Modelling the Performance of TCP/ARQ over MIMO Rayleigh Fading Channels

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***Abstract*— In this paper we describe a general framework for the modelling of the TCP performance over MIMO wireless systems including parameters such as fading, space-time transmission schemes, multiple antenna size, modulation schemes, channel coding and ARQ. We apply the framework to analyze the performance, the optimal channel coding rate and the effect of Doppler on the TCP throughput over the BLAST MIMO system and the orthogonal space-time block coded (STBC) system. We use the network simulator ns-2 to demonstrate the accuracy of the proposed analytical framework in characterizing various parameters of the TCP performance. We apply our framework to study of the buffer occupancy for TCP over MIMO systems and to a system that does not follow the AIMD TCP principle: CBR video transmission over MIMO channels.** **Keywords: Cross-layer analysis, MIMO, Rayleigh fading, TCP/ARQ.**

I. Introduction

In order to seamlessly support a variety of existing and emerging services over wireless networks, it is necessary for the higher layers to exchange information with the physical and the MAC layers, in order to exploit the network resources and to provide optimal inter-operation of applications. Moreover, there is a need for a model that enables the analytical characterization of the intricate tradeoffs (between throughput, time delay and packet loss probability, among others) that drive the network performance. Existing research in the cross-layer area [9], is either network centered that propose changes in TCP or in the network layer [8], or physical-layer centered, in which power adaptation and adaptive modulation are taken into account for increasing the physical layer throughput [6].

The main contribution of this paper is the introduction of a simple framework for the analysis of MIMO wireless channels combined with upper layer protocols such as TCP. Our framework takes into account various system parameters at different layers, such as MIMO fading channel characteristic, space-time transmission schemes, modulation and channel coding, ARQ schemes, etc., and it characterizes the performance at the TCP and UDP layer, including throughput and delay. We apply it to the analysis of MIMO wireless systems employing ARQ for both the BLAST and the orthogonal STBC systems, and study various performance metrics such as the optimal information rate that maximizes the TCP throughput, the effect of Doppler on the optimal TCP throughput, and the optimal channel coding rate that maximizes the TCP throughput under various modulation schemes. We show that the results obtained are very accurate when compared to simulations performed with the ns-2 network simulator, demonstrating the viability of the proposed modelling framework. As applications, we study the behavior of a CBR video transmission over BLAST and STBC systems, by calculating the maximum throughput of the video system and the optimal coding rate under various modulation schemes.

II. System Descriptions

Consider a MIMO system consisting of n_T transmit and n_R receive antennas, signalling through slow Rayleigh fading channels. The input-output relationship of this system is given by

$$\boldsymbol{y}(t) = \sqrt{\frac{\gamma}{n_T}}\boldsymbol{H}(t)\boldsymbol{s}(t) + \boldsymbol{n}(t), \tag{1}$$

where $s_i(t) \in \mathcal{A}$ is the transmitted symbol from antenna i, with $\mathcal{A}$ being a constellation set with unit energy, i.e., $E\left\{|s_i(t)|^2\right\} = 1$, $\boldsymbol{n}(t)$ is the received ambient noise vector, $n_i(t) \overset{i.i.d}{\sim} \mathcal{N}_c(0,1)$; $\gamma = \frac{E_s}{N_o}$ is the signal-to-noise ratio, and $h_{ij}(t)$ denotes the channel gain between the j-th transmit and the i-th receive antennas at time t, and it is a zero-mean complex Gaussian process with the Jakes' correlation model [4].

For the upper layers, a typical TCP/IP/LL/RLP stack is used on the wireless link between the radio network controller (RNC) and the mobile host (MH). The RLP layer implements a type of truncated hybrid retransmission-repeat request (ARQ) type-I, that performs retransmissions, fragmentation and reassembly. The TCP segment is correctly received when all the link layer frames are successfully received. In the case of a successful transmission a positive acknowledgement (ACK) is sent back to the transmitter over an error-free channel and the transmission of a new frame begins *immediately*. If, on the other hand, the frame is incorrectly received, a negative acknowledgement (NACK) is sent back to the transmitter, which will retransmit the frame in the next time block. If the maximum number of retransmissions is reached, the transmitter discards the frame silently (the upper layer, i.e., TCP, will eventually take care of the error). We assume the transmitter will always have a frame to transmit.

