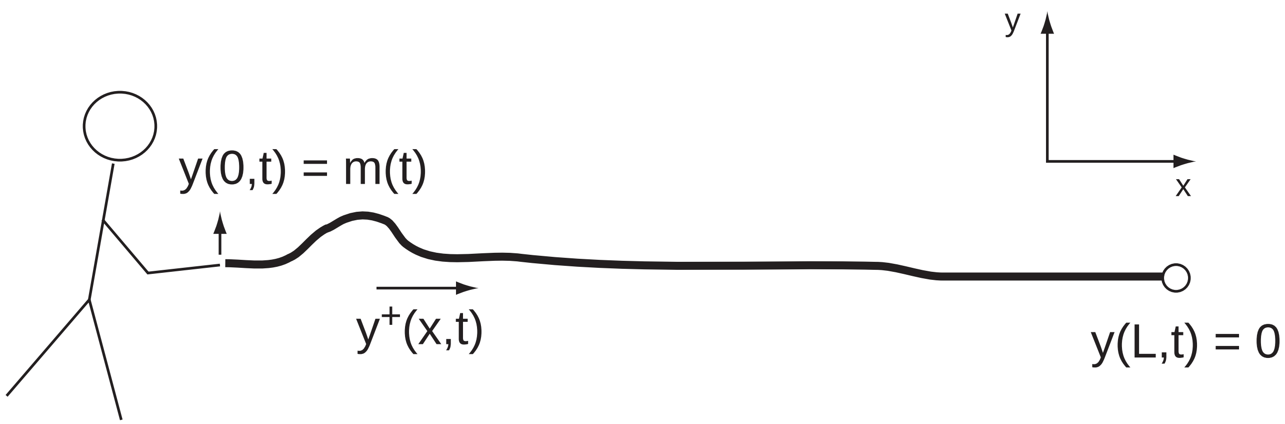
- $y^{+}(x-ct)$ and $y^{-}(x+ct)$ are ***travelling waves***
  - shape stays constant but changes position:



- c is travelling wave velocity (Δx / Δt)
- y^{+} moves right, y^{-} moves left
- resultant y(x) is *sum* of the two waves

Wave equation solutions (3)

- **What is the form of y^+, y^- ?**
  - any doubly-differentiable function will satisfy wave equation
- **Actual waveshapes dictated by *boundary conditions***
  - $y(x)$ at $t = 0$
  - constraints on y at particular x's
    e.g. input motion $y(0, t) = m(t)$
    rigid termination $y(L, t) = 0$



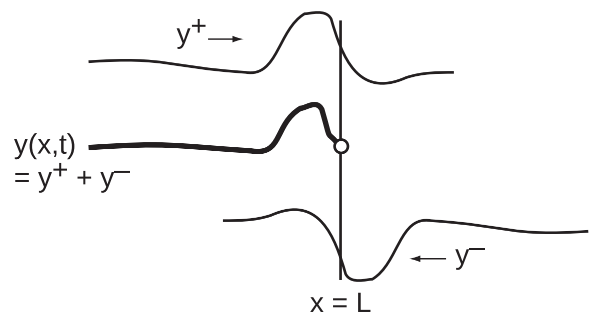
Terminations and reflections

- **System constraints:**
  - initial $y(x, 0) = 0$ (flat rope)
  - input $y(0, t) = m(t)$ (at agent's hand) ($\rightarrow y^+$)
  - termination $y(L, t) = 0$ (fixed end)
  - wave equation $y(x,t) = y^+(x - ct) + y^-(x + ct)$
- **At termination:**

  $y(L, t) = 0 \rightarrow y^+(L - ct) = -y^-(L + ct)$

  i.e. y^+ and y^- are mirrored in *time* and *amplitude* around $x=L$

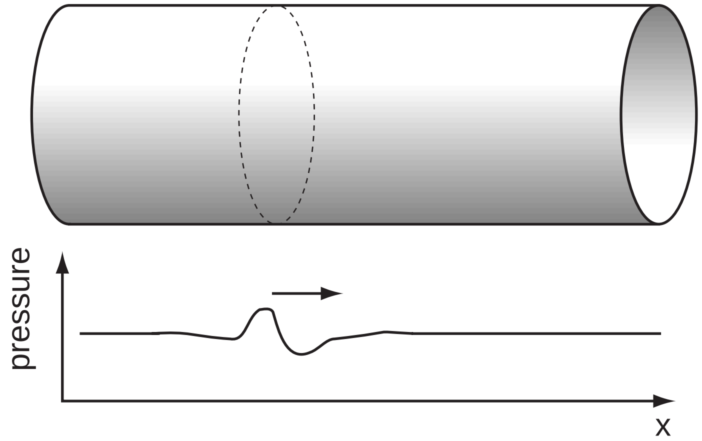
$\rightarrow$inverted reflection at termination



$\rightarrow$simulation [travel1.m]

