

# Image and Video Compression

#### Lecture 12, April 27th, 2009

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EE4830 Digital Image Processing http://www.ee.columbia.edu/~xlx/ee4830/

#### Announcements

- Evaluations on CourseWorks
  - please fill in and let us know what you think ☺
- Reminder for HW#6
  - you can choose between doing by hand or simple programming for problem 1 and problem 3
  - Problem 4 and 5 are optionally due next Wednesday May 6<sup>th</sup>

# outline

- image/video compression: what and why
- source coding basics
  - basic idea
  - symbol codes
  - stream codes
- compression systems and standards
  - system standards and quality measures
  - image coding JPEG
  - video coding and MPEG
  - audio coding (mp3) vs. image coding
- summary

### the need for compression

- Image: 6.0 million pixel camera, 3000x2000
  - 18 MB per image  $\rightarrow$  56 pictures / 1GB
- Video: DVD Disc 4.7 GB
  - video 720x480, RGB, 30 f/s  $\rightarrow$  31.1MB/sec
  - audio 16bits x 44.1KHz stereo  $\rightarrow$  176.4KB/s
    - $\rightarrow$  1.5 min per DVD disc
- Send video from cellphone: 352\*240, RGB, 15 frames / second
  - 3.8 MB/sec  $\rightarrow$  \$38.00/sec levied by AT&T

## Data Compression

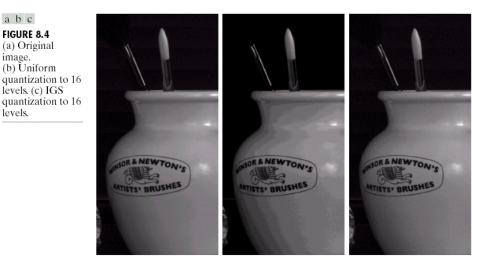
- Wikipedia: "data compression, or source coding, is the process of encoding information using fewer bits (or other information-bearing units) than an unencoded representation would use through use of specific encoding schemes."
- Applications
  - General data compression: .zip, .gz ...
  - Image over network: telephone/internet/wireless/etc
  - Slow device:
    - 1xCD-ROM 150KB/s, bluetooth v1.2 up to ~0.25MB/s
  - Large multimedia databases

Understanding compression:

- what are behind jpeg/mpeg/mp4 ... formats?
- what are the "good/fine/super fine" quality modifiers in my Canon 400D?
- why/when do I want to use raw/jpeg format in my digital camera?
- why doesn't "zipping" jpeg files help?
- what are the best ways to do compression?
- are we doing our best? (yes/no/maybe)

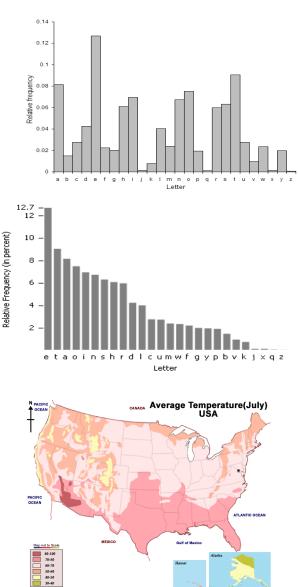
### what can we compress?

- Goals of compression
  - Remove redundancy
  - Reduce irrelevance
- irrelevance or perceptual redundancy
  - not all visual information is perceived by eye/brain, so throw away those that are not.



### what can we compress?

- Goals of compression
  - Remove redundancy
  - Reduce irrelevance
- redundant : exceeding what is necessary or normal
  - symbol redundancy
    - the common and uncommon values cost the same to store
  - spatial and temporal redundancy
    - Temperatures: tend to be similar in adjacent geographical areas, also tend to be similar in the same month over different years …



### symbol/inter-symbol redundancy

в

Ħ

N

T -

- Letters and words in English
  - e, a, i, s, t, ...
     q, y, z, x, j, ...
  - a, the, me, I ...
     good, magnificent, ...
  - fyi, btw, ttyl ...
- In the evolution of language we naturally chose to represent frequent meanings with shorter representations.

