

Camera Response Function Estimation from a Single-channel Image Using Differential Invariants

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Abstract

The camera response function (CRF) models the important characteristics of cameras and digital images. Recent work in [1] proposed a method for estimating CRF from a single grayscale image, by assuming the uniform distribution of the image irradiance in local edge regions. In this paper, we propose a different approach by exploring the fundamental properties of digital images – differential invariants. Specifically, the geometry invariants can be used to derive the analytical and computational solutions for estimating the CRF, based on the detailed analysis of the invariants. As the technique needs an analytic CRF model with good modeling power, we propose a general polynomial exponent model. The proposed method is tested using images from four different models of digital camera and achieves a very good accuracy – the average root mean square error (RMSE) for the estimated CRF is 0.0369. Being the basic properties of digital images, the differential invariants also show a great potential in solving other problems, such as modeling the cameras with a spatially varying CRF. This is a current ongoing work.

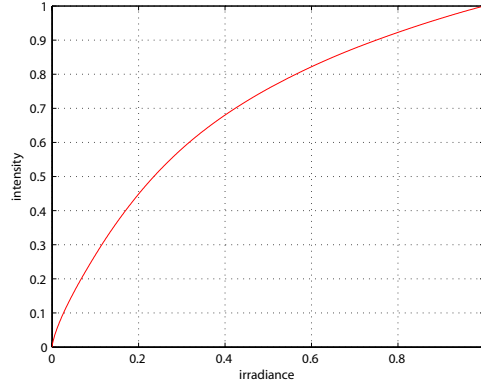


Figure 1: The averaged curve obtained from a database of real-world CRFs (DoRF).

1 Introduction

A camera response function (CRF) maps the image irradiance (light energy incident on the image sensors) to the image intensity (output image). In practice, the mapping is a collective effect of various camera internal operations [2] such as demosaicing, white-balancing, gamma correction and so on, apart from the random electronic noise. Therefore, the actual mapping could be multi-valued and noisy. However, the mapping can actually be approximated as a function, which is part of the design objectives of the camera manufacturers for the purpose of dynamic range compression and pleasing visual quality. It is well-known that CRFs are in general non-linear and its typical shape can be represented by the averaged curve from a database containing 201 real-world CRFs (DoRF) [3] as shown in Fig. 1.

The capability to estimate the CRF from an image or from a set of images is important, as various photometric computer vision algorithms, such as shape from shading, photometric stereo and so on, assume that the image intensity is linear with respect to the scene radiance. With the estimated CRF, the image intensity can be linearized. From a different perspective, the camera response can be considered a natural watermark for image authenticity [4]. Therefore, the estimation of the CRF is an important tool in image forensics for forgery detection. Our work is mainly motivated by the applications in image forensics.

In this paper, we assume that the CRF is uniform over the spatial dimension of an image, although some new models of camera may not follow this assumption. This type of CRF can be estimated from a set of images with different exposures by using the relationship of image intensity between the images [5, 6, 7, 2]. The function can also be estimated from a single RGB or grayscale image using the

property of pixel blending at the edge pixels [8, 1] or with a higher-order statistic [9]. In this work, we develop a set of differential invariants inspired by a simple observation in [4]: *image gradient contains information about CRFs*. The intuition is that the nonlinearity of the CRF modulates the image gradient with the slope of the CRF at different irradiance. This set of invariants can be generalized to any derivative order and any directional derivative. With the invariants, we develop an algorithm to estimate the CRF from a single-channel image. With a need for an analytic CRF model with good modeling power, we propose a general polynomial exponent model. Hence, the main contributions of this paper are the discovery of the differential invariants, a method of CRF estimation using the invariants and the proposal of a new analytic CRF model.

In Sec. 2, we provide a brief review of the CRF model and the CRF estimation techniques in the prior work. Then, we show a way of deriving the linear differential invariants in Sec. 3 and further elaborate on their properties in Sec. 4. In Sec. 5, we show the proposed analytic CRF model and provide an evaluation on its modeling power. The given properties of the invariants and the availability of the CRF model form the basic elements of the CRF estimation algorithm as presented in Sec. 6. We will show experimental results in Sec. 7 before the conclusions in Sec. 8.

1.1 Convention

In this paper, we use r for the image irradiance, R for the image intensity and f for the CRF, i.e., $R = f(r)$. The inverse of f is denoted as f^{-1} . For the first-order derivative, we denote $\frac{\partial R}{\partial x}$ as R_x and $\frac{df(r)}{dr}$ as $f'(r)$. A similar convention is used for the higher-order derivatives.

2 Prior Work Review

2.1 CRF Estimation Techniques

A conventional way of recovering the CRF is to photograph a chart, such as the Macbeth chart, with patches of known reflectance under uniform illumination. This method requires a manual scene setup and could be inconvenient. Automatic techniques using multiple images of a same scene taken with different exposure are proposed in [5, 6, 7, 2]. From these images, an *intensity mapping function* [10] can be derived by relating the intensity values of the images in the sequence. The intensity mapping function contains the complete information about the CRF and can be estimated from a *comparagram* [5], the joint histogram of the intensity values of corresponding points in two images of different exposures. In [5], the CRF

is estimated by exploiting the self-similar structure of the intensity mapping function. However in [6, 7], the estimation minimizes the differences of the linearized image intensity across the image sequence. All these techniques can be considered as indirectly fitting the intensity mapping function. As such techniques require multiple input images, they are not suitable for image forensics, in which only a single image is given for checking its authenticity.