Acoustic tubes

- **Sound waves travel down acoustic tubes:**

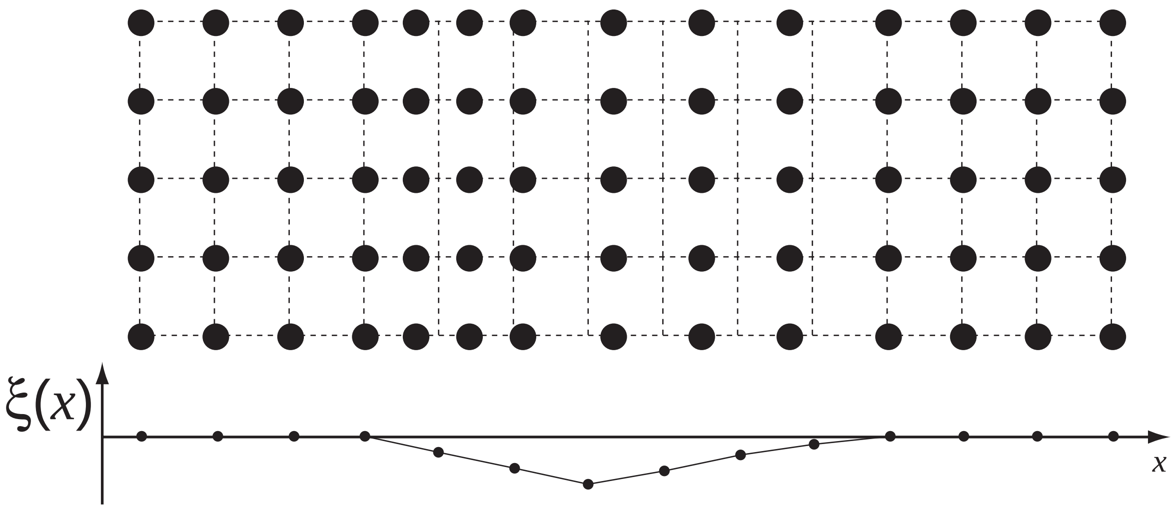


  - 1-dimensional; very similar to strings

- **Common situation:**
  - wind instrument bores
  - ear canal
  - vocal tract

Pressure and velocity

- **Consider air particle displacement** $\xi(x, t)$**:**



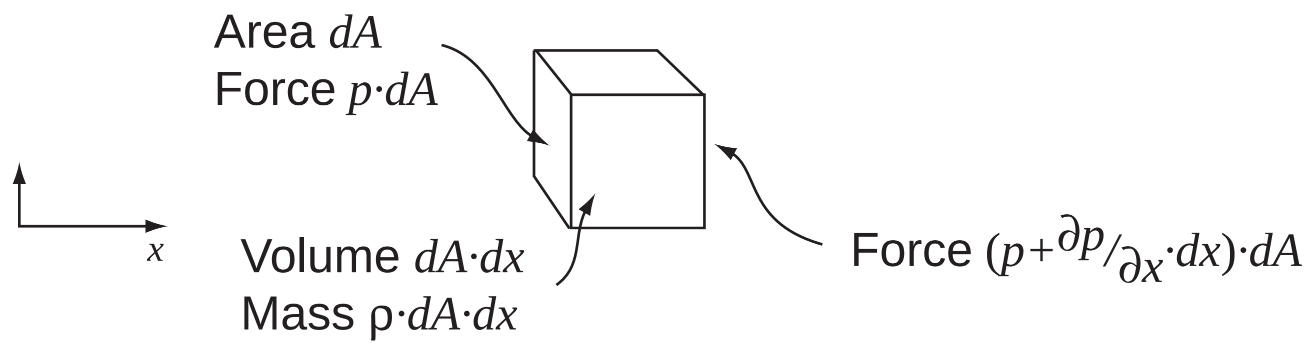
- **Particle velocity** $v(x, t) = \frac{\partial \xi}{\partial t}$

  hence volume velocity $u(x, t) = A \cdot v(x, t)$

- **Air pressure** $p(x, t) = -\frac{1}{\kappa} \cdot \frac{\partial \xi}{\partial x}$

Wave equation for a tube

- **Consider elemental volume:**



- **Newton's law:** $F = ma$

$$-\frac{\partial p}{\partial x} \cdot dx \cdot dA = \rho dA dx \cdot \frac{\partial v}{\partial t}$$

$$\Rightarrow \frac{\partial p}{\partial x} = -\rho \frac{\partial v}{\partial t}$$

- **Hence** $c^2 \cdot \frac{\partial^2 \xi}{\partial x^2} = \frac{\partial^2 \xi}{\partial t^2}$ $\qquad c = \frac{1}{\sqrt{\rho\kappa}}$

Acoustic tube traveling waves

- **Traveling waves in particle displacement:**

$$\xi(x, t) = \xi^{+}(x - ct) + \xi^{-}(x + ct)$$

- **Call** $u^{+}(\alpha) = -cA\frac{\partial}{\partial\alpha}\xi^{+}(\alpha)$

$$Z_0 = \frac{\rho c}{A}$$

- **Then pressure:**

$$p(x, t) = -\frac{1}{\kappa} \cdot \frac{\partial\xi}{\partial x} = Z_0 \cdot [u^{+}(x - ct) + u^{-}(x + ct)]$$