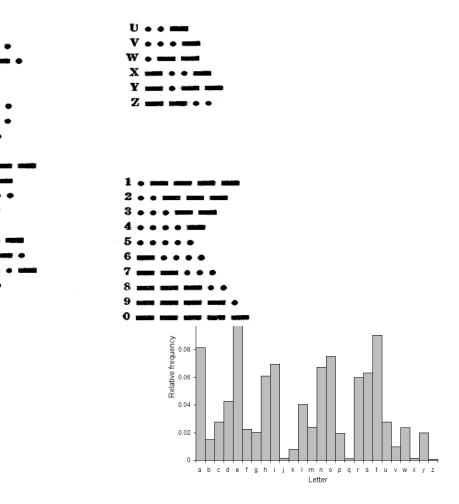
#### INTERNATIONAL MORSE CODE

1. A dash is equal to three dots.

2. The space between parts of the same letter is equal to one dot.

3. The space between two letters is equal to three dots.

4. The space between two words is equal to five dots.



## redundancy in an image

- Symbol redundancy in an image:
  - Some gray level value are more probable than others.
- Spatial redundancy:
  - Pixel values are not i.i.d.



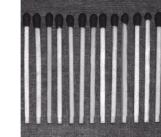
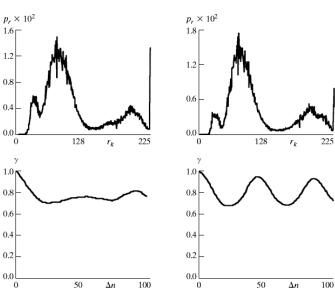




FIGURE 8.2 Two images and their gray-level histograms and normalized autocorrelation coefficients along one line.

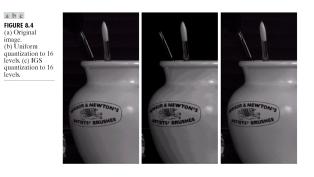


# modes of compression

- Lossless
  - preserve all information, perfectly recoverable
  - examples: morse code, zip/gz



- throw away perceptually insignificant information
- cannot recover all bits





## how much can we compress a picture?

MG_0470.jpg	944 KB	JPEG Image	11/11/2006 4:29 PM	11/11/2006	3:29 PM	1600 x 1200
🔟 IMG_0471.jpg	892 KB	JPEG Image	11/11/2006 4:30 PM	11/11/2006	3:30 PM	1600 x 1200
💼 IMG_0472.jpg	876 KB	JPEG Image	11/11/2006 4:31 PM	11/11/2006	3:31 PM	1600 x 1200
IMG_0473.jpg	1,214 KB	JPEG Image	11/11/2006 4:38 PM	11/11/2006	3:38 PM	1600 x 1200
💼 IMG_0474.jpg	1,117 KB	JPEG Image	11/11/2006 4:38 PM	11/11/2006	3:38 PM	1600 x 1200
💼 IMG_0475.jpg	1,208 KB	JPEG Image	11/11/2006 4:38 PM	11/11/2006	3:38 PM	1600 x 1200
1MG_0476.jpg	795 KB	JPEG Image	11/11/2006 4:39 PM	11/11/2006	3:39 PM	1600 x 1200
1MG_0477.jpg	1,042 KB	JPEG Image	11/11/2006 4:39 PM	11/11/2006	3:39 PM	1600 x 1200
💼 IMG_0478.jpg	1,027 KB	JPEG Image	11/11/2006 4:40 PM	11/11/2006	3:40 PM	1600 x 1200
💼 IMG_0479.jpg	1,010 KB	JPEG Image	11/11/2006 4:40 PM	11/11/2006	3:40 PM	1600 x 1200
💼 IMG_0480.jpg	790 KB	JPEG Image	11/11/2006 4:41 PM	11/11/2006	3:41 PM	1600 x 1200
1MG_0481.jpg	959 KB	JPEG Image	11/11/2006 4:41 PM	11/11/2006	3:41 PM	1600 x 1200
1MG_0482.jpg	1,073 KB	JPEG Image	11/11/2006 4:42 PM	11/11/2006	3:42 PM	1600 x 1200
💼 IMG_0483.jpg	990 KB	JPEG Image	11/11/2006 4:43 PM	11/11/2006	3:43 PM	1600 x 1200
💼 IMG_0484.jpg	1,046 KB	JPEG Image	11/11/2006 4:45 PM	11/11/2006	3:45 PM	1600 x 1200
💼 IMG_0485.jpg	878 KB	JPEG Image	11/11/2006 4:46 PM	11/11/2006	3:46 PM	1600 x 1200
💼 IMG_0486.jpg	774 KB	JPEG Image	11/11/2006 4-46 DM		2.46 DM	1600 - 1000
💼 IMG_0487.jpg	830 KB	JPEG Image	Explore / Interesting	jness /		
💼 IMG_0488.jpg	1,011 KB	JPEG Image				
1MG_0489.jpg	957 KB	JPEG Image	« March 2008		April 2008	
1MG_0490.jpg	961 KB	JPEG Image	SUN MON	TUE	WED	THU FF

