Lecture-9
Transition and Depletion Capacitance

Transition or Depletion or Space Charge Capacitance: During the reverse bias the minority carriers move away from the junction, thereby having uncovered immobile carriers on either side of the junction. Hence the thickness of the space-charge layer at the junction increases with reverse voltage. This increase in uncovered charge with applied voltage may be considered a capacitive effect. We may define an incremental capacitance C_T by

$$C_T = \left|\frac{dQ}{dV}\right| \tag{1}$$

where dQ is the increase in charge caused by a change dV in voltage. It follows from this definition that a change in voltage dV in a time dt will result in a current $i = dQ/dt$, given by

$$i = C_T \frac{dV}{dt} \tag{2}$$

Therefore a knowledge of C_T is important in considering a diode (or a transistor) as a circuit element. The quantity C_T is referred to as the *transition, space-charge, barrier, or depletion-region, capacitance*. For the step-graded[1] junction we know that

$$W = \sqrt{\frac{2\epsilon V_0}{q}\left(\frac{1}{N_D} + \frac{1}{N_A}\right)} \tag{3}$$

However under bias condition the junction voltage becomes $V_j = V_0 - V_D$ where $V_D = V_F$ for forward bias and $V_D = -V_R$ for reverse bias, thus the Eqn.(3) for the reverse bias becomes,

$$W = \sqrt{\frac{2\epsilon(V_0 + V_R)}{q}\left(\frac{1}{N_D} + \frac{1}{N_A}\right)} \tag{4}$$

This can also be written in terms of $V_j = V_0 + V_R$ as

$$V_j = \frac{q}{2\epsilon}\left(\frac{N_A N_D}{N_A + N_D}\right) W^2 \tag{5}$$

$$\Rightarrow \frac{dV_j}{dW} = \frac{q}{\epsilon}\left(\frac{N_A N_D}{N_A + N_D}\right) W \tag{6}$$

We know that at $Q = AqN_D x_n$, but x_n the depletion region in the n-side is given

[1] Refer to the lecture notes-3

by

$$x_n = \frac{WN_A}{N_A + N_D} \tag{7}$$

thus we have

$$Q = Aq\frac{N_A N_D}{N_A + N_D}W \tag{8}$$

Differentiating both sides with respect to V_j we have,

$$\frac{dQ}{dV_j} = Aq\frac{N_A N_D}{N_A + N_D}\frac{dW}{dV_j} \tag{9}$$

substituting Eqn.(6) in the above equation we get,

$$C_T = \frac{\epsilon A}{W} \tag{10}$$

$$= \frac{\epsilon A}{\sqrt{\frac{2\epsilon}{q}\left(\frac{N_A+N_D}{N_A N_D}\right)V_j}} \tag{11}$$

If $N_A \gg N_D$ then $x_p \approx 0$ and $x_n \approx W$ and the junction voltage V_j is given by,

$$V_j \approx \frac{qN_D W^2}{2\epsilon} \tag{12}$$

the equation for the transition capacitance given by Eqn.(10) still hold good, however now $W = x_n$. In general we can write, C_T as

$$C_T = \frac{\epsilon A}{x_n + x_p} \tag{13}$$

Diffusion Capacitance: For a forward bias a capacitance which is much larger than the transition capacitance C_T comes into play. The origin of the larger capacitance lies in the injected charge stored near the junction outside the transition region. It is convenient to introduce an incremental capacitance, defined as the rate of change of injected charge with voltage, called *diffusion, or storage, capacitance* C_D.

We now make the quantitative study of C_D,

$$C_D \equiv \frac{dQ}{dV} \tag{14}$$

But we have,

$$I = \frac{Q}{\tau_p} \Rightarrow Q = \tau_p \times I \tag{15}$$

In writing the above equation, we have considered only the effect of holes, i.e., $N_A \gg N_D$. Now substituting Eqn.(15) in (14) we get

$$C_D = \tau_p \frac{dI}{dV} = \tau_p g = \frac{\tau_p}{r} \tag{16}$$

where the diode incremental conductance $g \equiv dI/dV$. Substituting the expression for the diode incremental resistance $r = 1/g$ given in the Eqn.(5) of LN-8 we have,

$$C_D = \frac{\tau_p I}{\eta V_T} \tag{17}$$

We see that the *diffusion capacitance is proportional to the current I*. In the derivation above we have assumed that the diode current I is due to holes only. If this assumption is not satisfied, Eqn.(16) gives the diffusion capacitance of C_{D_p} due to holes only, and a similar expression can be obtained for the diffusion capacitance C_{D_n} due to the electrons. Thus the total diffusion capacitance can then be obtained as the sum of C_{D_p} and C_{D_n}, given by

$$\begin{aligned} C_D &= C_{D_p} + C_{D_n} \\ &= \frac{\tau_p I_{p_n}}{\eta V_T} + \frac{\tau_n I_{n_p}}{\eta V_T} \end{aligned} \tag{18}$$

For a reverse bias, g is very small and C_D may be neglected compared with C_T. For a forward current, on the other hand, C_D is usually much larger than C_T. For example for germanium ($\eta = 1$) at $I = 26\ mA$, $g = 1\mho$ and $C_D = \tau$. If say $\tau = 20\ \mu s$, then $C_D = 20\ \mu F$ a value which is about a million times larger than the transition capacitance.