The recovery of the CRF from a single RGB-color image was shown in [8] using the linear blending property of the image irradiance at the boundary of two homogenous regions with distinct irradiance values. The assumption of the uniform linear blending across the RGB color channels leads to the co-linearity (in the RGB space) among the image irradiance at an edge pixel and those of the two homogenous regions adjacent to the edge. As non-linear transformation on the image irradiance distorts the co-linearity, the CRF can be estimated by finding a function that best recovers the co-linearity. The same linear pixel blending property is employed in the case of using just a grayscale image, for which the uniform distribution for the image irradiance at the edge region is assumed [1]. The non-linear transformation on the image irradiance skews the distribution and the recovery of the CRF is performed by finding the best function that restores the uniformity of the distribution. In [9], a different property for non-linear transformation of the image irradiance was shown; a transformation of the image irradiance by a *gamma curve*, $f(r) = r^\gamma$, introduces harmonics of correlated frequencies. The occurrence of correlated harmonics can be measured by the magnitude of *bicoherence*, a third-order statistic, and the CRF can be estimated by the best γ which minimizes the bicoherence magnitude.

2.2 CRF Model

A CRF model plays an important role in CRF estimation. Two types of ambiguity, the *fractal ambiguity* and the *exponential ambiguity*, are shown to be present in the estimation of CRFs using the method of multiple exposure [10]. The presence of ambiguities implies the existence of multiple solutions for the estimation method. However, with a suitable CRF model, such as the polynomial model [7], these ambiguities can be resolved. Apart from this, from the perspective of model selection in machine learning, having a CRF model can greatly restrict the model space and prevent data overfitting, as a form of model regularization when estimating the CRF. This property is particularly crucial when the available data is noisy and scarce, as in the case of the single-image CRF estimation [8, 1, 9].

In general, a CRF has a few properties. Due to the unknown gain-offset of the image irradiance in a camera, the information about the absolute value of the image irradiance is permanently lost. Therefore, the value of the image irradiance r can

be scaled such that it occupies $[0, 1]$. Similarly, the range of the image intensity value R , such as $[0, 255]$ for an 8-bit image, can also be scaled to $[0, 1]$. Such scaling ensures that $f(1) = 1$. However, in general $f(0) \neq 0$ due to the bias from the thermal noise of a camera. However, for a low-noise camera or when a compensation is made using the averaged thermal noise (obtained from an image taken with the lens cap on), $f(0) = 0$ is a reasonable assumption.

The early model, $f(r) = \alpha + \beta r^\gamma$, is borrowed from the classic model of the photographic emulsion response function [5]. In this case, when the bias is compensated and (r, R) are both scaled to $[0, 1]$, the model becomes a gamma curve $f(r) = r^\gamma$. This model is known to have a limited flexibility in fitting the real-world CRFs. To overcome this limitation, a few other analytic models, with a better modeling flexibility but a more complex form, are proposed in [11]. Then, a more flexible and general model of a polynomial form is proposed in [7]. To achieve a tradeoff between specification and generality, an empirical model represented by a linear combination of the principle component analysis (PCA) bases is proposed [3]. The PCA bases are derived from the DoRF database which contains 201 real-world CRFs. As it is an empirical model, it lacks some nice properties of the analytic models, such as the monotonicity and differentiability of the curve.

3 Differential Invariants

In this section, based on the basic relation $R = f(r)$, we describe two types of differential invariants, namely the transform invariants and the geometry invariants, which have different properties and applications. In this paper, we assume that f , r and R are all differentiable, and f can have a general form and may not be monotonic. This paper focuses on the geometry invariants that we use to estimate the CRF. However, we will also briefly describe the transform invariants and one of its possible applications.

It is observed in [4] that the first-order derivatives of the image intensity differs from those of the image irradiance by a multiplicative factor as shown in Eq. (1).

$$R_x = f'(r)r_x, \quad R_y = f'(r)r_y \quad (1)$$

Note that R_x is the product of two factors; the first factor $f'(r)$ is purely related to the CRF while the second factor r_x is purely related to the geometry of the image irradiance. From this observation, a *transform invariant* can be obtained if the first factor can be eliminated. Similarly, a *geometry invariant* can be derived if the second factor can be cancelled off.

3.1 Transform Invariants

A *transform invariant* is a quantity which is independent of the CRF. It is easy to see from Eq. (1) that the ratio of R_x and R_y is a transform invariant, as shown in Eq. (2). Due to its independence to the CRF, the form of the transform invariant is the same for the image intensity and the image irradiance.

$$\frac{R_x}{R_y} = \frac{r_x}{r_y} \quad (2)$$

In fact, any function or derivative of Eq. (2), such as $\frac{R_x^2}{R_x^2 + R_y^2}$ and $\frac{R_y R_{xx} - R_x R_{yy}}{R_x R_{yy} - R_y R_{xx}}$, is also a transform invariant. In this way, we obtain a set of transform invariants for all derivative orders.