same dimensions (1600x1200), same original representation -- 3 bytes/pixel, 5.76MB uncompressed, same compressed representation (JPEG), same viewer sensitivity and subjective quality ...

different "information content" in each image!



May 2008 »

# characterizing information

#### i.i.d. random variable x

An ensemble X is a triple  $(x, \mathcal{A}_X, \mathcal{P}_X)$ , where the *outcome* x is the value of a random variable, which takes on one of a set of possible values,  $\mathcal{A}_X = \{a_1, a_2, \ldots, a_i, \ldots, a_I\}$ , having probabilities  $\mathcal{P}_X = \{p_1, p_2, \ldots, p_I\}$ , with  $P(x = a_i) = p_i, p_i \ge 0$  and  $\sum_{a_i \in \mathcal{A}_X} P(x = a_i) = 1$ .

#### information content

- characterize the surprising-ness
- related to probability
- additive for independent variables.

$$h(x = a_i) = \log_2 \frac{1}{p_i}$$

#### explanations

- cross-words: how many words have you "ruled out" after knowing that a word starts with an "a" or with a "z" ?
- #"a\*" 35,174 words #"z\*" 1,718 words English vocabulary: ~500K words

i	$a_i$	$p_i$	$h(p_i)$	
1	a	.0575	4.1	
2	b	.0128	6.3	
3	С	.0263	5.2	
4	d	.0285	5.1	
<b>5</b>	е	.0913	3.5	
6	f	.0173	5.9	
$\overline{7}$	g	.0133	6.2	
8	h	.0313	5.0	
9	i	.0599	4.1	
10	j	.0006	10.7	
11	k	.0084	6.9	
12	1	.0335	4.9	
13	m	.0235	5.4	
14	n	.0596	4.1	
15	0	.0689	3.9	
16	р	.0192	5.7	
17	q	.0008	10.3	
18	r	.0508	4.3	
19	s	.0567	4.1	
20	t	.0706	3.8	
21	u	.0334	4.9	
22	v	.0069	7.2	
23	W	.0119	6.4	
24	x	.0073	7.1	
25	у	.0164	5.9	
26	z	.0007	10.4	
27	-	.1928	2.4	
Σ	$\sum_{i} p_i$	$\log_2 \frac{1}{p_i}$	4.1	

FindTheWord.info

#### information content and entropy

 $H_2(p)$ 

0.011

0.081

0.47

0.72

1.0

Shannon information content

h(x = a<sub>i</sub>) = log<sub>2</sub> 1/p<sub>i</sub>

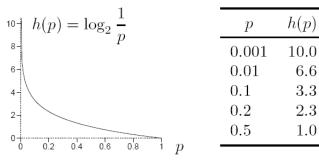
additive for independent variables:

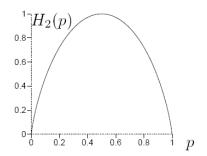
h(x = a<sub>i</sub>, y = a<sub>j</sub>) = log<sub>2</sub> 1/p<sub>i</sub>p<sub>j</sub>
h(x = a<sub>i</sub>) + h(y = a<sub>j</sub>)

Entropy: expected information content

H(X) = E{h(x)} = ∑a<sub>i</sub>∈Ax p<sub>i</sub> log<sub>2</sub> 1/p<sub>i</sub>

i	$a_i$	$p_i$	$h(p_i)$
1	a	.0575	4.1
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8	h	.0313	5.0
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Σ	$\sum_{i} p_i$	$\log_2 \frac{1}{p_i}$	4.1