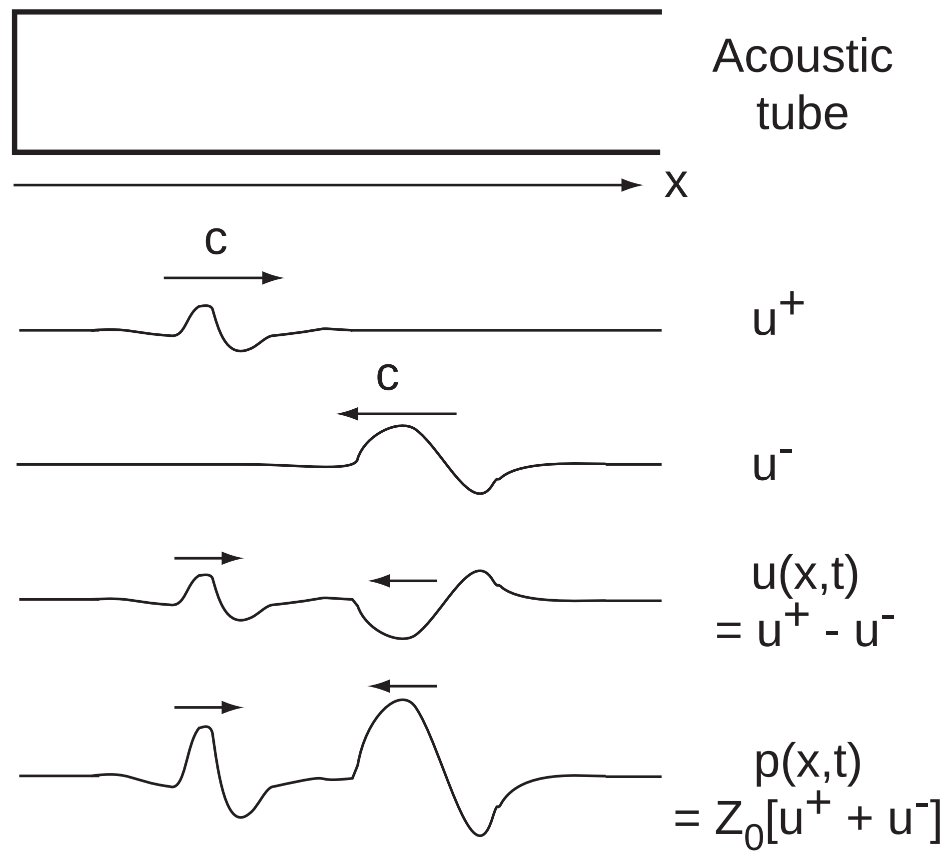
- **Volume velocity:**

$$u(x, t) = A \cdot \frac{\partial\xi}{\partial t} = u^{+}(x - ct) - u^{-}(x + ct)$$

- **(Scaled) sum and difference of traveling waves**

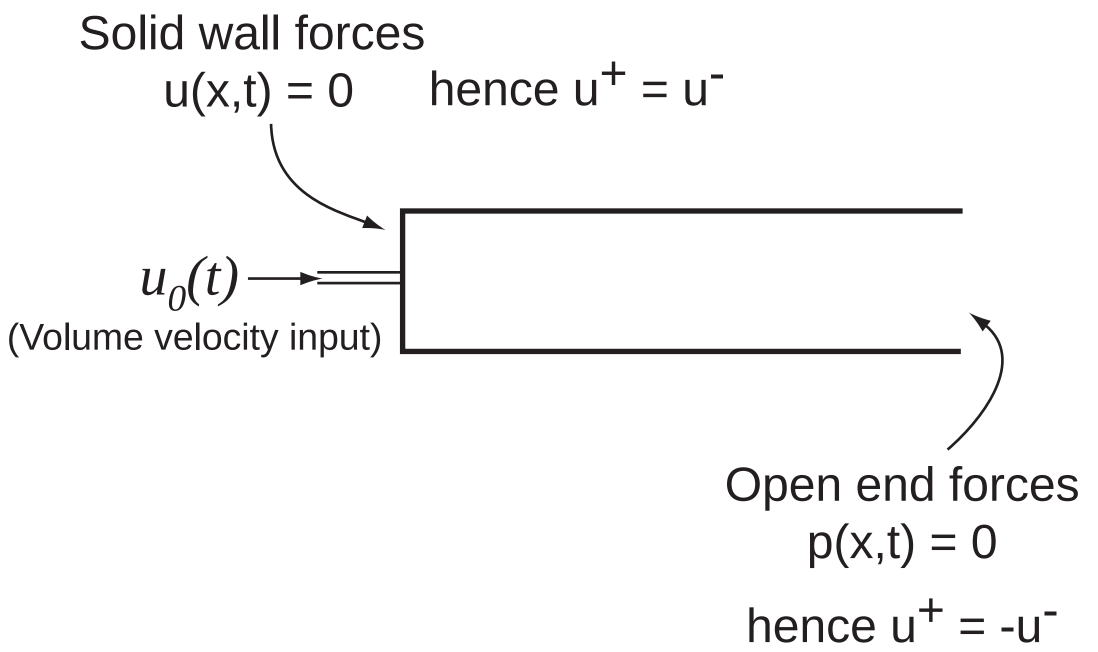
Acoustic tube traveling waves (2)