One possible but potentially non-straightforward application of the transform invariants is to provide a systematic way of locating image patches with a good co-linear property for the image irradiance of any three points at the edge regions in the RGB color space. Locating these patches is crucial for the CRF estimation method proposed in [8]. The co-linear property of the image irradiance in the RGB color space requires that the blending ratio α_R , α_G and α_B in Eq. (3) are same at the edge regions:

$$r_{iE}(x, y) = \alpha_i(x, y)r_{i1} + (1 - \alpha_i(x, y))r_{i2} \quad (3)$$

where r_{iE} is the image irradiance at an edge pixel and r_{i1} , r_{i2} are those for the pixels adjacent to the edge, $i = R, G, B$. With $\alpha_R = \alpha_G = \alpha_B = \alpha$, it can be shown that the transform invariants for the R, G and B channels are also equal in the local region:

$$\frac{(R_R)_x}{(R_R)_y} = \frac{(R_G)_x}{(R_G)_y} = \frac{(R_B)_x}{(R_B)_y} = \frac{\alpha_x}{\alpha_y} \quad (4)$$

where α_x and α_y are the spatial derivatives of the blending coefficients. Therefore, by checking the condition in Eq. (4), we can find pixels with the co-linearity property.

3.2 Geometry Invariants

A *geometry invariant* is a quantity only dependent on the CRF f but not the image irradiance r . Unlike the case for the transform invariants, it is non-trivial to eliminate the second factor in Eq. (1) as r is an unknown and spatially varying function. However, one way to obtain the geometry invariants is by making an assumption on the local geometry of r . In general, (R_x, R_y) is projectively equivalent to (r_x, r_y) as in Eq. (5) and (R_{xx}, R_{xy}, R_{yy}) is given by Eq. (6):

$$(R_x, R_y) = f'(r)(r_x, r_y) \quad (5)$$

$$(R_{xx}, R_{xy}, R_{yy}) = f''(r)(r_x^2, r_x r_y, r_y^2) + f'(r)(r_{xx}, r_{xy}, r_{yy}) \quad (6)$$

Let's assume that r is locally planar, i.e., $r = ax + by + c$. Then, due to $r_{xx} = r_{xy} = r_{yy} = 0$, (R_{xx}, R_{xy}, R_{yy}) have a similar projective form:

$$(R_{xx}, R_{xy}, R_{yy}) = f''(r)(r_x^2, r_x r_y, r_y^2) \quad (7)$$

From Eq. (5) and (7), we can derive three linear geometry invariants by taking their ratio:

$$\frac{R_{xx}}{R_x^2} = \frac{R_{yy}}{R_y^2} = \frac{R_{xy}}{R_x R_y} = \frac{f''(r)}{(f'(r))^2} = \frac{f''(f^{-1}(R))}{(f'(f^{-1}(R)))^2} \quad (8)$$

As shown above, the geometry invariants are independent of the local geometry of r . Instead, they depend on the derivatives of CRF only. Interestingly, all of them are equal to the same quantity, hereupon denoted as $A(R)$:

$$A(R) = \frac{f''(f^{-1}(R))}{(f'(f^{-1}(R)))^2} \quad (9)$$

Recall that the derivation of the geometry invariants is based on the assumption of the locally planar image irradiance. Given an image, how do we locate the local regions with planar image irradiance? One way is to use the equality constraints in Eq. (8) as a filter for selecting points with planar image irradiance.

We have extended the similar derivation to the case of locally quadratic image irradiance, $r = ax^2 + 2bxy + cy^2 + dx + ey + f$. In the quadratic case, we obtain two different sets of six quadratic geometry invariants, where the invariants within the same set are equal (equations not shown). Interestingly, the invariants in one of the sets are also equal to $A(R)$ given in Eq. (9). Similarly, the two sets provides two equality constraints for selecting points with quadratic image irradiance. The same procedure can be applied for deriving higher-order geometry invariants.

4 Properties of Geometry Invariants

The understanding on the properties of the geometry invariants is crucial for its application to the CRF estimation and helps in anticipating the potential difficulties. In this section, we describe four of the important properties concerning the geometry invariants.

4.1 Relationship to Gamma Curves

To have an intuitive understanding of the geometry invariants, we would like to associate a significant meaning to $A(R)$ in Eq. (9) by assuming a typical gamma

curve $R = f(r) = r^\gamma$ and compute $A(R)$:

$$A(R) = \left(\frac{\gamma - 1}{\gamma} \right) R^{-1} \quad (10)$$

Then, we can define a new quantity:

$$Q(R) = \frac{1}{1 - A(R)R} = \gamma \quad (11)$$

The above expression shows that $Q(R)$ is equal to the γ of the gamma curve. Hence, if we approximate the shape of the CRF at the local region of a specific R by a gamma curve, its exponent $\gamma(R)$ can be approximated by $Q(R)$. Note in computing $A(R)$ and $Q(R)$, we do not have to know the CRF and the geometry invariants are computed by the derivatives of R , as shown in Eq. (8).