#### source coding

consider ensemble  $X : (x, \mathcal{A}_x, \mathcal{P}_x)$ 

- source code  $c(x) : \mathcal{A}_x$
- length of a codeword

$$c(x): \mathcal{A}_x 
ightarrow \mathcal{C}_x$$
  
 $l(c(x)), \ x \in \mathcal{A}_x$ 

expected length of a code

$$L(C,X) = \sum_{a_i \in \mathcal{A}_x} p_i l(c(a_i))$$

an example

$$\mathcal{A}_{x} = \{a, b, c, d\} \qquad \begin{array}{lll} P(X = a) &=& 1/2 & C(a) &=& 1 \\ P(X = b) &=& 1/4 & C(b) &=& 10 \\ P(X = c) &=& 1/8 & C(c) &=& 110 \\ P(X = d) &=& 1/8 & C(d) &=& 111 \end{array}$$

H(X) = ?, L(C, X) = ?

#### source coding theorem

Source coding theorem -

N outcomes from a source X can be compressed into roughly NH(X) bits.

Proved by counting the typical set

When a source X produces N independent outcomes  $\mathbf{x} = x_1 x_2 \dots x_N$ this string is very likely to be one of the  $\sim 2^{NH(X)}$  typical outcomes all of which have probability  $\sim 2^{-NH(X)}$ 

informal shorthand:  $L(C, X) \ge H(X)$ 

[Shannon 1948]

#### revisiting Morse code

i  $a_i$   $p_i$   $h(p_i)$ 

#### INTERNATIONAL MORSE CODE

		1	a	.0575	4.1
1. A dash is equal to three dots.		2	b	.0128	6.3
2. The space between parts of the sa	ame letter is equal to one dot.	3	С	.0263	5.2
3. The space between two letters is	-	4	d	.0285	5.1
<ol><li>The space between two words is</li></ol>	equal to five dots.	5	е	.0913	3.5
		6	f	.0173	5.9
		7	g	.0133	6.2
		8	h	.0313	5.0
B	V • • • ====	9	i	.0599	4.1
	W •	10	j	.0006	10.7
D	X	11	k	.0084	6.9
E •	Y	12	1	.0335	4.9
		13	m	.0235	5.4
F • • mm •		14	n	.0596	4.1
G		15	0	.0689	3.9
H • • • •		16	р	.0192	5.7
1		17	q	.0008	10.3
3		$\frac{18}{19}$	r	.0508 .0567	4.3
•••		20	s	.0567 .0706	$4.1 \\ 3.8$
K	1 •	$\frac{20}{21}$	t u	.0700.0334	$\frac{3.8}{4.9}$
L • • • •	2 • • • • • • • • • • •	$\frac{21}{22}$	u V	.0334 .0069	$\frac{4.9}{7.2}$
M	3	22	w	.0009.0119	6.4
N mme +	A	23 24	x	.0073	7.1
0		$\frac{2}{25}$	у	.0164	5.9
	5	26	z	.0007	10.4
P • mm · · · ·	6	$\frac{20}{27}$	_	.1928	2.4
Q man man + unit	7 • • • •				
R • • •	8	7	$\sum p_i$	$\log \frac{1}{2}$	4.1
Seee	9	2	$P_i$	$\frac{\log_2}{p_i}$	4.1
T -	0		e		

 $L(C,X) = \sum_{a_i} p_i l(c(a_i)) \qquad H(X) = \sum_i p_i \log_2 \frac{1}{p_i}$ 

Left as excercise

# desired properties of symbol codes

- good codes are not only short but also easy to encode/decode
  - Non-singular: every symbol in X maps to a different code word
  - Uniquely decodable: every sequence {x<sub>1</sub>, ... x<sub>n</sub>} maps to different codeword sequence
  - Instantaneous: no codeword is a prefix of any other codeword

#### English in less than 26 letters (just kidding)

The European Union commissioners have announced that agreement has been reached to adopt English as the preferred language for European communications, rather than German, which was the other possibility. As part of the negotiations, Her Majesty's Government conceded that English spelling had some room for improvement and has accepted a five-year phased plan for what will be known as Euro-English (Euro for short).

In the first year, 's' will be used instead of the soft 'c'. Sertainly, sivil servants will resieve this news with joy. Also, the hard 'c' will be replaced with 'k.' Not only will this klear up konfusion, but typewriters kan have one less letter.

There will be growing publik enthusiasm in the sekond year, when the troublesome 'ph' will be replaced by 'f'. This will make words like 'fotograf' 20 per sent shorter.