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III. Model for MIMO Wireless Links

We consider two MIMO transmission schemes, namely the BLAST method [2], and the orthogonal space-time block coding (STBC) method [11]. We assume that an ideal channel coding is employed so that we can characterize the physical layer link loss behavior using the maximum information rate for BLAST and STBC systems. The basis of our analytical framework is as follows. By the Shannon's channel coding theorem, a channel code rate R is achievable if it is such that $R < C$, where C is the capacity of the channel. Considering the instantaneous mutual information of the channel at time t as a measure of its 'instantaneous capacity', we can say that if $R < C$ the data at time t can be transmitted with an arbitrarily low probability of error. Then we will consider the channel to be in a 'good' state, in which errorless transmission is feasible. Conversely, if the rate of transmission at time t is greater than the 'instantaneous capacity', then we consider the channel to be in a 'bad' state, in which errorless transmission is not possible. For a MIMO system, a good channel state G corresponds to

$$\mathsf{G} \equiv \{\mathcal{I}(\boldsymbol{y}(t); \boldsymbol{x}(t) \mid \boldsymbol{H}(t)) > R\}, \qquad (2)$$

where $\mathcal{I}(\boldsymbol{y}(t); \boldsymbol{x}(t) \mid \boldsymbol{H}(t))$ is the instantaneous mutual information of the channel at time t given the realization of the channel $\boldsymbol{H}(t)$, and R is the information rate in bps/Hz. If (2) is not satisfied we consider the system to be in a bad state B.

As we are interested in Rayleigh fading channels, we generate the instances of $\boldsymbol{H}(t)$ according to the Jakes' model [4]. As a result of the time-correlated Doppler fading, once the MIMO system has moved to a certain state (G or B), it may stay over a period of time. We use Monte-Carlo to simulate the state of the system for a long enough period of time, and estimate p and q by counting the events of the system going from state G to state B and vice versa. For a BLAST system [2] with n_T transmit and n_R receive antennas, the instantaneous mutual information between the output $\boldsymbol{y}(t)$ and input $\boldsymbol{x}(t)$ for a system like (1) with MPSK or MQAM constellations and uniformly distributed symbol probabilities is computed by $\mathcal{I}(\boldsymbol{y}; \boldsymbol{x}) = \mathcal{H}(\boldsymbol{y}) - \mathcal{H}(\boldsymbol{y} \mid \boldsymbol{x})$ [3], where $\mathcal{I}$ and $\mathcal{H}$ denote the instantaneous mutual information and entropy function respectively. It is known that $\mathcal{H}(\boldsymbol{y} \mid \boldsymbol{x}) = n_R \log(\pi e)$ for a Gaussian channel. The entropy of the received signal $\boldsymbol{H}(y)$, however, has to be computed by Monte Carlo as follows [3]

$$\mathcal{I}(\boldsymbol{y}; \boldsymbol{x} \mid \boldsymbol{H}) = \mathcal{H}(\boldsymbol{y} \mid \boldsymbol{H}) - \mathcal{H}(\boldsymbol{y} \mid \boldsymbol{x}, \boldsymbol{H})$$
$$= \underbrace{-\mathcal{E}_n \log_2 \left[\frac{1}{2^{n_T M_c} \pi^{n_R}} \sum_{\mathcal{A}^{n_T}} \exp\left(-\|\boldsymbol{y} - \sqrt{\frac{\rho R}{n_T}} \boldsymbol{H}\boldsymbol{x}\|^2 \right) \right]}_{\mathcal{H}(\boldsymbol{y}|\boldsymbol{H})} - \underbrace{n_R \log_2(\pi e)}_{\mathcal{H}(\boldsymbol{y}|\boldsymbol{x},\boldsymbol{H})}, \qquad (3)$$

with $R \equiv n_T M_c r$, where $r \leq 1$ is the coding rate, 2^{M_c} is the finite-constellation size (e.g. $M_c = 4$ for 16-QAM), $\boldsymbol{y}$ is defined in (1); the expectation $\mathcal{E}$ is taken over the random noise $\boldsymbol{n}$ also defined in (1); and the summation $\sum_{\mathcal{A}}$ is over all possible values of $\boldsymbol{x} \in \mathcal{A}$, in the total amount of $2^{M_c n_T}$ possibilities. If $M_c n_T$ is too large the computation in (3) can be approximated using Monte Carlo as follows

