Problem #1
Assume that a BPSK signal is transmitted over a slow flat Rayleigh fading channel with additive WGN. We use one antenna at the transmitter and one antenna at the receiver (SISO).

a. Calculate the average bit error probability, $Pr_b\{\varepsilon\}$ for BPSK (we actually started this in class), as a function of the average $E_b/N_0$.

b. Repeat (a) for the symbol error probability, $Pr_s\{\varepsilon\}$, for QAM (ignoring the $Q^2$ terms in the error probability equation), as a function of the average $E_s/N_0$. 
Problem #2
Assume that a BPSK signal is transmitted over a slow flat Rayleigh fading channel with additive WGN, in a SISO system.

a. Calculate the Outage Probability for BPSK (we actually did this in class), as a function of the average $E_b/N_0$ and $E_{b,req}/N_0$, for this SISO system.

b. For BPSK, find the required average $E_b/N_0$, if the outage probability, $Pr\{\text{out}\}$ is equal to $10^{-3}$, and if the desired instantaneous, $Pr_b\{\varepsilon\}$ equals $10^{-5}$.

c. Compare the value found in (b) with the average $E_b/N_0$, required when the average $Pr_b\{\varepsilon\}$ equals $10^{-5}$.

d. Repeat parts (b) and (c) if the desired instantaneous $Pr_b\{\varepsilon\}$ remains the same (i.e., equals $10^{-5}$) but the $Pr\{\text{out}\}$ equals $10^{-1}$ and also $10^{-5}$. 
Modern Digital Modulation Techniques  
ELEN E6909  
Columbia University  
Spring Semester 2008

PROBLEM SET # 6 (New Set Number)  
Due Date: 2 April 2008

Read the following articles on BLAST and MIMO

The theoretical background behind MIMO.


The BLAST Algorithm


These articles may be downloaded from the following website

http://www1.bell-labs.com/project/blast/
Problem #1
This problem concerns Maximal Ratio Combining (MRC) – SIMO techniques

a) The modulation technique is BPSK. Find the outage probability at the output of a maximal-ratio combining receiver (with two receiving antennas) as a function of the average received energy per bit per antenna, divided by the noise spectral density, $\frac{E_{b,\text{avg, ant}}}{N_0}$, and the required instantaneous, $\frac{E_{b,\text{req, ant}}}{N_0}$, for the required instantaneous probability of error. Assume that the receiving antennas receive independent signals of the same average power.

b) Now find the outage probability of a maximal-ratio combining receiver (with two receiving antennas) as a function of the total average received energy per bit at both antennas divided by the noise spectral density, $\frac{E_{b,\text{avg, total}}}{N_0}$. Assume that the antennas receive independent signals of the same average power.
c) For BPSK, compare the results of (a) and (b) with those for a single receiving antenna at an outage probabilities, of $10^{-3}$ and $10^{-1}$, if the desired instantaneous $Pr_b{\epsilon}=10^{-5}$. How many dB have been gained in each case by using MRC-SIMO techniques?

**Problem #2**
This problem concerns MRC techniques when the number of receiving antennas is “L”.

For BPSK, show that the probability density function, $f(x)$, for the combined received signal for $L$ antennas, with maximal-ratio combining, is given by the equation below.

$$f(x)= \frac{1}{(L-1)! (2\sigma^2)^L} x^{L-1} \exp\{-x/2\sigma^2\}; \quad x \geq 0$$

(\text{where } x = x_1 + x_2 + \ldots + x_L) ; x_i= r_i^2.

The variable, $r_i$ represents the random Rayleigh variable at each receiving antenna.

**Hint:** This is similar to what we did in class for two receiving antennas.