4.2 Directional Equivalence of Rotational Invariance

The geometry invariants in Eq. (8) can be further generalized by considering the directional derivative instead of just the derivative in the x and y direction predetermined by the coordinate frame. A directional derivative in a θ direction can be written as a linear operator below:

$$D_s^\theta = \cos(\theta) \frac{\partial}{\partial x} + \sin(\theta) \frac{\partial}{\partial y} \quad (12)$$

Given D_s^θ , we can define another directional derivative D_q^θ in quadrature phase to D_s^θ :

$$D_q^\theta = \cos\left(\theta + \frac{\pi}{2}\right) \frac{\partial}{\partial x} + \sin\left(\theta + \frac{\pi}{2}\right) \frac{\partial}{\partial y} = -\sin(\theta) \frac{\partial}{\partial x} + \cos(\theta) \frac{\partial}{\partial y} \quad (13)$$

By the linearity of the operators, we can derive the following second-order directional derivatives:

$$D_{ss}^\theta = \cos^2(\theta) \frac{\partial^2}{\partial x^2} + \sin^2(\theta) \frac{\partial^2}{\partial y^2} + 2 \cos(\theta) \sin(\theta) \frac{\partial^2}{\partial x \partial y} \quad (14)$$

$$D_{sq}^\theta = \cos(\theta) \sin(\theta) \left(\frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial x^2} \right) + (\cos^2(\theta) - \sin^2(\theta)) \frac{\partial^2}{\partial x \partial y} \quad (15)$$

Let $R_s^\theta = D_s^\theta R$, $R_{ss}^\theta = D_{ss}^\theta R$ and similar notations for quadrature-phase derivatives. Then, for the locally planar regions, it can be shown using Eq. (5) and (7) that:

$$\frac{R_{ss}^\theta}{(R_s^\theta)^2} = \frac{R_{qq}^\theta}{(R_q^\theta)^2} = \frac{R_{sq}^\theta}{R_s^\theta R_q^\theta} = \frac{f''(r)}{(f'(r))^2} \quad (16)$$

Eq. (16) is indeed a generalization of Eq. (8) for all directions of θ . When s and q are set to x and y respectively, we readily get the invariants in Eq. (8).

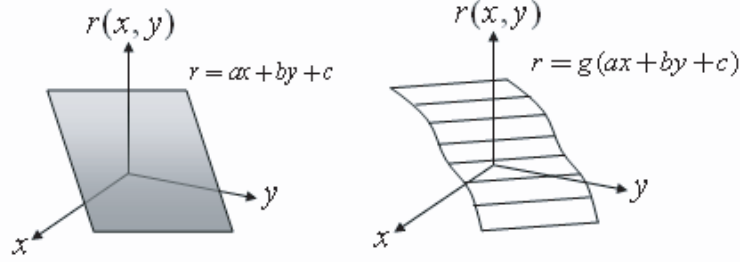


Figure 2: The transform ambiguity implies the geometry invariants hold in locally planar regions (as shown in the left graph) as well as the more general regions (as shown in the right graph).

4.3 The Transform Ambiguity

In Sec. 3.2, we have shown that:

$$r = ax + by + c \Rightarrow \frac{R_{xx}}{R_x^2} = \frac{R_{yy}}{R_y^2} = \frac{R_{xy}}{R_x R_y} \quad (17)$$

Namely, the locally planar property leads to the directional equivalence constraint shown above. However, a question worth asking is: Does every $R(x, y)$ that satisfies the directional equivalence constraints would correspond to a locally planar image irradiance (the reverse implication)? The answer is unfortunately no, therefore there exists an ambiguity in locating the locally planar $r(x, y)$. Intuitively, we could see that for $r = g(ax + by + c)$ with any arbitrary function g and $R = f(r)$, the directional equivalence constraints will be satisfied and the resultant quantities of Eq. (17) are equal to $\frac{h''(r)}{(h'(r))^2}$, where $h = f \circ g$, a composition of function f and function g . This idea is graphically illustrated in Fig. 2. The left graph of Fig. 2 shows a locally planar region, while the right graph shows a case in which all points having the same $ax + by + c$ values have the same irradiance, r . Both cases satisfy the directional equivalence constraints. We call such an ambiguity (in distinguishing f and its composition with any other function g) the *transform ambiguity*.