$$\mathcal{I}(\boldsymbol{y}; \boldsymbol{x} \mid \boldsymbol{H}) = \underbrace{-\mathcal{E}_n \log_2 \left[\frac{1}{\pi^{n_R}} \mathcal{E}_x \exp\left(-\|\boldsymbol{y} - \sqrt{\frac{\rho R}{n_T}} \boldsymbol{H}\boldsymbol{x}\|^2 \right) \right]}_{\mathcal{H}(\boldsymbol{y}|\boldsymbol{H})} - \underbrace{n_R \log_2(\pi e)}_{\mathcal{H}(\boldsymbol{y}|\boldsymbol{x},\boldsymbol{H})}, \qquad (4)$$

with $R \equiv n_T M_c r$, where the expectation $\mathcal{E}_x$ is taken over the symbols $\boldsymbol{x} \in \mathcal{A}$. In computing (4), we carry out a large enough number of Monte Carlo runs with two loops: the outer loop generates complex Gaussian noise $\boldsymbol{n}$ according to (1), and the inner loop uniformly generates finite-constellation symbol $\boldsymbol{x}$. The average of all Monte Carlo runs result in an estimate of $\mathcal{I}(\boldsymbol{y}; \boldsymbol{x}|\boldsymbol{H})$.

On the other hand, the mutual information of a system employing orthogonal space-time block codes (STBC) [11] of rate r_c symbols per transmission with MPSK or MQAM constellations and uniformly distributed symbol probabilities is computed by:

$$\mathcal{I}(\boldsymbol{y}; \boldsymbol{x} \mid \boldsymbol{H}) = -r_c \mathcal{E}_n \log_2 \left[\frac{1}{2^{M_c} \pi} \sum_{\mathcal{A}} \exp\left(-\left| \hat{y}_1 - \sqrt{\frac{\rho R}{n_T} \sum_{j=1}^{n_T} \sum_{i=1}^{n_R} |h_{i,j}|^2} \cdot x_1 \right|^2 \right) \right] - r_c \log_2(\pi e) \qquad (5)$$

with $R \equiv n_T M_c r_c r$, where y_1 is the first element of vector $\boldsymbol{y}$ and x_1 is the first element of vector $\boldsymbol{x}$. Conversely, if $M_c n_T$ is too large, (5) can be computed as

$$\mathcal{I}(\boldsymbol{y}; \boldsymbol{x} \mid \boldsymbol{H}) = -r_c \mathcal{E}_n \log_2 \left[\frac{1}{\pi} \mathcal{E}_{x_1} \exp\left(-\left| \hat{y}_1 - \sqrt{\frac{\rho R}{n_T} \sum_{j=1}^{n_T} \sum_{i=1}^{n_R} |h_{i,j}|^2} \cdot x_1 \right|^2 \right) \right] - r_c \log_2(\pi e). \qquad (6)$$

with $R \equiv m M_c r_c r$.

IV. TCP/ARQ Performance Model

We use a two-state Markov model (i.e. Gilbert model) to model the physical layer transmission success/failure. Specifically, let $S(t)$ be the state of the physical layer link corresponding to the t-th frame transmission. Then $S(t) \in \{\mathsf{G}, \mathsf{B}\}$, where $S(t) = \mathsf{G}$ if the t-th transmission is successful and $S(t) = \mathsf{B}$ if the i-th transmission is erroneous. That is, the physical layer link is modelled by a two-state Markov chain with transition matrix $\boldsymbol{Q} = \begin{pmatrix} 1-p & p \\ q & 1-q \end{pmatrix}$, where $p \triangleq P[S(t) = \mathsf{B}|S(t-1) = \mathsf{G}]$ and $q \triangleq P[S(i) = \mathsf{G}|S(i-1) = \mathsf{B}]$. The values p and q, calculated in the previous

