Second Midterm, EE3701, Nov. 13, 2002

1. The two major sources of error in a delta modulator are granular noise and slope overload.
   a. Describe briefly what slope overload is.
      Ans: Slope overload occurs when the signal changes at a faster rate than the maximum rate of change of the delta modulator.
   b. Suppose that, at each modulator output time, the magnitude $d$ of the difference between the modulator output and the input signal $m(t)$ is uniformly distributed between 0 and the modulator step size, $\delta$. Find the granular noise power, i.e., the second moment of the error $d$, and then the signal-to-noise (power) ratio for granular noise assuming $m(t) = A\sin \omega t$.
      Ans: The granular noise power is
      \[
      N = \int_0^\delta x^2 \, dx = \frac{\delta^2}{3}
      \]
      and the signal power is
      \[
      \frac{1}{T} \int_T A^2 \sin^2(\omega t) \, dt = \frac{A^2}{2}
      \]
      where the integral is over one period. Then the signal to noise ratio is
      \[
      SNR = \frac{3A^2}{2\delta^2}
      \]

2. There are several basic functions performed in digital communication systems. Among them are, in alphabetical order, quantization, sampling, source encoding, and time-division multiplexing.
   a. Put these functions in the order in which they are performed in such systems.
      Ans: Sampling, quantizing, encoding, TDM
   b. Implementing these functions in practical systems encounters problems. Indicate a problem that arises for each.
      Ans: Sampling: signal never bandlimited so get aliasing. Quantizing: quantization error from discretization of a continuous signal. Encoding: minimum-length codes rarely achievable, if ever, in practice, and efficiency increases at expense of complexity. TDM: synchronization at receiver is usually difficult.

3. A source emits symbols from an alphabet consisting of four symbols A, B, C, and D, with probabilities .4, .4, .1, .1 respectively. An encoder uses Huffman encoding to produce binary codes sent over a noiseless channel that transmits at a rate of $10^6$ binary digits per second.
a. Give the Huffman encoding of the four symbols.
Ans: In the tree construction bottom-up combine the $p(C)$ and $p(D)$, then combine $p(B)$ with $p(C) + p(D)$, and finally combine $p(A)$ with $p(B) + p(C) + p(D)$. "Combine" means make them siblings of a parent node labeled with the sum of their probabilities. Now label edges out of each node with 0 and 1 in one order or the other, and finally read the codes for the letters by taking the path from the root down to the leaf corresponding to the letter. For example, one such code is A:0 B:10 C:110 D:111

b. Compute the average length of the code and determine the maximum symbol/sec rate at the input of the encoder, if the channel is to transmit within its given rate?
Ans: The average code length is 

$$ \frac{A \times 1 + A \times 2 + 1 \times 3 + 1 \times 3}{A} = 1.8 $$

The source rate must satisfy

$$ Source \ rate \leq \frac{10^6 \ bits/sec}{1.8 \ bits/symbol} = \frac{5}{9} \times 10^6 \ symbols/sec $$

c. What does Shannon’s source coding theorem say about this average code length? (The answer needs no calculations.)
Ans: The average code length must be at least the source entropy.

4. A source outputs 2 symbols called a dot and a dash. The source emits a dot in one time unit, but takes two time units to emit a dash. If the dash is half as probable as the dot, what is

a. the source entropy (bits/symbol)? (log 3 = 1.585)
Ans: The probabilities must be $p(dash) = 1/3$, $p(dot) = 2/3 = 2p(dash)$ so the entropy is

$$ H = \frac{1}{3} \log \frac{1}{1/3} + \frac{2}{3} \log \frac{1}{2/3} = \log 3 - \frac{2}{3} = .918 \ bits/time \ unit $$

b. the source information rate in bits per time unit?
Ans: Average symbol duration is $\frac{1}{2} \cdot 1 + \frac{1}{3} \cdot 2 = \frac{5}{3}$ time units, so the information rate is

$$ \frac{.918 \ bits/symbol}{4/3 \ time \ units/symbol} = .69 \ bits/time \ unit $$

5. The outputs of 10 PCM sources are multiplexed onto a single channel that can transmit one binary digit every 2 $\mu$sec. Each source emits digitized sound-wave signals quantized into 128 levels. In an ideal system, if the sound-wave signal bandwidth $W$ is small enough, quantization is the only
source of error in reconstructing the original signals at the receiver. How small must $W$ be?

Ans: Channel transmits 500 Kbits/sec which is

$$\frac{500 \text{ Kbits/sec}}{7 \text{ bits/sample}} = 71,430 \text{ samples/sec}$$

Each source gets 1/10 of them so each source gets 7,143 samples/sec which by sampling theorem must be at least twice the bandwidth of the signal, so

$$W \leq \frac{1}{2} \cdot 7,143 \text{ Hz}$$

or $W \leq 3.57 \text{ kHz}$