- **Different residuals for pressure and vol. veloc.:**



Terminations in tubes

- **Equivalent of 'fixed point' for tubes?**



- ***Open end*** **is like fixed point for rope: reflects wave back inverted**
- **Unlike fixed point,** ***solid wall*** **reflects traveling wave** ***without*** **inversion**

Standing waves

- **Consider (complex) sinusoidal input:**

$$u_0(t) = U_0 \cdot e^{j\omega t}$$

- **At any point, values will have form** $Ke^{j(\omega t + \phi)}$

- **Hence traveling waves:**

$$u^+(x - ct) = |A| e^{j(-kx + \omega t + \phi_A)}$$

$$u^-(x + ct) = |B| e^{j(kx + \omega t + \phi_B)}$$

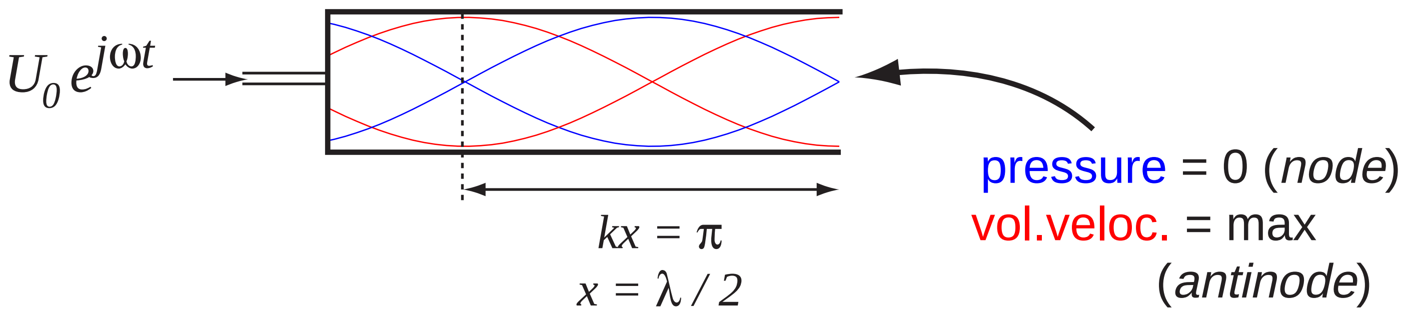
where $k = \omega / c$ **(spatial frequency, rad/m)**

(wavelength $\lambda = c / f = 2\pi c / \omega$ **)**

- **Pressure / vol. veloc. resultants show stationary pattern: *standing waves***
  - even when $|A| \neq |B|$

  $\rightarrow$simulation [sintwavemov.m]

Standing waves (2)



- **For lossless termination ($|\bar{u}^+| = |\bar{u}^-|$), have true nodes & antinodes**
- **Pressure and vol. veloc. are phase shifted**
  - in space and in time

*

Transfer function

- **Consider tube excited by** $u_0(t) = U_0 \cdot e^{j\omega t}$**:**
  - sinusoidal traveling waves must satisfy termination 'boundary conditions'
  - satisfied by complex constants A and B in

$$
\begin{aligned}
u(x, t) &= u^{+}(x - ct) + u^{-}(x + ct) \\
&= Ae^{j(-kx + \omega t)} + Be^{j(kx + \omega t)} \\
&= e^{j\omega t} \cdot (Ae^{-jkx} + Be^{jkx})
\end{aligned}
$$

  - standing wave pattern will scale with input magnitude
  - point of excitation makes a big difference

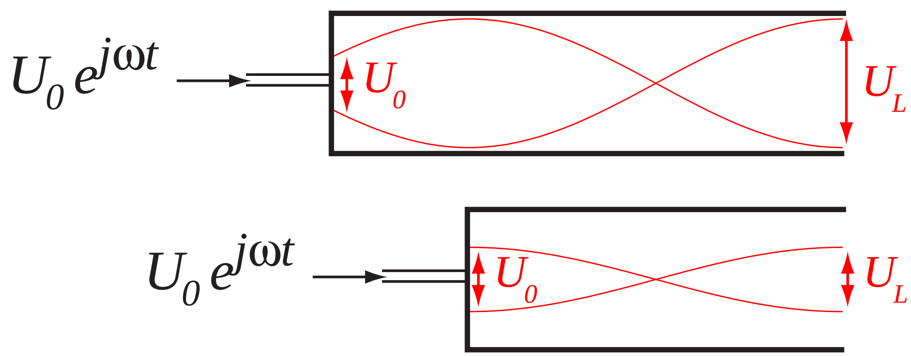
Transfer function (2)

- **For open-ended tube of length L excited at x = 0 by $U_0 e^{j\omega t}$:**

$$u(x,t) = \frac{\cos k(L-x)}{\cos kL} \cdot U_0 e^{j\omega t} \qquad \left(k = \frac{\omega}{c}\right)$$

(works at x = 0)