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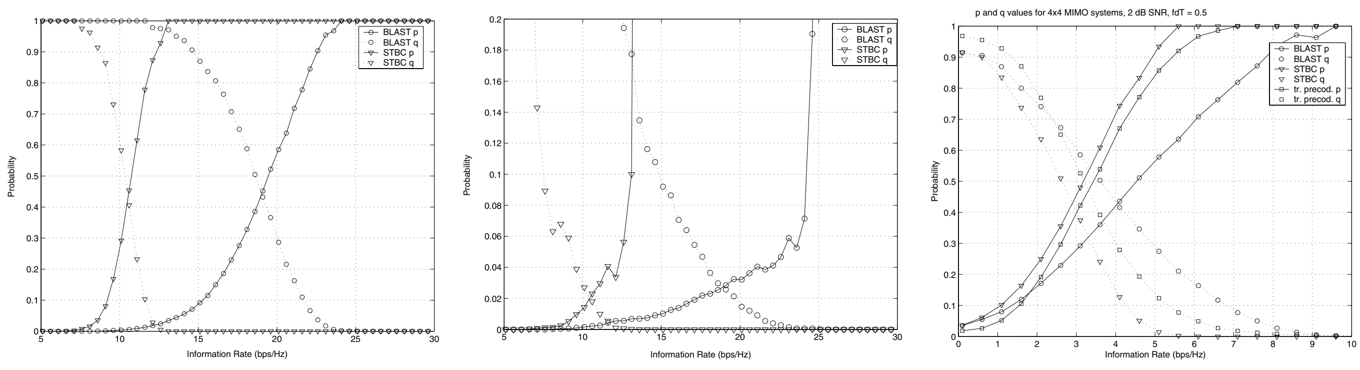

(a) 2×2 $f_dT = 0.5$. $Es/No = 20$ dB. (b) 2×2 $f_dT = 0.01$. $Es/No = 20$ dB. (c) 4×4 $f_dT = 0.5$. $Es/No = 2$ dB.

Fig. 1. Gilbert model parameters for BLAST and STBC for different Doppler values.

section, summarize the physical layer characteristics such as space-time transmission schemes, MIMO fading channels, modulation and channel coding, etc.

Probability of Packet Loss under ARQ: To calculate the probability of loss of a packet, and the number of frames sent per packet we follow the method in [12] (other methods have been proposed, e.g., [1]). Given the maximum number of retransmissions N on the link layer and the number of frames per packet L, define $\bar{p}_i$ as the probability of the i-th link layer frame of the current packet being lost in this frame slot given the previous slot state was 'good'. Also define $\bar{q}_i$ as the probability of the i-th link layer frame corresponding to the current packet being lost in this frame slot, including retransmissions, given that the previous slot state was 'bad'.Then we have $\bar{p}_1$ and $\bar{q}_1$ given by

$$\bar{p}_1 = p(1-q)^{N-1}, \quad \bar{q}_1 = (1-q)^N. \tag{7}$$

By induction it is easy to calculate those probabilities for the L frames that constitute a TCP packet as [12]

$$\bar{p}_i = \bar{p}_1 + (1-\bar{p}_1)\bar{p}_{i-1}, \quad \bar{q}_i = \bar{q}_1 + (1-\bar{q}_1)\bar{p}_{i-1}, \qquad i = 2, ..., L. \tag{8}$$

Denote π_G and π_B as the steady-state probabilities of $\boldsymbol{Q}$. Then the probability of a packet loss in the current time slot is

$$P_e = \pi_G \bar{p}_L + \pi_B \bar{q}_L. \tag{9}$$

Average Number of Frames per Packet: We now calculate the average number of frames transmitted per packet, taking into account retransmissions [12]. Let n_i be the average number of actual frames sent for a packet consisting of i link layer frames given that the first link frame found the channel in a 'good' state; and let m_i^k be the average number of actual frames sent for a packet consisting of i link layer frames, given that the first frame has already undergone k ARQ retransmissions and the current channel state is (still) 'bad'. Then, the average number of frames sent per packet is given by

$$F = \pi_G n_L + \pi_B m_L^0. \tag{10}$$

In (10), n_L and m_L^0 can be calculated recursively as follows: the initialization step

$$n_1 = 1, \tag{11}$$

$$m_1^{N-1} = 1, \tag{12}$$

$$m_1^k = 1 + q\ n_1 + (1-q)\ m_1^{k+1}, \quad k = N-2, ..., 0, \tag{13}$$

and the recursion steps for $i = 2, ..., L$

$$n_i = 1 + (1-p)n_{i-1} + p\ m_{i-1}^0, \tag{14}$$

$$m_i^{N-1} = 1 + q\ n_{i-1} + (1-q)\ m_{i-1}^0, \tag{15}$$

$$m_i^k = 1 + q\ n_i + (1-q)\ m_i^{k+1}, \quad k = N-2, ..., 0. \tag{16}$$

TCP Throughput: Using the above results, the delay of a TCP packet will be directly proportional to the number of link layer transmissions needed. In the absence of buffer delay the round trip time for a TCP segment is [12]