In the third year, publik akseptanse of the new spelling kan be expekted to reach the stage where more komplikated changes are possible. Governments will enkourage the removal of double letters, which have always ben a deterent to akurate speling. Also, al wil agre that the horible mes of silent 'e's in the languag is disgrasful, and they would go.

By the fourth year, peopl wil be reseptiv to steps such as replasing 'th' by 'z' and 'W' by 'V'. During ze fifz year, ze unesesary 'o' kan be dropd from vords kontaining 'ou', and similar changes vud of kors; be aplid to ozer kombinations of leters.

After zis fifz yer, ve vil hav a reli sensibl riten styl. Zer vil b no mor trubls or difikultis and evrivun vil find it ezi tu understand ech ozer. Ze drem vil finali kum tru.

### desired properties of symbol codes

- Non-singular: every symbol in X maps to a different code word
- Uniquely decodable: every sequence {x<sub>1</sub>, ... x<sub>n</sub>} maps to different codeword sequence
- Instantaneous: no codeword is a prefix of any other codeword

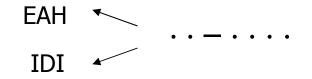
3. The space between two lett 4. The space between two wor	
K	

INTERNATIONAL MORSE CODE

2. The space between parts of the same letter is equal to one dot.

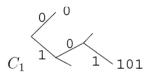
1. A dash is equal to three dots.

Morse code without blanks:

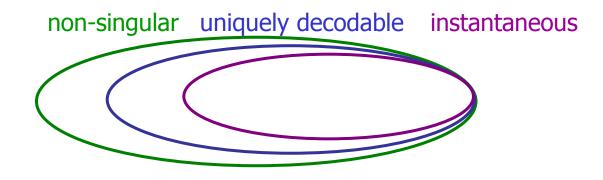


### desired properties of symbol codes

- Non-singular: every symbol in X maps to a different code word
- Uniquely decodable: every sequence {x<sub>1</sub>, ... x<sub>n</sub>} maps to different codeword sequence
- Instantaneous: no codeword is a prefix of any other codeword a.k.a. prefix code, self-punctuating code, prefix-free code.
  - Example 5.4. The code  $C_1 = \{0, 101\}$  is a prefix code because 0 is not a prefix of 101, nor is 101 a prefix of 0.



Example 5.5. Let  $C_2 = \{1, 101\}$ . This code is not a prefix code because 1 is a prefix of 101.



good news: being unique decodable + instantaneous do not compromise coding efficiency (much)

The optimal symbol code's expected length L satisfies  $H(X) \le L < H(X) + 1$ 

## Huffman codes

#### optimal symbol code by construction

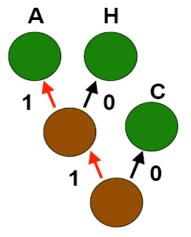
Binary (Huffman) tree

Represents Huffman code

• Edge  $\Rightarrow$  code (0 or 1)

- Leaf ⇒ symbol
- Path to leaf ⇒ encoding
- Example

■ A = "11", H = "10", C = "0"



#### Encoding

- 1. Calculate frequency of symbols in file
- 2. Create binary tree representing "best" encoding
- 3. Use binary tree to encode compressed file
  - For each symbol, output path from root to leaf
  - Size of encoding = length of path
- 4. Save binary tree

#### construct Huffman codes

#### a recursive algorithm in two steps

- 1. Take the two least probable symbols in the alphabet. These two symbols will be given the longest codewords, which will have equal length, and differ only in the last digit.
- 2. Combine these two symbols into a single symbol, and repeat.

#### Example 1

$$\mathcal{A}_{X} = \{ a, b, c, d, e \}$$
  

$$\mathcal{P}_{X} = \{ 0.25, 0.25, 0.2, 0.15, 0.15 \}.$$
  
x step 1 step 2 step 3 step 4  
a  $0.25 - 0.25 - 0.25 - 0.25 - 0.55 - 0 = 1.0$   
b  $0.25 - 0.25 - 0.45 - 0.45 - 1 = 0.45 - 1$   
c  $0.2 - 0.2 = 0.2 - 1 = 0.3$ 

$a_i$	$p_i$	$h(p_i)$	$l_i$	$c(a_i)$
a	0.25	2.0	2	00
Ъ	0.25	2.0	2	10
С	0.2	2.3	2	11
d	0.15	2.7	3	010
е	0.15	2.7	3	011

Table 5.5. Code created by theHuffman algorithm.