When $r = g(ax + by + c)$, we can derive the following by using Eq. (5) and Eq. (6) and the directional equivalence constraints (right hand side of Eq. (17)):

$$\frac{R_{xx}}{R_x^2} = \frac{R_{yy}}{R_y^2} = \frac{R_{xy}}{R_x R_y} = \frac{f''}{(f')^2} + \frac{g''}{f'(g')^2} \quad (18)$$

where $R = f(g(ax + by + c))$. Note the above equality conditions differ from that shown in Eq. (8) by a new term at the end, $\frac{g''}{f'(g')^2}$, which vanishes if g is

an identity function. Unlike the first term, this term depends on the irradiance geometry. In other words, when $r = g(ax + by + c)$, the directional equivalence constraints hold, but not the geometry invariants. Therefore, given an intensity image, R , we are only able to verify the directional equivalence constraints, but not the geometry invariants $\frac{f''}{(f')^2}$. However, we will show through experiments later that majority of the points satisfying the directional equivalence constraints also satisfy the geometry invariants. This allows us to explore the geometry invariants in estimation of the CRF, f .

4.4 An Analytic Solution for CRF

If there is not the transform ambiguity, we can actually obtain the response function directly from the geometry invariant by solving the following differential equation.

$$\frac{f''(f^{-1}(R))}{(f'(f^{-1}(R)))^2} = \frac{d}{dR} (\ln f'(f^{-1}(R))) = A(R) \quad (19)$$

Then the inverse of the CRF can be shown to be

$$f^{-1}(R) = \int \exp \left(- \int A(R) dR \right) dR \quad (20)$$

Note in case of local planarity, $A(R)$ can be obtained by computing the derivative ratios $\frac{R_{xx}}{(R_x)^2}$ or its equivalents (Eq (8)). However, in the presence of the transform ambiguity, there is a deviation term introduced (Eq. (18)) and an alternative solution is needed.

5 Curve Model and Parameter Estimation

One way to recover the CRF, as suggested in Sec. 4.4, is to solve Eq. (19). In the presence of the transform ambiguity, we not only need to handle a stochastic differential function, we also need a way to incorporate a CRF model in order to cope with the multiple solutions arising from the transform ambiguity. Therefore, we adopt an alternative approach; we look for a suitable CRF model for which the parameters of the model can be estimated directly from $Q(R)$ or $A(R)$. Observing the close relationship of $Q(R)$ to the parameter of the gamma curve in Sec. 4.1, we derive a general model, which we call *the polynomial exponent model*:

$$f(r) = r^{\sum_{i=0}^n c_i r^i} \quad (21)$$

When the exponent is a constant, the polynomial exponent model is reduced to a gamma curve. We evaluate the model by performing the linear least square fit of

Table 1: Mean RMSE ($\times 10^{-2}$) for Model Fitting

Model	Number of model parameters			
	1	2	3	4
polynomial exponent	5.18	2.34	1.16	0.60
EMOR [3]	4.00	1.73	0.63	0.25
polynomial [7]	7.37	3.29	1.71	1.06

the model to the 201 real-world curve in the DoRF database. The average root mean square error (RMSE) of the fit is shown in Table 1, with a comparison to the EMOR [3] model and the polynomial model [7] (the numbers are extracted from [3]). Note that the polynomial exponent model¹ performs slightly worse than the empirical EMOR model but outperforms the polynomial model. In this paper, we mainly use the linear exponent model (i.e., two parameters only) for our experiments. The parameters of the linear exponent model $f(r) = r^{c_0+c_1r}$ are related to $Q(R)$ as follows.

$$Q(R) = \frac{(c_1r \ln(r) + c_1r + c_0)^2}{c_0 - c_1r} \quad (22)$$

6 Estimation Algorithm

We first summarize our CRF estimation algorithm, and then provide the details in the following subsections.

1. Select image points using the directional equivalence constraints by computing the geometry invariants as shown in Eq. (8). [Sec. 6.1]
2. Refine the estimation of the geometry invariants ($A(R)$ and $Q(R)$) using the rotational profile of the generalized geometry invariant. [Sec. 6.2]
3. Refine the image point selection using a support vector machine (SVM) trained on the features such as R , and characteristics of the rotational profile of the generalized geometry invariants. [Sec. 6.3]
4. Estimate the model parameters by fitting $Q(R)$ with a parametric curve given in Eq. (22) using a recursive non-linear least square method. [Sec. 6.4]

¹The mean RMSE for the polynomial exponent model are from fitting all 201 curves in the DoRF database, but those for other models are from a subset of the entire database.

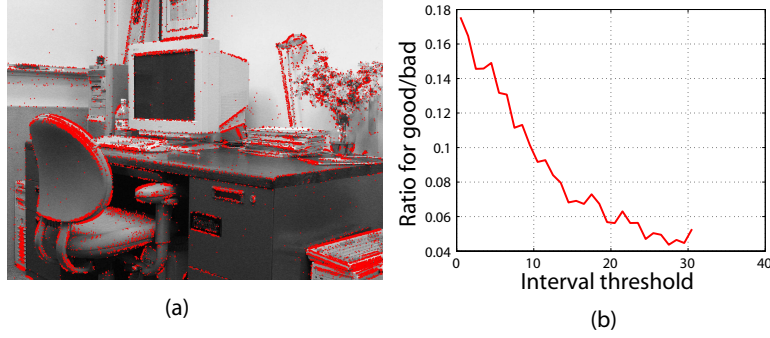


Figure 3: (a) Points selected on two images by the geometry constraint criteria. (b) A plot of the ratio of the number of good points and that of bad points over the threshold interval $T = [n, n + 1]$ where $n = 0, \dots, 30$.

