Problem Set #8 Solutions

1. a. Old telephone line $P/N_0 W \geq 28db$, $W = 3khz$, $28db = 10^{2.8} = 630.96$, then the minimum capacity is

$$C_{min} \geq 3 \times 10^3 \log_2 [1 + 630.96] = 27.911kbps.$$ b. $W \rightarrow \infty, P/N_0 W \rightarrow 0$, then

$$C_\infty = \lim_{W \rightarrow \infty} W \log_2 \left(1 + \frac{P}{N_0 W}\right)$$

$$= \frac{P}{N_0 \log 2}.$$ $P/N_0 W = 3 \times 10^3 \times 630.96 = 1.893MHZ$, so $C_\infty = 2.73Mbps$. Compared with the 3khz channel, we gain capacity by $2730 - 27.9 = 2702.1kbps$

c. $0.99 \times 2.73 \times 10^6 = W \log_2 \left[1 + \frac{1.893 \times 10^6}{W}\right]$, assuming $W$ big we get

$$W = \frac{(1.893 \times 10^6)^2}{2 \log 2 \times 27.308 \times 10^3} = 94.64MHz.$$ From the results, we can see that in order to improve the capacity, we have to increase the BW. But the capacity tend asymptotically to the value $P/(N_0 \log 2)$.

a. When $N_R P/(N_0 W) \gg 1$,

$$C_a = W \log_2 \left(1 + \frac{N_R P}{N_0 W}\right) bps$$

$$\approx W \log_2 \left(\frac{N_R P}{N_0 W}\right).$$

b. When $P/(N_0 W) \gg 1$,

$$C_b = M_T W \log_2 \left[1 + \frac{N_R P}{M_T N_0 W}\right]$$

$$\approx W \log_2 \left(\frac{N_R P}{M_T N_0 W}\right).$$

c. If $M_T = N_R = N$,

$$C_b = NW \log_2 \left(\frac{P}{N_0 W}\right),$$

$$C_a = W \log_2 \left(\frac{NP}{N_0 W}\right).$$
$C_b$ is clearly linear in $N$, $C_a$ is a logarithmic function of $N$. When $P/(N_0W) \gg 1$ constant, the linear function grows much faster than the log function. In fact, we have $\lim_{N \to \infty} C_b/C_a = +\infty$, i.e. $C_b \gg C_a$ when $N \to \infty$.

d. When $N_R P/N_0 W \ll 1$,

$$C_a = W \log_2 \left( 1 + \frac{N_R P}{N_0 W} \right)$$

$$= \frac{W}{\log 2} \log \left( 1 + \frac{N_R P}{N_0 W} \right)$$

$$\simeq \frac{N_R P}{N_0 \log 2}.$$ 

When $P/N_0 W \ll 1$,

$$C_b = M_T W \log_2 \left[ 1 + \frac{N_R P}{M_T N_0 W} \right]$$

$$\simeq \frac{N_R P}{\log 2 N_0}.$$ 

At least in first order approximation $C_a = C_b$. For $P/N_0 W \ll 1$, MIMO does not affect any improvement over SIMO.