Code created by greedy top-down splits

00
01
10
110
111

We have  $H(X) \le L(C) \le H(X)+1$ 

H(X)=2.2855, L(C)=2.30

# Greedy division can be suboptimal

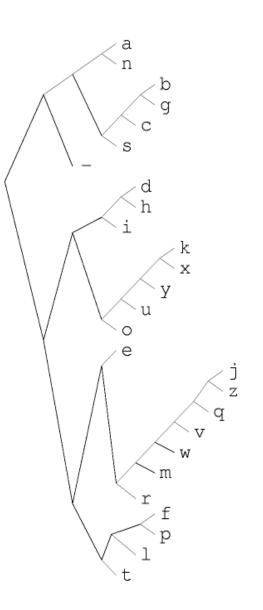
**example 2** Find the optimal binary symbol code for the ensemble:

$$\mathcal{A}_X = \{ a, b, c, d, e, f, g \} \mathcal{P}_X = \{ 0.01, 0.24, 0.05, 0.20, 0.47, 0.01, 0.02 \}$$
(5.24)

$a_i$	$p_i$	Greedy	Huffman
а	.01	000	000000
Ъ	.24	001	01
С	.05	010	0001
d	.20	011	001
е	.47	10	1
f	.01	110	000001
g	.02	111	00001

Table 5.7. A greedily-constructed code compared with the Huffman code.

$a_i$	$p_i$	$\log_2 \frac{1}{p_i}$	$l_i$	$c(a_i)$
a	0.0575	4.1	4	0000
Ъ	0.0128	6.3	6	001000
с	0.0263	5.2	5	00101
d	0.0285	5.1	5	10000
е	0.0913	3.5	4	1100
f	0.0173	5.9	6	111000
g	0.0133	6.2	6	001001
h	0.0313	5.0	5	10001
i	0.0599	4.1	4	1001
j	0.0006	10.7	10	1101000000
k	0.0084	6.9	$\overline{7}$	1010000
1	0.0335	4.9	5	11101
m	0.0235	5.4	6	110101
n	0.0596	4.1	4	0001
о	0.0689	3.9	4	1011
Р	0.0192	5.7	6	111001
q	0.0008	10.3	9	110100001
r	0.0508	4.3	5	11011
s	0.0567	4.1	4	0011
t	0.0706	3.8	4	1111
u	0.0334	4.9	5	10101
v	0.0069	7.2	8	11010001
W	0.0119	6.4	$\overline{7}$	1101001
х	0.0073	7.1	$\overline{7}$	1010001
У	0.0164	5.9	6	101001
z	0.0007	10.4	10	1101000001
_	0.1928	2.4	2	01



4

3h.

## why do we need stream codes

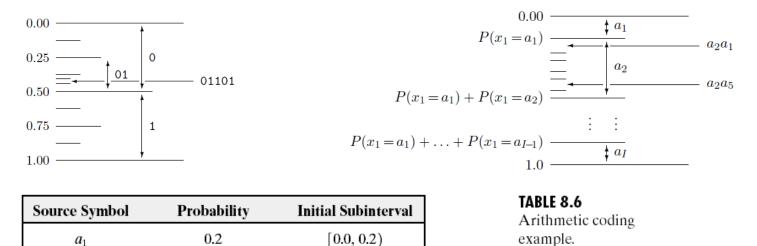
- Huffman code is optimal but must be integer length.
   each symbol x → codeword c(x)
- the interval [H(X), H(X)+1) can be loose.
- consider the following optimal symbol code:

а	0.001	00000			
b	0.001	00001			
С	0.990	1			
d	0.001	00010			
е	0.001	00011			
f	0.001	0100			
g	0.001	0101	expected	length	1.034
h	0.001	0110	entropy		0.11401
i	0.001	0111	Спстору		0.11401
j	0.001	0010	length /	entropy	9
k	0.001	0011			

Stream code: mapping a stream of symbols X:  $x_1x_2$ , ...,  $x_n \rightarrow$  codeword c(X)

### arithmetic coding

Observation #1: Symbol distributions split the [0,1) into intervals.



