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§~~4~~3.4. Signal Transmission through LTI systems.

① $f(t) \Leftrightarrow F(\omega) \longrightarrow \boxed{h(t) \Leftrightarrow H(\omega)} \longrightarrow y(t) \Leftrightarrow Y(\omega)$

$$y(t) = h(t) * f(t) \Rightarrow h(t) = y(t)\Big|_{f(t)=\delta(t)}$$

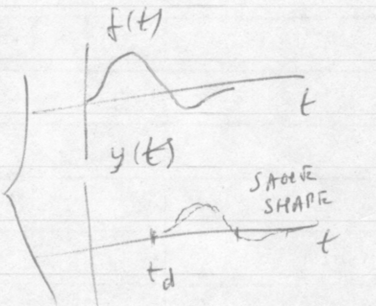
$$\Rightarrow Y(\omega) = \underbrace{H(\omega)}_{\text{freq. resp.}} \cdot F(\omega)$$

$$\therefore \ |Y(\omega)| = |H(\omega)| \cdot |F(\omega)| \quad \text{— amplitude resp. } |H(\omega)|$$

$$\measuredangle Y(\omega) = \measuredangle H(\omega) + \measuredangle F(\omega) \quad \text{— phase resp. } \measuredangle H(\omega)$$

②

<u>Distortionless transmission:</u>



$$y(t) = \underset{\text{const.}}{\underset{\uparrow}{k}} \cdot f(t - \underset{\text{delay}}{\underset{\uparrow}{td}})$$

$$\Longrightarrow Y(\omega) = k \cdot F(\omega)\, e^{-j\omega td}$$

$$\therefore \ H(\omega) = k \cdot e^{-j\omega td}.$$

$$\Longrightarrow \begin{cases} |H(\omega)| = k \\ \measuredangle H(\omega) = -\omega \cdot td \end{cases} \qquad \omega \in B_{f(t)}$$

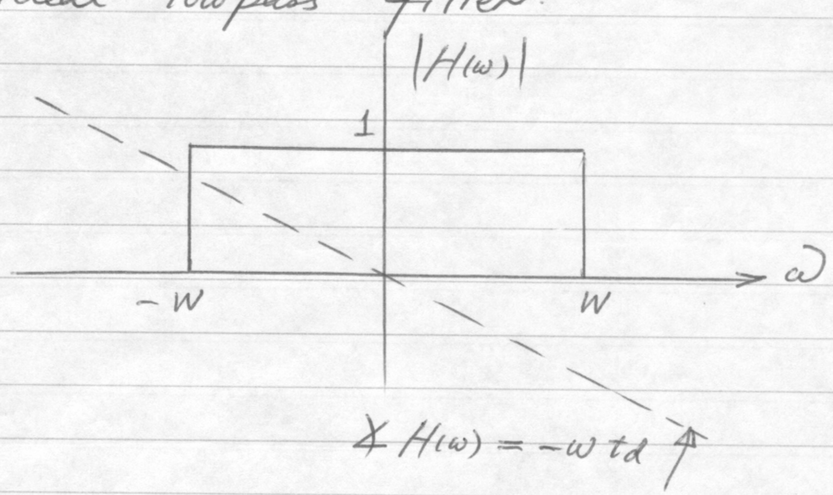
Human ear : sensitive to amplitude distortion
insensitive to phase distortion

Human eye : opposite .

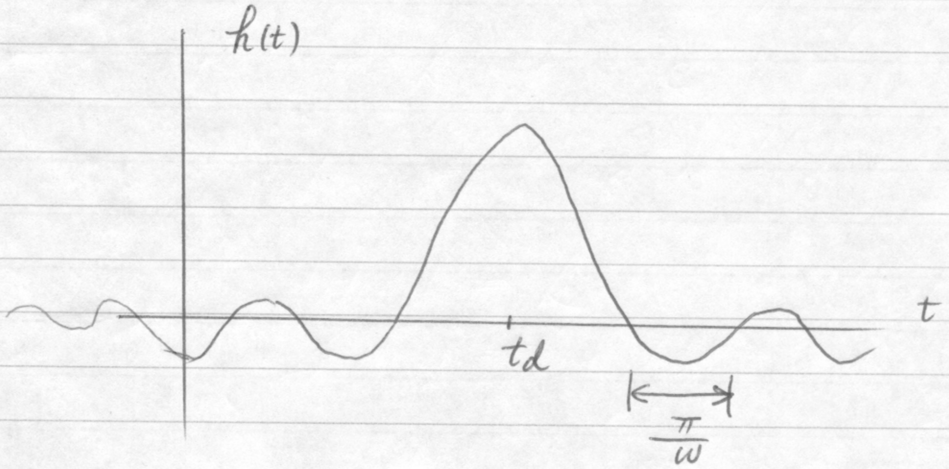
③ Ideal filters

Allow distortionless transmission of a certain band of freq's and suppress all the remaining freq's.