6.1 Point Selection

Given a single-channel image, we compute the geometry invariants at every pixel and select the points potentially corresponding to the locally planar image irradiance by checking the following condition.

$$\left| \frac{R_{xx}}{R_x^2} - \frac{R_{yy}}{R_y^2} \right| + \left| \frac{R_{xx}}{R_x^2} - \frac{R_{xy}}{R_x R_y} \right| + \left| \frac{R_{yy}}{R_y^2} - \frac{R_{xy}}{R_x R_y} \right| < T \quad (23)$$

where the threshold value, T , is empirically set. We first use a loose threshold in anticipation of the subsequent point selection refinement step. Fig. 3 (a) shows the locations of the points selected by Eq. (23) and it is interesting to note that most points are from the edges. To further evaluate the usefulness of the constraint, we conduct an experiment using the artificially generated non-linear images (intensity) obtained by transforming a set of Canon G3 linear images (irradiance, generated by the software accompanying the camera) with a gamma curve. Then, if we compute $Q(R)$ using these gamma images, as shown in Sec. 4.1, the good points would fall in a range centered on the gamma value, $|Q(R) - \gamma| < \epsilon$. Other points out of the region are considered as bad points. Fig. 3 (b) shows the ratio of the count for the good points and the bad points for the threshold interval $T = [n, n + 1]$ where $n = 0, \dots, 30$. From the figure, we see that proportion of good points is falling off rapidly as the threshold increases. This indicates the importance of enforcing the threshold condition in Eq. (23). As an implementation detail, we compute the required derivatives in the linear Gaussian scale-space in order to counter for the quantization effect, specifically, we convolve the images with a Gaussian kernel which turns a non-differentiable staircase function into a differential real-value function.

6.2 Robust Estimation of Geometry Invariants using Directional Derivative Profile

Here we explore the direction-equivalent property of the geometry invariants (as discussed in Sec. 4.2) to derive estimates of $Q(R)$ and additional constraints for refining the selection of points. In order to improve the estimation of $Q(R)$, we compute the rotational profile for the generalized geometry invariants $A_1^\theta = \frac{R_{ss}^\theta}{(R_s^\theta)^2}$ and $A_2^\theta = \frac{R_{sq}^\theta}{R_s^\theta R_q^\theta}$, with a resolution of $\Delta\theta = 11.25^\circ$, $\theta \in [0^\circ, 360^\circ]$, and then transform A_i^θ to Q_i^θ according to Eq. (11) for $i = 1, 2$. Finally, $Q(R)$ is estimated by the average of the 10% trimmed mean (over θ) of $Q_1^\theta(R)$ and $Q_2^\theta(R)$, as in Eq. (24). The trimmed mean estimator discounts the 10% data at the two extremes of the real $\mathbb{R}$ line, therefore it is robust for estimating the mean value in the presence of singularities in the rotational profile. The singularity is due to the fact that one can always find a specific θ for which the first-order directional derivative of r is zero, if r is planar. This will cause division by zero operation in calculating terms in Eq. (16).

$$Q(R) = \frac{1}{2}(\text{trimmean}(Q_1^\theta(R)) + \text{trimmean}(Q_2^\theta(R))) \quad (24)$$

For implementation, we need to have a derivative kernel that provides an accurate computation of the directional derivative computation. In [12], a set of optimally rotation-equivariant derivative kernels are derived by minimizing the squared errors in the directional derivatives, while ensuring the derivative kernel to be separable and compact for a good computational efficiency.

6.3 Refinement on Point Selection using SVM

To ensure the geometry invariance over all directions, we further compute the following features at each point.

1. The entropy and the variance of Q_i^θ over θ .
2. The 10% trimmed mean of the first-order finite difference (over θ) on Q_i^θ , i.e., $\text{trimmean}(\Delta * Q_i^\theta)$, where $*$ is the convolution operator and $\Delta = [-1, 1]$ is the first-order finite difference kernel.
3. The disagreement between Q_1 and Q_2 , i.e., $|Q_1 - Q_2|$.

One straightforward way of using the above features would be to compare each of the above features against a certain threshold value. However, such simple approaches have resulted in insufficient performance in our experiments. To rectify

such a problem, we use a learning-based method to develop a robust classifier that discriminates good points from bad points by considering attributes of the rotational profile of the generalized geometry invariants. Specifically, we adopt a support vector machine (SVM) classifier that is trained using a pool of 3000 good points and 3000 bad points extracted from the artificially generated gamma image, as described in Sec. 6.1. In the implementation of SVM, we use a radial basis function (RBF) kernel and a 2-to-1 weighting on the false positive errors. In addition to the three features listed above, the feature vector input to the SVM also includes the absolute error of the geometry invariant (left hand side of Eq. (23)), and the intensity value of the point, R . The former ensures the geometry invariants in the horizontal and vertical directions, while the latter allows exploration of the potential dependence of the geometry invariants on the image intensity.

