Problem # 1
You are given a binary communications system, using the two signals shown below. Assume that the noise is additive white gaussian noise, with power spectral density equal to $\frac{N_0}{2}$ watts/Hz.

a. Find and draw the optimum two-branch correlation receiver.

b. What is the average energy per bit, in terms of $E$?

c. Are these signals orthogonal? Explain.

d. What is the probability of bit error for this receiver as function of the average energy per bit?

\[ s_1(t) \]
\[ \sqrt{\frac{E}{T}} \]

\[ s_2(t) \]
\[ \sqrt{\frac{E}{T}} \]

(continued on next page)
d. Repeat parts (a) through (d) if \( s_2(t) = 0 \).

**Hint:** In this part maybe it is worthwhile to find the average probability of error using the minimum distance equation, i.e., \( \Pr\{ \varepsilon \} = Q[ d_{\text{min}}/\sqrt{2N_0}]. \)

**Problem # 2**

a. Show that the general equation for the average energy, \( E_s \), of a square QAM constellation is given by

\[
E_s = [(M-1)/6] \ d_{\text{min}}^2
\]

where \( M=2^n \) and \( n=\text{number of bits/symbol} \)

b. Find the peak-to-average energy ratio. This ratio is defined as the ratio of the energy of the strongest signal in the signal set divided by the average energy of the constellation set.

c. Find the peak-to-average ratio defined in (b) as \( M \) becomes very large.
Problem # 3 (This is a typical final exam problem!!)

We have decided to use the following modulation 32-ary (five bits/symbol) modulation technique, operating at a baud rate of 1/T bauds.

We send one of 32 signals every T seconds
The signal set is made up of the following 32 signals.

We have one 16-QAM square constellation centered around frequency $f_0$ Hz, and one 16-QAM square constellation centered around frequency $f_0 + 1/T$ Hz (with $f_0 T=10$). Every T seconds, we transmit one of the 32 signals described above.

a. What is the average energy per signal of this 32-ary modulation technique, as a function of d?
b. How many dimensions are there in this modulation technique?
c. What is the minimum distance between any pair of signals in this modulation technique as a function of “d”? Also keep in mind that this minimum distance is the big factor in determining the error probability.
d. What is the maximum distance between any pair of signals in this modulation technique as a function of “d”?
e. If the second frequency set is centered at $(f_0 + 2/T)$ Hz, instead of $(f_0 + 1/T)$ Hz, do the answers to parts (c), (d) and (e) change? Explain!
16-QAM Costellation

\[ x(t) = a_1 \sqrt{2/T} \cos 2\pi f_0 t \]
\[ + a_2 [-\sqrt{2/T} \sin 2\pi f_0 t] \quad ;0 \leq t \leq T \]
\[ a_1, a_2 = \pm d, \pm 3d/2 \]

\[ \phi_1(t) = \sqrt{2/T} \cos 2\pi f_0 t \]
\[ \phi_2(t) = -\sqrt{2/T} \sin 2\pi f_0 t \]