$$RTT = 2 \cdot T_f + F \cdot T_w, \tag{17}$$

where T_f is the delay of the whole TCP segment through the internet, T_w is the delay of a link layer frame through the wireless interface, and F is the average number of link layer frames sent per TCP segment calculated in (10). We use the TCP Reno approximation in [7] to estimate the TCP throughput as

$$B_{TCP} \approx f(W_{\max}, RTT, b, P_e, T_o), \tag{18}$$

where $W_{\max}$ is the maximum congestion window size, b is the number of packets acknowledged by a received TCP ACK (usually 2), T_o is the initial time-out for the TCP sender, and RTT and P_e are the round-trip time and the TCP loss probability calculated in (17) and (9) respectively.

V. Numerical Results

Fig. 1(a) shows the values of p and q as a function of the information rate R, for 2×2 BLAST and STBC (rate 3/4) and a normalized Doppler fading $f_dT = 0.5$ and $E_s/N_o = 20$ dB. It is clear that the BLAST system has a larger capacity, as it is able to transmit at a higher information rate without errors. Also we can see that the STBC system is more reliable, because the parameter p draws a steeper curve than the BLAST case. Interestingly we can see the effect of mobility

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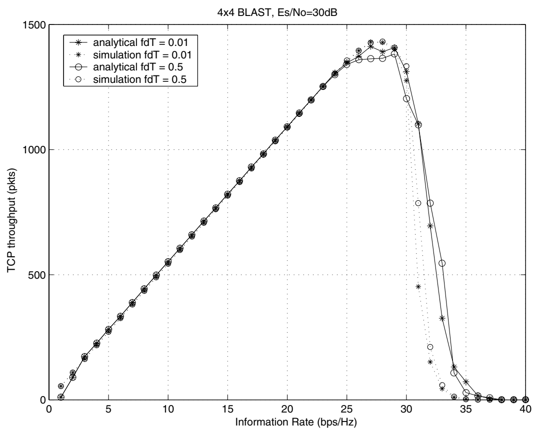

Fig. 2. TCP throughput for analytical and simulation results.

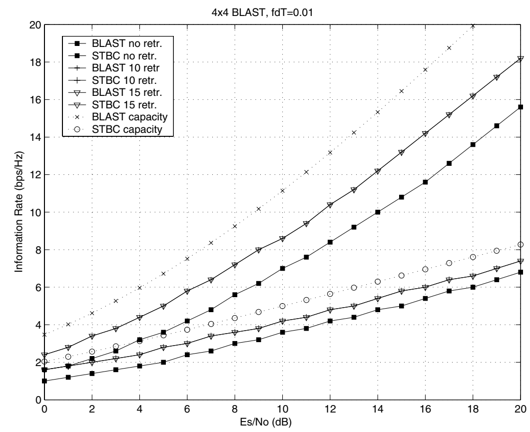

Fig. 3. Information rate that maximizes TCP throughput vs. ARQ persistence.

in Fig. 1(b). Note that the points where the p and q values cross for $f_dT = 0.01$ is much lower in probability (around 2%, compared to the 50% in the $f_dT = 0.5$ case), i.e, for the same SNR the fadings are less frequent but larger for $f_dT = 0.01$. For TCP with ARQ, this implies that the effect of the mobility is positive: at $f_dT = 0.01$, although the capacity of the channel does not change compared to $f_dT = 0.5$, and the probability of going to the 'bad' state p is lower, the probability of getting back to the 'good' state q is only 1%. This implies that when the probability of error blocks starts to increase, the average *length* of the burst errors is much larger for $f_dT = 0.01$ than for $f_dT = 0.5$. This, as we will see, makes the fading correlation (i.e. high mobility) very attractive for FEC and ARQ schemes that will require less retransmissions to approach the capacity curve. Fig. 1(c) shows the p and q values for the three systems when $f_dT = 0.5$ and $SNR = 2dB$. As expected the rates at which the transmission is possible without errors is much lower.

Comparison with ns-2 simulations: Fig. 2 compares the results obtained for the TCP throughput calculated in (18) and the simulations for a 4×4 BLAST system with $\frac{E_s}{N_o} = 30$ dB. For the simulations we modified RLC module in the GPRS implementation by Richa Jain at IITB (India) to implement a link layer retransmission mechanism for the ns-2 simulator [10]. For the physical layer we generated a sequence of states according to the model in (2). We consider T to be large enough to accommodate a link layer frame, and the transmissions to be synchronized at the beginning of the frame time. For every frame to be transmitted the link controller checks the state of the channel and discards the frame if the state is 'bad', and transmit the frame if the state is 'good'. Fig. 2 shows that our framework is indeed very accurate, as the simulations match almost perfectly the analytical results.