6.4 Curve Fitting using Recursive Nonlinear Least Square Method

From Eq. (22), we perform the curve-fitting on the $(R, Q(R))$ space based on the following criteria:

$$(c_0^*, c_1^*) = \arg \min_{(c_0, c_1)} \sum_{j=0}^J \sum_{k=0}^K J(j, k) \left| Q_j - \frac{(c_1 r_k \ln(r_k) + c_1 r_k + c_0)^2}{c_0 - c_1 r_k} \right|^2 \quad (25)$$

where $J(j, k) = P(Q_j | R_k)$, a normalized conditional histogram on the $(R, Q(R))$ space, with Q_j and R_k being the fixed discrete values of Q and R , and $r_k = f^{-1}(R_k)$. The conditional histogram is used so that the curve-fitting is not affected by $P(R)$ which is content-dependent. Observe that if we know (c_0, c_1) , we can easily find r_i from R_i using the tabulated values of the inverse function f^{-1} and, similarly, if we know r_i , we can minimize the above objective function using a nonlinear least square method. Therefore, (c_0^*, c_1^*) is computed recursively as below:

1. **Initialize:** Let $r_i = R_i$, $i = 1, 2, \dots, K$, $n = 0$, and set (c_0^n, c_1^n) to some initial values such as $(100, 100)$.
2. **Non-linear Least Square:** increment n and by assuming that r_i are known, find new estimates of (c_0^n, c_1^n) according to Eq. (25) and thereby determine f .
3. **Refine r_i :** Find new r_i from the tabulated f^{-1} based on the current (c_0^n, c_1^n) .
4. **Iterations:** Repeat step 2 until $\|(c_0^n, c_1^n) - (c_0^{n-1}, c_1^{n-1})\|_{L_2} < \epsilon$.

7 Experiments

We perform experiments on four models of camera, i.e., Canon G3, Canon Rebel XT, Nikon D70 and Kodak DCS330². For each of the models, we estimate their response function using two different methods of multiple exposures [6, 7], where their software implementation are respectively known as HDR Shop³ and RASCAL⁴. We use the average of the curves estimated from the two techniques as the ground-truth for our experiments. As a caveat, the ground-truth can only serve as a reference but not an absolute gold standard for the CRF. The estimation from the conventional methods can yield results with an observable discrepancy. The RMS difference between the CRFs estimated by the two multi-exposure methods is 0.0239.

We have five test images from each model of camera. For each image, three curves correspond to the R, G and B color channels are estimated. In this experiment, in order to provide a transparent assessment to our technique, no specific selection of test images is made. Table 2 shows the mean RMSE over the five images for each model of camera. The overall mean RMSE is about 0.0413. We consider such performance quite satisfactory, given that our current implementation uses a CRF analytical model (i.e., polynomial exponent model) with two parameters only. As shown earlier in Table 1, direct fitting of the real-world CRF's in the DoRF database with two parameters results in a RMSE bound of 0.0234. This indicates the performance is very close to the model limit and the proposed estimation method is indeed effective. We also experimented with the quadratic exponent model (with three parameters) and we obtain a mean RMSE of 0.0369. However, the mean RMSE does not improve further when the exponent model goes to higher orders, indicating a limitation for the planar geometry invariant method. Despite this limitation, the estimation robustness and accuracy may be improved if we incorporate higher-order geometry invariants. As a comparison, prior work [1] reported an average RMSE of 0.156 over a different test set but with only two models of camera. In that work, experiments are only performed on the color-to-grayscale images and the CRF estimation using the single-color-channel images is stated as future work.

Fig. 4 shows the estimated curves for the red color channel, superposed with the ground-truth curves. Except for the Canon G3 estimations with a consistent deviation from the ground-truth curve at the mid-intensity range, the estimation for other cameras matches well with the ground-truth curve. Besides this, the algorithm has a good computational efficiency, it takes about 2.5 minutes to estimate

²The Kodak DCS330 images are also used in [8]. We thank the authors for sharing the images.

³Downloading from <http://gl.ict.usc.edu/HDRShop/>

⁴Downloading from <http://www1.cs.columbia.edu/CAVE/>

Table 2: RMSE mean ($\times 10^{-2}$) and standard derivation ($\times 10^{-2}$ and in the bracket) for the camera curve estimation

camera	R	G	B
Nikon D70	3.65 (0.75)	2.05 (0.75)	3.11 (0.81)
Canon G3	5.11 (1.87)	4.95 (1.68)	4.25 (0.90)
Canon XT	3.76 (2.16)	5.04 (2.58)	4.70 (2.32)
Kodak DCS330	4.14 (1.84)	4.45 (1.94)	4.33 (0.94)