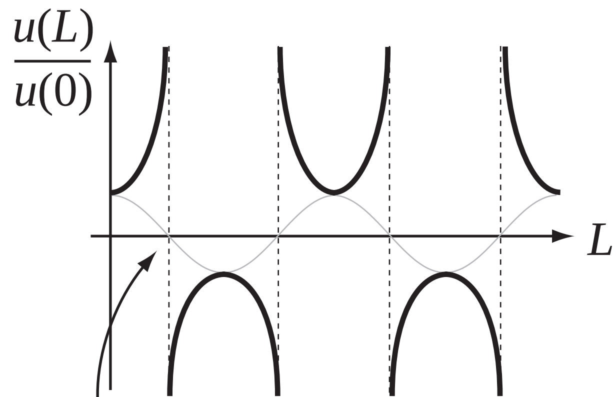
- **i.e. standing wave pattern**
e.g. varying L for a given ω (and hence k):



Transfer function (3)

- **Varying ω for a given L:**

  - at $x = L$, $\dfrac{u(L,t)}{u(0,t)} = \dfrac{1}{\cos kL} = \dfrac{1}{\cos(\omega L/c)}$



$$\left|\frac{u(L)}{u(0)}\right| \rightarrow \infty$$ at $\omega L/c = (2r+1)\pi/2$, $r = 0,1,2...$

- **Output vol. veloc. always larger than input**

- **Unbounded for** $L = (2r+1)\dfrac{\pi c}{2\omega} = (2r+1)\dfrac{\lambda}{4}$

  i.e. *resonance*

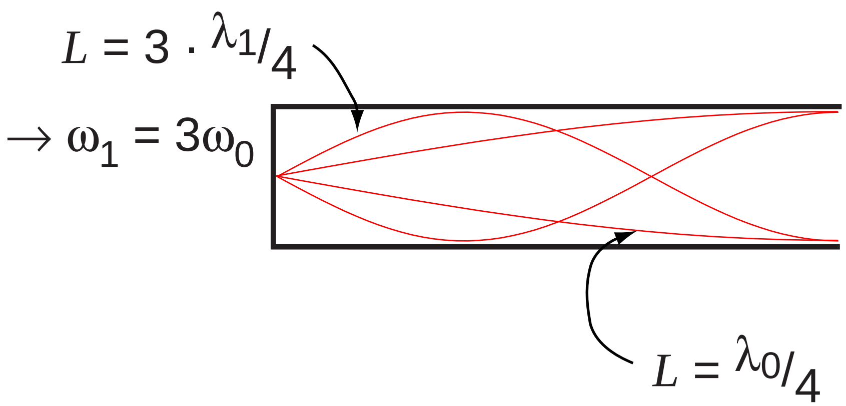
Resonant modes

- **For lossless tube**

  with $L = m \cdot \frac{\lambda}{4}$**,** m **odd,** λ **wavelength,**

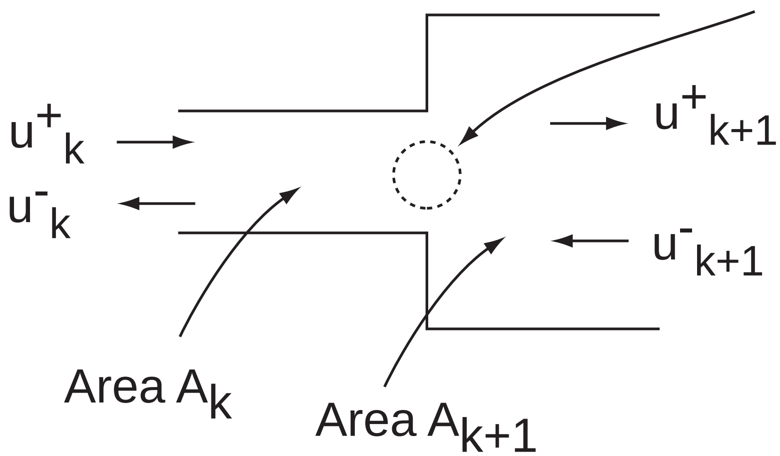
  $\left|\frac{u(L)}{u(0)}\right|$ **is unbounded, meaning:**

  - transfer function has pole on frequency axis
  - energy at that frequency sustains indefinitely



- **e.g 17.5 cm vocal tract,** c **= 350 m/s**
  $\to$ ω_0 **= 2**π **· 500 Hz (then 1500, 2500 ...)**

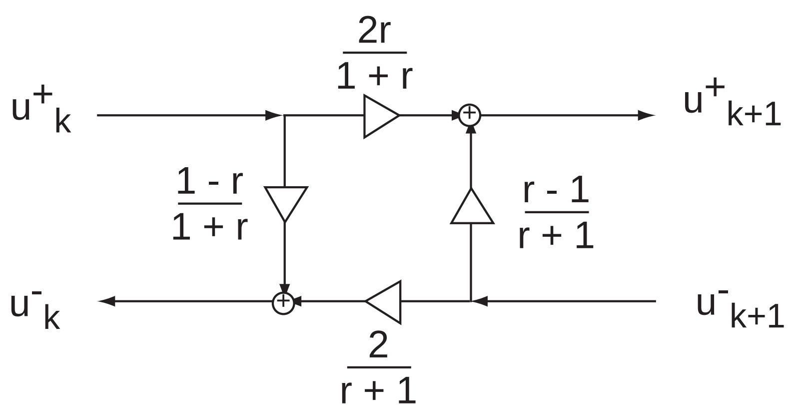
Scattering junctions



At abrupt change in area:

- pressure must be continuous
  $p_k(x, t) = p_{k+1}(x, t)$
- vol. veloc. must be continuous
  $u_k(x, t) = u_{k+1}(x, t)$
- traveling waves
  $u^+_k, u^-_k, u^+_{k+1}, u^-_{k+1}$
  will be different

- **Solve e.g. for u^-_k and u^+_{k+1}: (generalized term.)**

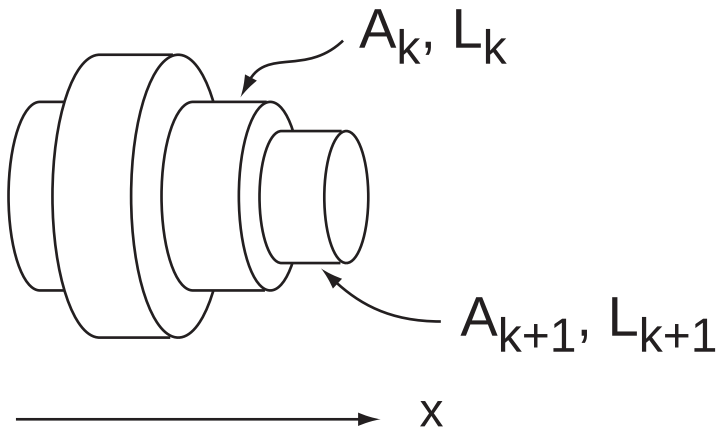


$$r = \frac{A_{k+1}}{A_k}$$

"Area ratio"

Concatenated tube model

- **Discrete approximation to varying-diameter tube:**

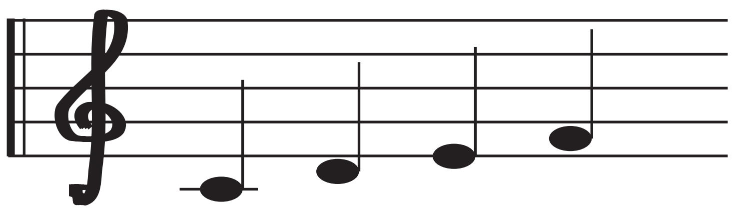


- **Can solve for resonances**
- **Reasonable approx to human vocal tract**
- **Vowel formants from tube resonances**

sound example? ah ee oo

3 Oscillations & musical acoustics

- **Pitch (periodicity) is essence of music:**



  - why? why music?

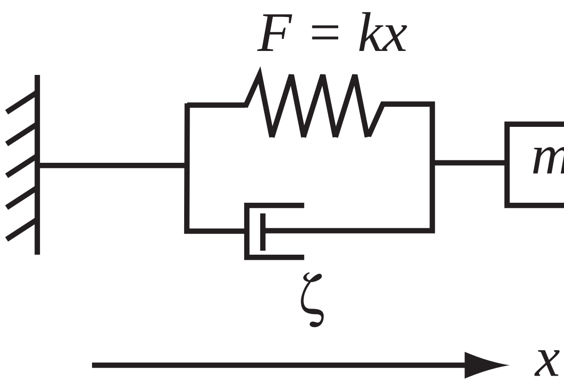
- **Different kinds of oscillators:**
  - simple harmonic motion (tuning fork)
  - relaxation oscillator (voice)
  - string traveling wave (plucked/struck/bowed)
  - air column (nonlinear energy element)

