1 9.3–5

a)
\[ E^n y[k] = (b_n E^n + b_{n-1} E^{n-1} + \cdots + b_0) f[k] \]

or

\[ y[k] = b_n f[k] + b_{n-1} f[k-1] + \cdots + b_0 f[k-n] \]

When \( f[k] = \delta[k] \), \( y[k] = h[k] \). Therefore

\[ h[k] = b_n \delta[k] + b_{n-1} \delta[k-1] + \cdots + b_0 \delta[k-n] \]

b) Here \( n = 3, b_3 = 3, b_2 = -5, b_1 = 0, b_0 = -2 \). Therefore

\[ h[k] = 3\delta[k] - 5\delta[k-1] - 2\delta[k-3] \]

2 9.4–4

The (zero state) response \( y[k] \) of is the convolution of the input signal and the unit impulse response

\[ y[k] = f[k] * h[k] \]

\[ = h[k] * f[k] \]

\[ = 3k(2)^k u[k] * (3)^{-k} u[k] \]

\[ = 3k(2)^k u[k] * (1/3)^k u[k] \]

Use Table 9.1 (p 590) No 6. Set \( \gamma_1 = 2 \) and \( \gamma_2 = 1/3 \). Then

\[ y[k] = 3 \frac{2^{2/3}}{(2 - 1/3)^2} [(1/3)^k - (2)^k + \frac{2 - 1/3}{1/3} k(2)^k] u[k] \]

\[ = \frac{18}{25}((3)^{-k} - (2)^k + 5k(2)^k] u[k] \]

3 9.4–5

The (zero state) response \( y[k] \) of is the convolution of the input signal and the unit impulse response

\[ y[k] = f[k] * h[k] \]

\[ = h[k] * f[k] \]

\[ = (3)^k \cos(\frac{\pi}{3} k - 0.5) u[k] * (2)^k u[k] \]
Use Table 9. No 10. Set $\gamma_1 = 3$, $\beta = \pi/3$, $\theta = -0.5$ and $\gamma_2 = 2$. Then

$$R = [(3)^2 + (2)^2 - 2(3)(2)\cos(\pi/3)]^{1/2}$$
$$= [9 + 4 - 2(3)(2)(.5)]^{1/2}$$
$$= \sqrt{7}$$

$$\phi = \tan^{-1}\left[\frac{3\sin(\pi/3)}{3\cos(\pi/3) - 2}\right]$$
$$= \tan^{-1}\left[\frac{3\sqrt{3}/2}{1.5 - 2}\right] = 1.761 \text{ rad}$$

Therefore

$$y[k] = \frac{1}{\sqrt{7}} [(3)^{k+1}\cos\left(\frac{\pi}{3}(k + 1)\right) - 2.261) - (2)^{k+1}\cos(2.261)]u[k]$$
$$= \frac{1}{\sqrt{7}} [(3)^{k+1}\cos\left(\frac{\pi}{3}(k + 1)\right) - 2.261) + 0.637(2)^{k+1}]u[k]$$

4 9.4–6

**First**, determine the zero-input component.

From the equation $y[k + 1] + 2y[k] = f[k + 1]$, we see that the characteristic root is $-2$. Therefore

$$y_0[k] = c(-2)^k$$

Setting $k = -1$ and substituting $y[-1] = 10$, yields

$$10 = -c/2 \Rightarrow c = -20$$

Therefore

$$y_0[k] = -20(-2)^k, k \geq 0$$

**Second**, find the unit impulse response, $h[k]$ of the system.

The characteristic root is $-2$, $b_0 = 0$ and $a_0 = 2$. Therefore

$$h[k] = c(-2)^ku[k] \quad (1)$$

We need one value of $h[k]$ to determine $c$. This is done by solving iteratively

$$h[k + 1] + 2h[k] = \delta[k + 1]$$

Setting $k = -1$, and substituting $h[-1] = 0$, $\delta[0] = 1$, yields

$$h[0] = 1 = c(-2)^0u[0] \Rightarrow c = 1$$

Therefore,

$$h[k] = (-2)^ku[k]$$

**Third**, using the impulse response, find the zero-state response.

$$y[k] = f[k] * h[k] = e^{-k}u[k] * (-2)^ku[k]$$
Using Table 9.1 No 4, \( \gamma_1 = 1/e \) and \( \gamma_2 = -2 \). Therefore