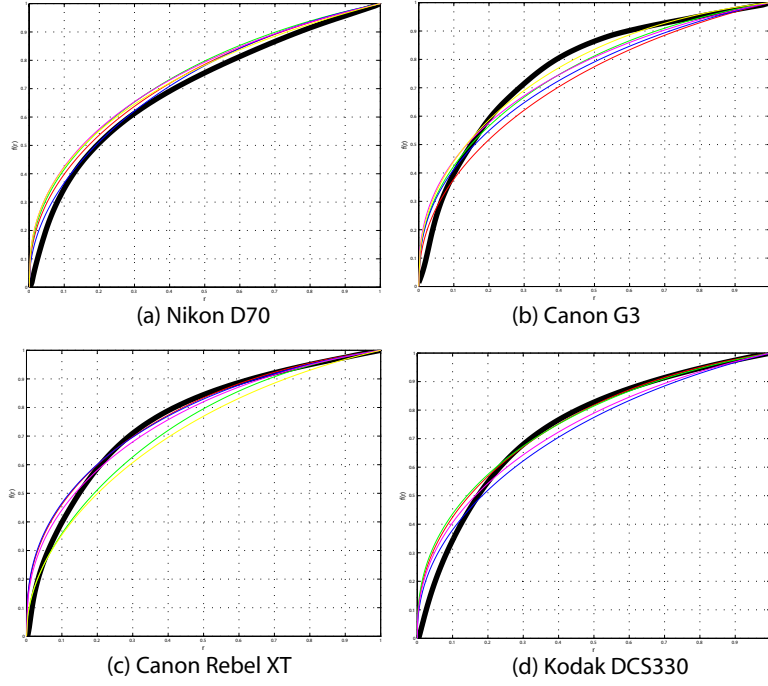


Figure 4: Estimated camera curves for the red color channel. The thick black lines are the ground-truth curves and the fine lines of other colors are the estimated curves using our techniques.

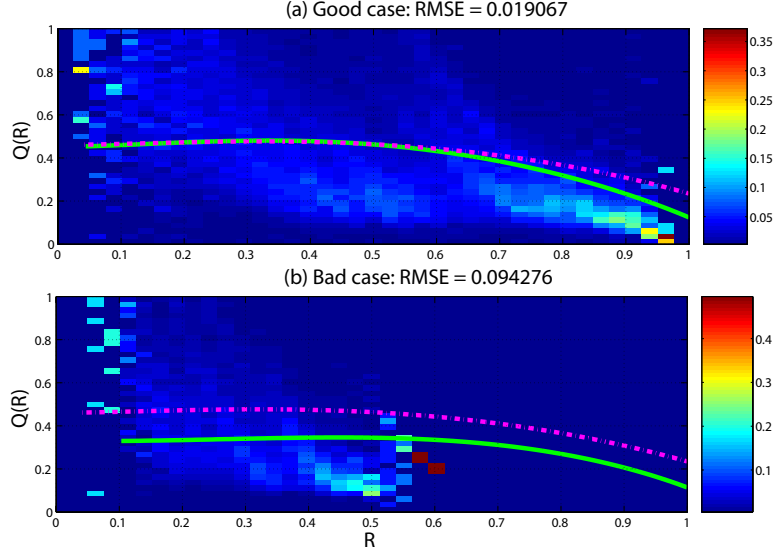


Figure 5: Examples of curve fitting for the good and bad cases. The green solid curve is the ground-truth curve while the magenta dashed line is the fitted curve. The brightness of the background indicates the conditional histogram.

a curve from a single-channel image of size 1000×700 pixels using Matlab on a single-CPU PC. Fig. 5 shows the ground truth and estimated values of $Q(R)$ over different R values for a good case and a bad one together with the conditional histogram $P(Q|R)$. We can see that, as shown in the lower plot, the insufficient data-point coverage of the R -axis introduces major errors in curve-fitting.

As discussed earlier, theoretically the use of the geometry invariants alone is not sufficient for the correct estimation of CRFs, due to the transform ambiguity. The feasibility of a solution lies in the structure of the arbitrary g function in $r = g(ax + by + c)$. It is fortunate that, in natural-scene images, majority of the computed points $(R, Q(R))$ (recall that we perform curve-fitting on the $(R, Q(R))$ space) do fall on the ground-truth curve area as can be observed from Fig. 5. This implies that most of the ambiguous g functions are close to an identity function, i.e., $z = g(z)$. To investigate further, we inversely transform an intensity image using its ground-truth curve and then compute the $|r_{xx}| + |r_{yy}| + |r_{xy}|$ at the data point locations for assessing the local planarity of the linearized image. We have found that indeed majority of the points are locally planar and thus fall on the ground truth curve area.

8 Conclusion

In this paper, we present two types of differential invariants, i.e., the transform invariants and the geometry invariants. With the geometry invariants, we develop a method to recover the CRF using a single-channel image. The method employs a new response function model, i.e., the polynomial exponent model. Finally, we present experimental results to demonstrate the effectiveness of the proposed method. We believe that the differential invariants can do more than estimating the CRF. For example, it may be used for selecting image patches with good collinearity in the image irradiance [8] (as discussed in Sec. 3.1). In general, the invariants are the basic properties of digital images and can be used for many other applications such as image forgery detection. Furthermore, the invariants are from the local structure of images, we believe that the invariants may be used for developing methods for estimating the spatially varying CRF, which can be found in the new generation of digital cameras such as the Sony Cybershot.

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