Simple harmonic motion

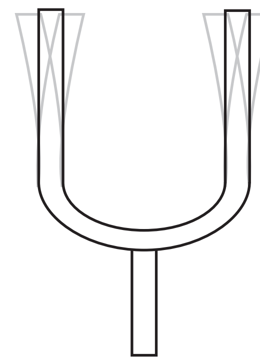
- **Basic mechanical oscillation:**

$$\ddot{x} = -\omega^2 x \qquad x = A\cos(\omega t + \varphi)$$

- **Spring + mass (+ damper)**



$$\omega^2 = \frac{k}{m}$$

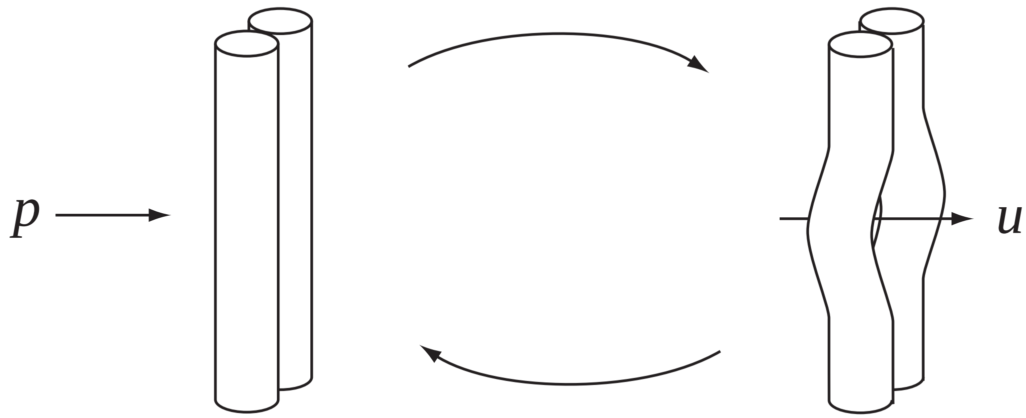


- **e.g. tuning fork**
- **Not great for music:**
  - fundamental ($\cos\omega t$) only
  - relatively low energy

Relaxation oscillator

- **Multi-state process:**
  - one state builds up potential (e.g. pressure)
  - switch to second (release) state
  - revert to first state etc.

- **e.g. vocal folds:**

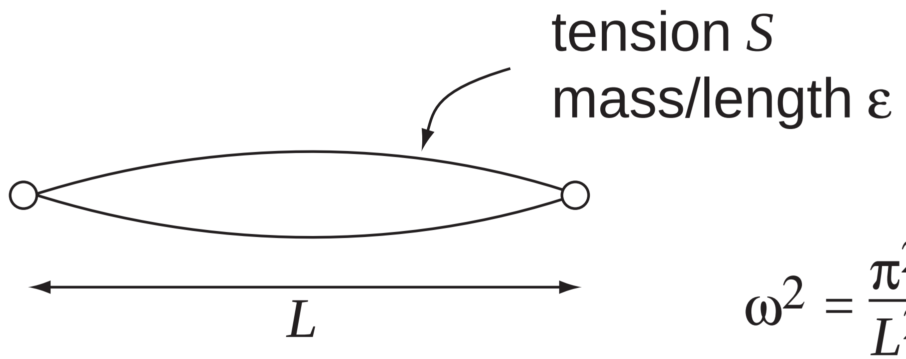


- **Oscillation period depends on force (tension)**
  - easy to change
  - hard to keep stable

  →less used in music

Ringing string

- **e.g. our original 'rope' example**

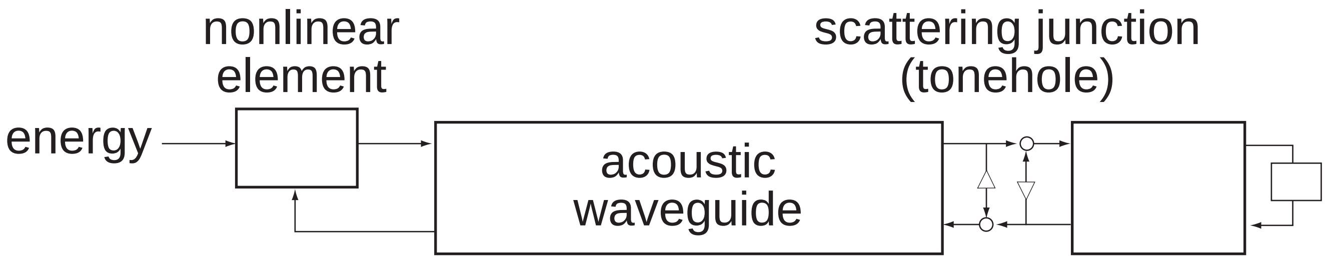


$$\omega^2 = \frac{\pi^2 S}{L^2 \varepsilon}$$

- **Many musical instruments**
  - guitar (plucked)
  - piano (struck)
  - violin (bowed)
- **Control period (pitch):**
  - change length (fretting)
  - change tension (tuning piano)
  - change mass (piano strings)
- **Influence of excitation ... [pluck1a.m]**

Wind tube

- **Resonant tube + energy input**



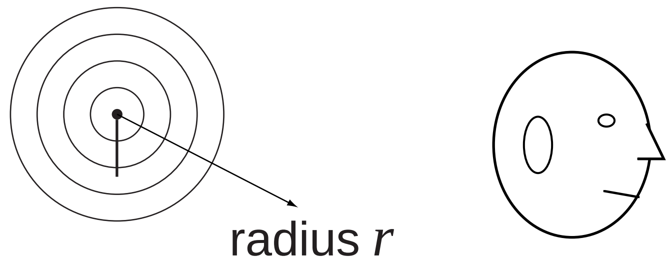
$$\omega = \frac{\pi c}{2L}$$ (quarter wavelength)

- **e.g. clarinet**
  - lip pressure keeps reed closed
  - reflected pressure wave opens reed
  - reinforced pressure wave passes through
- **Finger holes determine first reflection $\rightarrow$ effective waveguide length**

e.g.