\[
y[k] = \left[ \frac{(1/e)^{k+1} - (-2)^{k+1}}{1/e - (-2)} \right] u[k] \\
= \frac{e}{2e + 1} \left[ e^{-k} - (-2)^{k+1} \right] u[k] \\
= \frac{e}{2e + 1} \left[ (1/e)e^{-k} + 2(-2)^{k} \right] u[k] \\
= \left[ \frac{1}{2e + 1}(e^{-k} - \frac{2e}{2e + 1}(-2)^{k}) \right] u[k]
\]

Finally, the total response is the sum of the zero-input response and the zero-state response

\[
total\ response = y_0[k] + y[k] \\
= \left[ -20(-2)^{k} + \frac{1}{2e + 1}(e^{-k} - \frac{2e}{2e + 1}(-2)^{k}) \right] u[k]
\]

5 9.6–1

(a) \( \gamma^2 + 0.6\gamma - 1.6 = (\gamma - 0.2)(\gamma + 0.8) \)

Roots are 0.2 and -0.8.

Both roots are inside the unit circle. Therefore, the system is asymptotically stable.

(b) \( (\gamma^2 + 1)(\gamma^2 + \gamma + 1) = (\gamma - j)(\gamma + j)(\gamma + \frac{1}{2} - \frac{j\sqrt{3}}{2})(\gamma + \frac{1}{2} + \frac{j\sqrt{3}}{2}) \)

Roots are \( \pm j, \ -\frac{1}{2} \pm \frac{j\sqrt{3}}{2} = e^{\pm j\pi/3} \).

All the roots are simple and on unit circle. Therefore, the system is marginally stable.

(c) \( (\gamma - 1)^2(\gamma + \frac{1}{2}) \)

Roots are 1 (repeated twice) and -0.5.

The repeated root is on unit circle. Therefore, the system is unstable.

(d) \( \gamma^2 + 2\gamma + 0.96 = (\gamma + 0.8)(\gamma + 1.2) \)

Roots are -0.8 and -1.2.

One of the roots is outside of unit circle. Therefore, the system is unstable.

(e) \( (\gamma^2 - 1)(\gamma + 1)^2 = (\gamma + 1)(\gamma - 1)(\gamma + j)(\gamma - j) \)

Roots are \( \pm 1 \) and \( \pm j \).

All the roots are simple and on unit circle. Therefore, the system is marginally stable.

6 9.6–2

Assume that a system exists that violates (9.75), and yet produces bounded output for every bounded input. The system response at \( k = k_1 \) is

\[
y[k_1] = \sum_{m=-\infty}^{\infty} h[m] f[k_1 - m]
\]
Consider a bounded input \( f[k] \) such that

\[
f[k] = \begin{cases} 
1 & \text{if } h[m] > 0 \\
-1 & \text{if } h[m] < 0
\end{cases}
\]

In this case

\[ h[m]f[k_1 - m] = |h[m]| \]

and

\[ y[k_1] = \sum_{m=-\infty}^{\infty} |h[m]| = \infty \]

This violates the assumption.

### 7 9.6–3

For a marginally stable system, at least one characteristic root is on unit circle and \( h[k] \) does not decay. For large \( k \), it is either constant or oscillates with constant amplitude. Clearly

\[ \sum_{m=-\infty}^{\infty} |h[m]| = \infty \]

Thus, the system is BIBO unstable.

Consider the simple case:

\[ y[k+1] - y[k] = f[k+1] \]

Here the characteristic root is 1, \( b_0 = 0 \) and \( a_0 = -1 \). Therefore

\[ h[k] = c(1)^k u[k] \]

To determine \( c \),

\[ h[k+1] - h[k] = \delta[k+1] \]

Setting \( k = -1 \), and substituting \( h[-1] = 0 \), \( \delta[0] = 1 \), yields

\[ h[0] = 1 = c \]

Therefore,

\[ h[k] = (1)^k u[k] = u[k] \]

If \( f[k] = (1)^k = 1 \) (bounded input), then

\[ y[k] = \sum_{m=-\infty}^{\infty} u[m]1^{k-m} = \sum_{m=0}^{\infty} 1 = \infty \]