Optimal information rate that maximizes the TCP throughput: Fig. 3 shows the information rate that maximizes TCP throughput in a 4×4 BLAST and STBC (rate $3/4$) for $f_dT = 0.01$. The dashed lines denote the ergodic capacities of BLAST and STBC, and the other lines show the rate at which TCP operates at maximum throughput for different maximum number of retransmissions. It is seen that the optimal rate for TCP is far from capacity irrespective of the number of ARQ retransmission. From a cross-layer design point of view it is an indication that increasing spectral efficiency does not always result in an increment of the TCP throughput. For TCP, it may be preferable a more reliable system because TCP, even when ARQ and combining is used, cannot make use of the additional bit rate. Also, increasing the maximum number of retransmissions over 10 does not provide any benefit to the overall TCP throughput, however it may increase complexity and buffer occupancy, so it may be interesting to truncate the ARQ persistence.

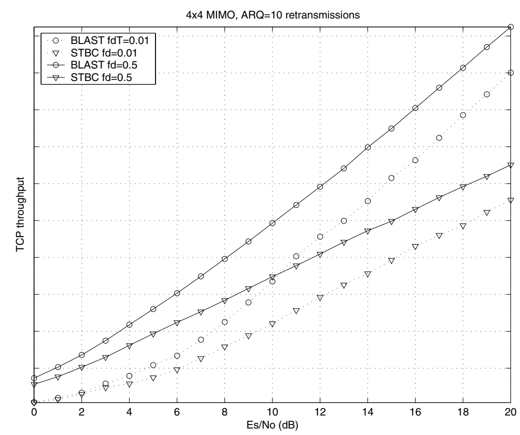

Fig. 4. Maximum TCP throughput for BLAST and STBC vs. Doppler.

Effect of Doppler on optimal TCP throughput: Fig. 4 shows the maximum TCP throughput for BLAST and STBC for both $f_dT = 0.5$ and $f_dT = 0.01$ and 10 retransmissions. As expected, the values for the TCP throughput are substantially better in the case of high mobility ($f_dT = 0.5$), and only when the SNR reaches 6 dB the TCP throughput for $f_dT = 0.01$ begins to grow significatively. Our framework effectively captures the effect that mobility has on TCP: while the capacity of a MIMO system does not change with the Doppler, the effects of the time correlation and the deep fadings on TCP are significant.

Optimal channel coding rate with finite constellations: Fig. 5 shows the optimal coding rates that maximizes the TCP

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throughput for QPSK and 16QAM with a maximum ARQ persistence of 10 retransmissions. Note that QPSK needs less coding than the 16QAM, especially for low SNR. Also it is interesting to note that, for optimal operation, the transmission under $f_d T = 0.5$ needs less coding protection.

VI. Applications of the Framework

Analysis of TCP Buffer Occupancy: Assume each TCP packet is formed by L link layer frames that arrive in bursts with a Bernoulli distribution. Also assume there is a queue at the base station for link layer frames that can accommodate up to K TCP packets, i.e., LK link layer frames. At the beginning of time slot i, let Q_i denote the number of frames in the buffer and S_i the state of the channel with $S_i = 1$ if the channel is 'good' and $S_i = 0$ if the channel is 'bad' . Then $\{(Q_i, S_i), i \geq 0\}$ is a two-dimension discrete-time Markov process. This model has been extensively used in the literature [5], [6]. A TCP packet is lost due to a packet overflow if any of the constituting link layer frames cannot be accommodated in the buffer (i.e., buffer is full). Let P_{enter} be the probability of a TCP packet entering the queue, then the probability of a TCP packet arrival is [5]

$$P_a = B \cdot T \cdot P_{enter}, \tag{19}$$

where B is the TCP throughput, and T is the time slot corresponding to the duration time of 'good' and 'bad' states of the Gilbert model. If the channel is bad there can be no departures and if the channel is good, there will be one departure with probability one if there is at least one frame in the buffer.

