\[ p = p_1 + p_2 = \frac{C_1^2 + C_2^2}{\Delta} \]

\[ = \frac{10^2}{\Delta} + \frac{16^2}{\Delta} \]

\[ = 178. \]

Rms value = \[ \sqrt{p} = \sqrt{178}. \]

\[ f(t) = \frac{e^{j\omega t} \cos \omega_0 t}{\Delta} = \frac{e^{j\omega t}}{\Delta} \left( e^{j\omega t} + e^{-j\omega t} \right) \]

\[ f(t) = \frac{1}{2} \left( e^{j(\omega + \omega_0) t} + e^{j(\omega - \omega_0) t} \right) \]

\[ p = \lim_{T \to \infty} \frac{1}{T} \int_{-T}^{T} f(t) \overline{f(t)} \, dt \]

\[ = \lim_{T \to \infty} \frac{1}{T} \int_{-T}^{T} \frac{1}{4} \left( e^{2j\omega t} + e^{2j\omega_0 t} + e^{-2j\omega t} + e^{-2j\omega_0 t} \right) \, dt \]

\[ = \lim_{T \to \infty} \frac{1}{4T} \left( 2T + \int_{-T}^{T} \cos (2j\omega_0 t) \, dt \right) \]

\[ = \lim_{T \to \infty} \frac{2}{4T} + \lim_{T \to \infty} \frac{1}{4T} \left( \int_{-T}^{T} \cos (2j\omega_0 t) \, dt \right) \]

\[ = \frac{1}{\Delta^2} \text{ (The second term is a finite quantity, divided by } T \to \infty \text{ gives zero).} \]
(a) \( f_1(t) = 4(t+1) [u(t+1)-u(t)] + (-2t+4) [u(t)-u(t-2)] \)

\[ = 4(t+1)u(t+1) - 6u(t) + 4u(t) + (2t-4)u(t-2) \]

(b) \( f_2(t) = t^2 [u(t) - u(t-2)] + (2t-8)[u(t-2) - u(t-4)] \)

\[ = t^2u(t) - (t^2-2t+8)u(t-2) - (2t-8)u(t-4). \]
\[ e^{-t} \cos(3t-60^\circ) \delta(t) \]
\[ = \cos(-60^\circ) \delta(t) \]
\[ = \frac{1}{2} \delta(t) \]

\[
\left( \frac{\sin kw}{w} \right) \delta(w) \\
= \lim_{w \to 0} \left( \frac{\sin kw}{w} \right) \delta(w) \\
= \k \delta(w)
\]

**Note:** In both problems above, we used the fact that
\[ f(t) \delta(t) = f(0) \delta(t) \]

1.4.5 (g)
\[ \int_{-\infty}^{\infty} f(2-t) \delta(2-t) \, dt \]
\[ = f(2-(3)) \]
\[ = f(-1). \]

(h)
\[ \int_{-\infty}^{\infty} e^{(-2t)} \cos \left( \frac{\pi}{2} (x-5) \right) \delta(x-3) \, dx \]
\[ = \frac{e^{(3-1)}}{e^{(-2)}} \cos \left( \frac{\pi}{2} (3-5) \right) \delta(x-3) \]
\[ = e^{2} \cos (-x) = -e^{2} \]
Note: \[ f_0(t) = \frac{f(t) - f(-t)}{2} \quad \text{(odd part)} \]
\[ f_e(t) = \frac{f(t) + f(-t)}{2} \quad \text{(even part)} \]

(b) \[ f(t) = tu(t). \]

\[ f_0(t) = tu(t) - \frac{t}{\alpha^2} \quad \text{(odd part)} \]
\[ = \frac{t}{\alpha^2} [u(t) + u(-t)] \]
\[ = \frac{t}{\alpha^2} \]

\[ f_e(t) = tu(t) + \frac{t}{\alpha^2} \quad \text{(even part)} \]
\[ = \frac{|t|}{\alpha^2} \]

(e) \[ f(t) = \sin \omega t \]
\[ f_0(t) = \sin \omega t \]
\[ f_e(t) = 0. \]  
Note that \( \sin \omega t \) is an odd function.
b) $f_o(t) = t/2$

$c. (1.5-1) Sketches$

e) $f(t) = \sin(\omega_0 t)$

$f_o(t) = \sin(\omega_0 t)$

$f_e(t) = 0$
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#7. (1,7-1)
e) let \( f_1(t) \rightarrow y_1(t) \) and \( f_2(t) \rightarrow y_2(t) \)

\[
f_1(t) = \left(\frac{dy_1(t)}{dt}\right)^2 + 2y_1(t) \Rightarrow \text{mult. by } k_1 \Rightarrow k_1f_1(t) = k_1 \left(\frac{dy_1(t)}{dt}\right)^2 + 2k_1y_1(t)
\]

If the system was linear, then \( k_1f_1(t) = \left(\frac{dk_1y_1(t)}{dt}\right)^2 + 2k_1y_1(t) \)

Since the system does not satisfy the homogeneity (scaling) property, it is **NONLINEAR**.

c) Does this system satisfy homogeneity?

\[
\text{mult. by } k \Rightarrow 3ky(t) + 2k = kf(t) \neq 3ky(t) + 2
\]

Answer: No, \( \Rightarrow \) **NONLINEAR**

f) let \( f_1(t) \rightarrow y_1(t) \) and \( f_2(t) \rightarrow y_2(t) \)

\[
\frac{dy_1(t)}{dt} + \sin(t)y_1(t) = \frac{df_1(t)}{dt} + 2f_1(t) \Rightarrow k_1\frac{dy_1(t)}{dt} + k_1\sin(t)y_1(t) = \frac{k_1df_1(t)}{dt} + 2k_1f_1(t)
\]

\[
\frac{dy_2(t)}{dt} + \sin(t)y_2(t) = \frac{df_2(t)}{dt} + 2f_2(t) \Rightarrow k_2\frac{dy_2(t)}{dt} + k_2\sin(t)y_2(t) = \frac{k_2df_2(t)}{dt} + 2k_2f_2(t)
\]

\[
\frac{d}{dt}(ky_1(t) + ky_2(t)) + \sin(t)[ky_1(t) + ky_2(t)] = \frac{d}{dt}[k_1f_1(t) + k_2f_2(t)] + 2[k_1f_1(t) + k_2f_2(t)]
\]

This is the same system response to an input of \( f(t) = k_1f_1(t) + k_2f_2(t) \)

\( \therefore \) **LINEAR**
#8. (1.7-2)

d) \( y(t) = t f(t-2) \). Since the coefficient of the input is a function of \( t \), (it is \( t \)), this system is time-varying.

e) \( y(t) = \int_{-5}^{5} f(\xi) d\xi \).

For a time-invariant system, if the input is delayed by \( T \) sec, then the output is the same as before but delayed by \( T \). Therefore, this system is time varying because as the input is delayed, the limits remain the same, so the output is different.

We can check this graphically. An integral gives the area under a curve:

![Graphs showing different areas](image.png)