4 Room acoustics

- **Sound in free air expands spherically:**



- **Spherical wave equation:**

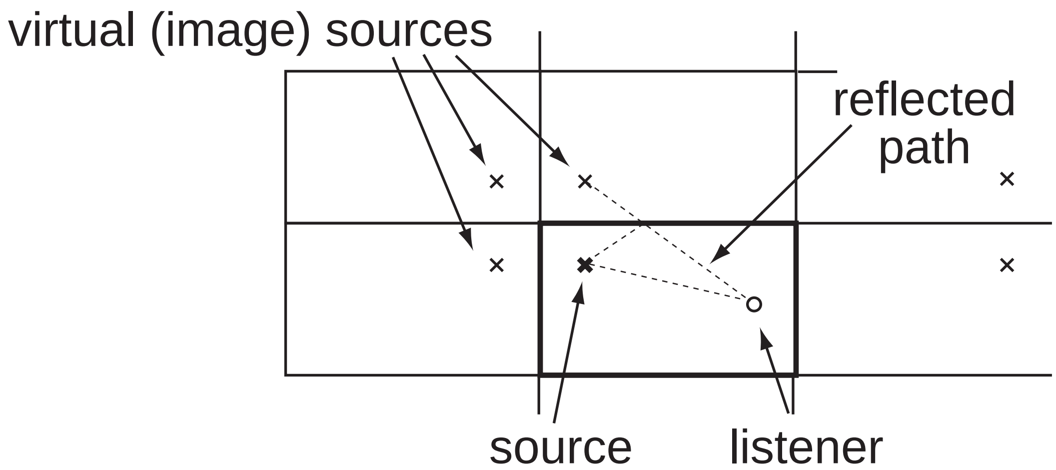
$$\frac{\partial^2 p}{\partial r^2} + \frac{2}{r} \cdot \frac{\partial p}{\partial r} = \frac{1}{c^2} \cdot \frac{\partial^2 p}{\partial t^2}$$

solved by $p(r, t) = \frac{P_0}{r} \cdot e^{j(\omega t - kr)}$

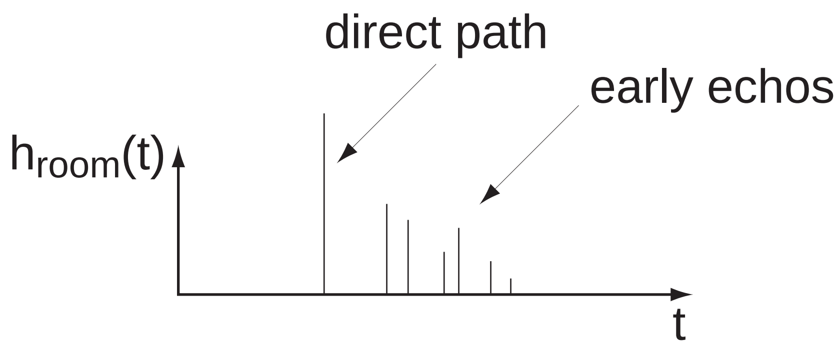
- **Intensity** $\propto p^2$ **falls as** $\frac{1}{r^2}$

Effect of rooms (1): Images

- **Ideal reflections are like multiple sources:**



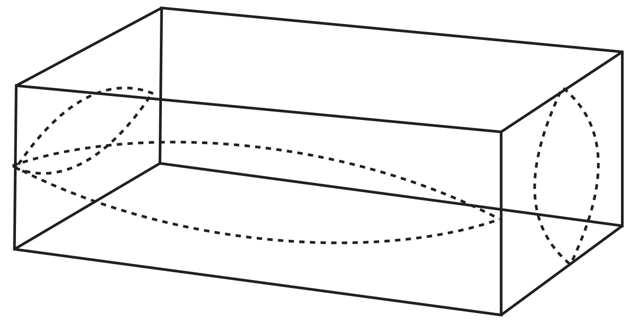
- **'Early echos' in room impulse response:**



- **Actual reflection may be $h_{ref}(t)$, not $\delta(t)$**

Effect of rooms (2): modes

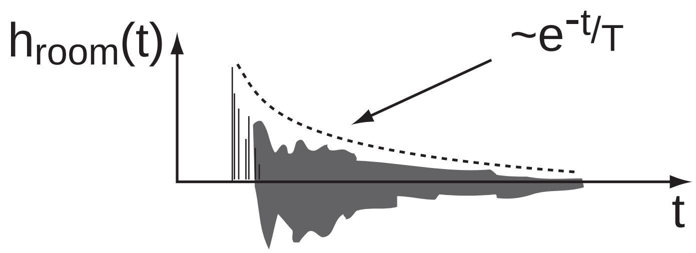
- **Regularly-spaced echos behave like acoustic tubes:**



- **Real rooms have lots of modes!**
  - dense, sustained echos in impulse response
  - complex pattern of peaks in frequency response

Reverberation

- **Exponential decay of reflections:**



- **Frequency-dependent**
  - greater absorption at high frequencies $\rightarrow$ faster decay
- **Size-dependent**
  - larger rooms $\rightarrow$ longer delays $\rightarrow$ slower decay
- **Sabine's equation:**

$$RT_{60} = \frac{0.049V}{S\bar{\alpha}}$$

- **Time constant as size, absorption [e.g.]**

Summary

- **Travelling waves**
- **Acoustic tubes & resonance**
- **Musical acoustics & periodicity**
- **Room acoustics & reverberation**