Let $\boldsymbol{\pi}$be the stationary distribution of the queue. Then the average probability of a TCP packet drop in the queue is given by $P_{drop} = \sum_{Q_i=LK-L+1}^{LK} \left(\pi_{0,Q_i} + \pi_{1,Q_i}\right)$ i.e., the probability of finding the buffer with more than L slots free, in which case one of the frames would be dropped and the TCP packet will be lost. The average buffer occupancy is given by $E\{Q\} = \sum_{Q_i=0}^{LK} Q_i \cdot \left(\pi_{0,Q_i} + \pi_{1,Q_i}\right)$ and the average length of the queue for a TCP packet is $E\{Q_{TCP}\} = \sum_{Q_i=0}^{LK} \left\lceil \frac{Q_i}{L} \right\rceil \left(\pi_{0,Q_i} + \pi_{1,Q_i}\right)$ [5]. Applying Little's law we

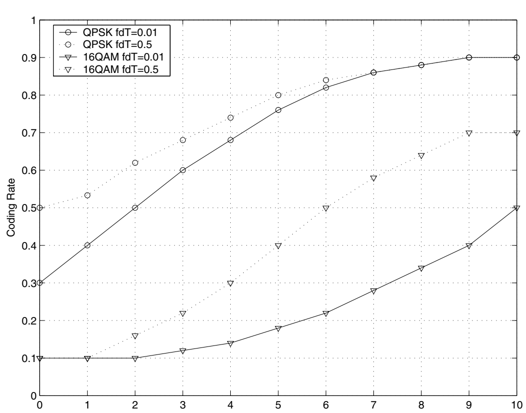

Fig. 5. Optimal coding rate for TCP over 2×2 BLAST.

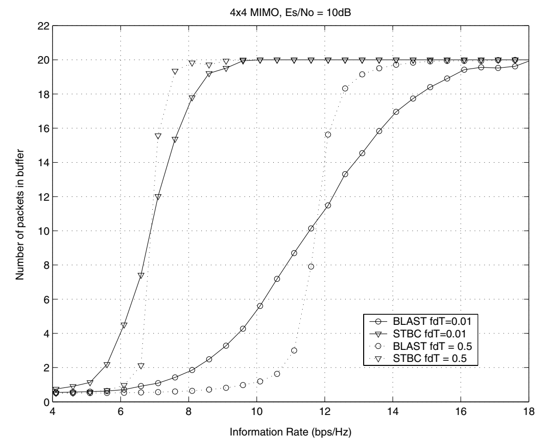

Fig. 6. Comparison of buffer occupancy for 4×4 BLAST and STBC for different Doppler values. Buffer is 20 packets and $E_s/N_o = 10$ dB.

can obtain the average waiting time in the buffer

$$T_{buffer} = \frac{E\{Q_{TCP}\}}{B(1 - P_{drop})}. \tag{20}$$

The expressions for the TCP packet loss probability and RTT are as follows

$$\bar{P}_e = P_e + P_{drop} - P_e \cdot P_{drop} \tag{21}$$

$$RTT_q = 2 \cdot T_f + F \cdot T_w + T_{buffer}, \tag{22}$$

where P_e is the probability of TCP loss including the maximum number of retransmissions (9), P_{drop} is the probability of a TCP packet finding the buffer full, F is given by (10) and T_{buffer} is the delay at the buffer. The final expression for the TCP throughput is obtained by substituting the values of (21) and (22) in (18).

Figs. 6 show the behavior of a 20 packets buffer for 4×4 BLAST and STBC (rate 3/4) systems, at $E_s/N_o = 10$ dB and normalized Doppler fading of $f_d T = 0.01$ and $f_d T = 0.5$ respectively. The maximum number of ARQ retransmissions in all cases is 10. The highly correlated fading ($f_d T = 0.01$) have a strong impact on TCP, as it requires more retransmissions, and the queues start to fill quickly. At $f_d T = 0.5$ the queues start to fill at a higher information rate and more abruptly, usually when the optimal capacity of the channel for TCP is reached. In general, BLAST has a higher capacity as the buffer saturates at a higher information rate. Also we can see that STBC is more tolerant to the Doppler, as even for $f_d T = 0.01$ the buffer starts to fill in closer to capacity relative to BLAST.