E.g. Ideal lowpass filter:



$$h(t) = \frac{W}{\pi} \operatorname{sinc}\left[W(t - t_d)\right]$$



Note: $h(t)$ is noncausal

$\longrightarrow$ physically unrealizable physically realizable system: $h(t)$ must be causal

$$h(t) = 0 \quad \text{for } t < 0$$

frequency-domain condition: Paley-Wiener criterion:

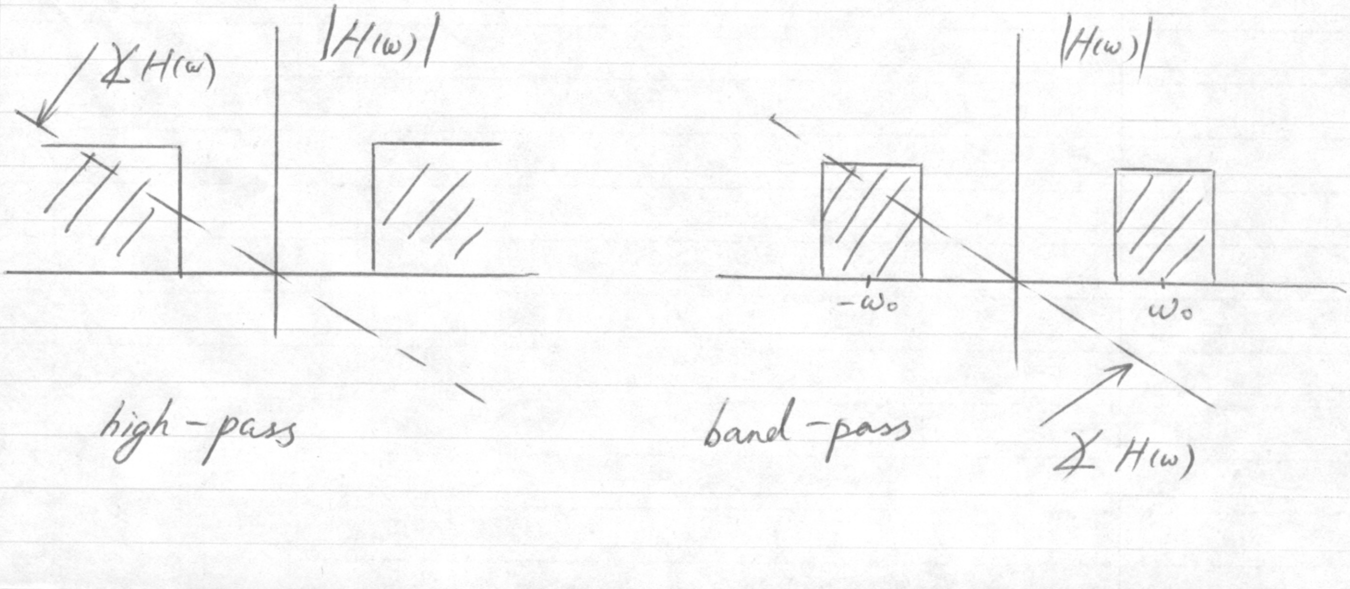
The necessary & sufficient condition for $|H(\omega)|$ to be realizable is

$$\int_{-\infty}^{+\infty} \frac{|\ln |H(\omega)||}{1+\omega^2} \, d\omega < \infty$$

Note: If $|H(\omega)| = 0$ over a finite band, then $|\ln|H(\omega)|| = \infty$ over that band $\longrightarrow$ the P-W condition is violated.

$\therefore$ for a physically realizable system, $H(\omega)$ may be zero at some discrete freq's but it can not be zero over any finite band.

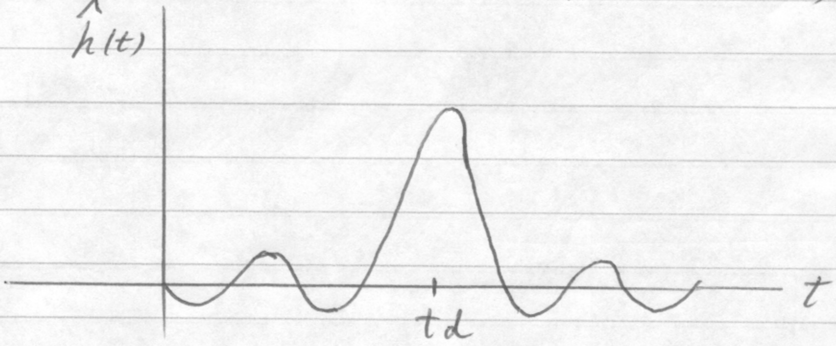
$\rightarrow$ ideal filters are physically unrealizable.



④ Practical filters

truncate ideal filter:

$$\hat{h}(t) = h(t)\, u(t)$$



The lager is the delay td
the closer is the approximation

LTI system response to periodic signals:

$f(t)$ is periodic w/ period T_0.

$$f(t) = \sum_{n=-\infty}^{+\infty} D_n e^{jn\omega_0 t}, \quad \omega_0 = \frac{2\pi}{T_0}$$

$$\Rightarrow F(\omega) = 2\pi \sum_{n=-\infty}^{+\infty} D_n \delta(\omega - n\omega_0)$$

$$Y(\omega) = F(\omega) \cdot H(\omega)$$

$$= 2\pi \sum_{n=-\infty}^{+\infty} \underbrace{D_n H(n\omega_0)}_{\tilde{D}_n} \cdot \delta(\omega - n\omega_0)$$

$$\Rightarrow y(n) = \sum_{n=-\infty}^{+\infty} \tilde{D}_n e^{jn\omega_0 t}$$

$\therefore$ output is periodic w/ same period T_0