Analysis of CBR Video Transmission: We consider CBR video source with a fixed maximum delay D and frame error rate P_v, where the video frames are split into L link layer frames. We also consider a buffered ARQ system on the wireless side similar to the one presented in Section IV. Then, the maximum number of retransmissions allowed by the system N_{video} is calculated as

$$T = T_f + F_{video} \cdot (T_w + \hat{T}_{buffer}) \leq D, \tag{23}$$

where F_{video} is the average number of transmissions per frame under the delay constraint, T_f is the TCP packet delay in the fixed link, T_w is the delay of the frame in the wireless link and

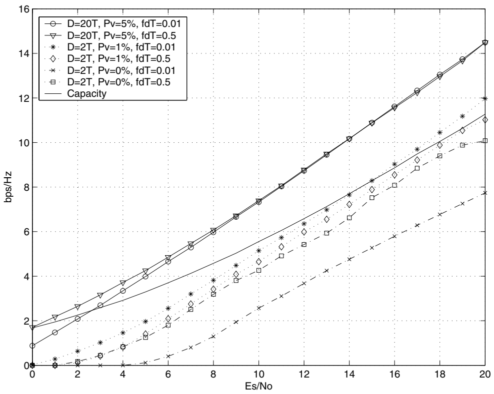

Fig. 7. Rate that maximizes the video throughput for 2×2 BLAST under different delay and error constraints.

T is the packet end-to-end delay. $\hat{T}_{buffer}$ is the buffer delay for each frame, depending on the video rate B_v, and can be calculated by substituting the values of B_v in (19) and (20). We calculate the average number of frames per video packet with the delay constraint considering that

$$\pi_G \hat{n}_L + \pi_B \hat{m}_L^0 = F_{video} \leq \frac{D - T_f}{T_w + \hat{T}_{buffer}}, \tag{24}$$

where $\hat{n}_L$ and $\hat{m}_L^0$ can be numerically calculated following the scheme in section IV. Solving (24) gives the maximum number of retransmissions per video frame N_{video}. The frame error rate for the video transmission is then obtained by substituting the value of N_{video} in (11-16) to obtain the average number of frames sent per video frame, and in (21) to calculate the probability of a video packet error P. Assuming a maximum tolerable frame error rate for the video of P_v we can compare the systems in Sections II for different video codecs, or simply calculate the maximum throughput of a video source to meet the delay and loss requirements. For the optimization we follow an iterative approach: we first optimize the system to reduce the error rate as much as possible by increasing the number of ARQ retransmissions until the delay constraint is reached or a maximum number of retransmissions is reached. Then, if the error rate is acceptable for the requirement, the system is considered to be in its optimal point; otherwise the video throughput is reduced until it is such that all the requirements are met.

Consider a video transmission over UDP where the base station has a buffer for 20 video packets and ARQ has a maximum of 20 retransmissions. For generality, the delay constraint D is expressed in end-to-end packet transmission delay T (23). We consider three different video applications: a streaming video tolerant to errors in video quality ($D = 20T$, $P_v = 5\%$) (web-based video streaming); a real-time video with near-unimpaired video quality ($D = 2T$, $P_v = 1\%$), (a video-conference); and a real-time video with virtually no errors ($D = 2T$, $P_v \sim 0\%$). Fig. 7 shows the information rate that maximizes the CBR video throughput for the three applications over BLAST (we obtained similar results for STBC), for $f_dT = 0.01$ and $f_dT = 0.5$. The most noticeable result is that the information rate that maximizes the video throughput is greater than the channel capacity for applications that *tolerate* errors and delay. Also, the mobility does not significantly affect the performance because the system is allowed a large delay that absorbs the fading with retransmissions, except for low SNRs, in which the $f_dT = 0.5$ performs better. The video conference application tolerates a 1% error rate which moves the optimal rate closer to capacity. Finally, the optimal rate for the real-time application is below capacity in order to maintain the zero error rate. In this case, the low delay requirement prevents ARQ from recovering most of the long burst errors, which makes the $f_dT = 0.5$ to have better performance.

VII. Conclusions

We have presented a simple modelling framework for TCP over coded multi-antenna wireless systems. We showed that the type of application plays a crucial role in the optimization of a wireless system (for example, increasing the channel bandwidth efficiency is not always a good strategy). By direct framework application, we showed that the more uncorrelated the channel (higher doppler) the more the TCP/ARQ system can benefit from larger buffers without performance penalty. Finally, we showed that the application requirements affect the optimal rate of transmissions (for error and delay-tolerant video applications, increasing the transmission rate beyond capacity is a good strategy). Parameters such as the delay tolerance or the TCP AIMD feedback scheme drive the system performance, so systems should take these into account and accommodate the information rate at which the physical layer is transmitting to the real system demands.

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