[0.2, 0.4)

[0.4, 0.8)

[0.8, 1.0)

Observation #2: interval division can be recursive.

0.2

0.4

0.2

 $a_2$ 

 $a_3$ 

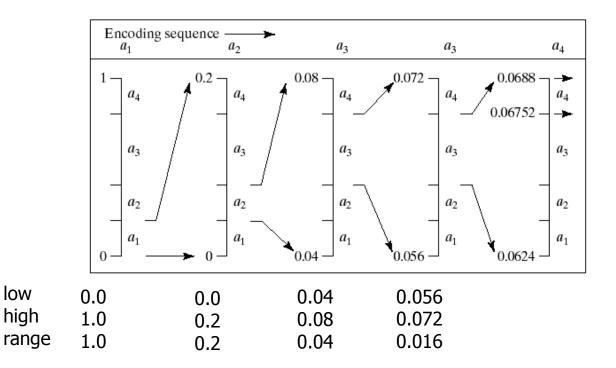
 $a_{A}$ 

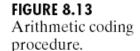
Arithmetic coding can treats the whole message as one unit. A message is represented by a half-open interval [a; b) where a and b are real numbers between 0 and 1. Initially, the interval is [0; 1). When the message becomes longer, the length of the interval shortens and the number of bits needed to represent the interval increases.

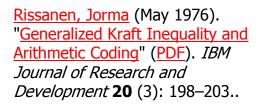
### Arithmetic Encoding Agorithm

```
BEGIN
low = 0.0; high = 1.0; range = 1.0;
while (symbol != terminator)
{
    get (symbol);
    low = low + range * Range_low(symbol);
    high = low + range * Range_high(symbol);
    range = high - low;
    }
output a code so that low <= code < high;
END</pre>
```

Source Symbol	Probability	Initial Subinterval
$a_1$	0.2	[0.0, 0.2)
$a_2$	0.2	[0.2, 0.4)
$a_3$	0.4	[0.4, 0.8)
$a_4$	0.2	[0.8, 1.0)







### universal data compression

What if the symbol probabilities are unknown?LZW algorithm (Lempel-Ziv-Welch)

#### encoding

```
w = NIL;
while ( read a character k )
{
    if wk exists in the dictionary
    w = wk;
    else
        add wk to the dictionary;
        output the code for w;
        w = k;
    }
```

#### decoding

```
read a character k;
output k;
w = k;
while ( read a character k )
/* k could be a character or a code. */
{
    entry = dictionary entry for k;
    output entry;
    add w + entry[0] to dictionary;
    w = entry;
}
```

- Widely used: GIF, TIFF, PDF ...
- Its royalty-free variant (DEFLATE) used in PNG, ZIP, ...
  - Unisys U.S. LZW Patent No. 4,558,302 expired on June 20, 2003 <u>http://www.unisys.com/about\_unisys/lzw</u>

#### LZW

#### Example

39

126

393912612639391261263939126126

126

39

Currently Recognized Sequence	Pixel Being Processed	Encoded Output	Dictionary Location (Code Word)	Dictionary Entry	<b>TABLE 8.7</b> LZW codi example.
	39				
39	39	39	256	39-39	
39	126	39	257	39-126	
126	126	126	258	126-126	
126	39	126	259	126-39	
39	39				
39-39	126	256	260	39-39-126	
126	126				
126-126	39	258	261	126-126-39	
39	39				
39-39	126				
39-39-126	126	260	262	39-39-126-126	
126	39				
126-39	39	259	263	126-39-39	
39	126				
39-126	126	257	264	39-126-126	
126		126			

 Exercise: verify that the dictionary can be automatically reconstructed during decoding. (G&W Problem 8.20)

### **Run-Length Coding**

- Encode the number of consecutive `0's or `1's
- Used in FAX transmission standard
- Why is run-length coding with p = P(X=0) >> P(X=1) actually beneficial?
  - See Jain Sec 11.3 (at courseworks)

probability of a run

average run-length

 $g(l) = \begin{cases} p^{l}(1-p), & 0 \le l \le M-1\\ p^{M}, & l = M \end{cases}$  $\mu_{l} = \frac{1-p^{M}}{1-p} \qquad \text{e.g.}\\ p=0.9\\ M=15 \end{cases}$ 

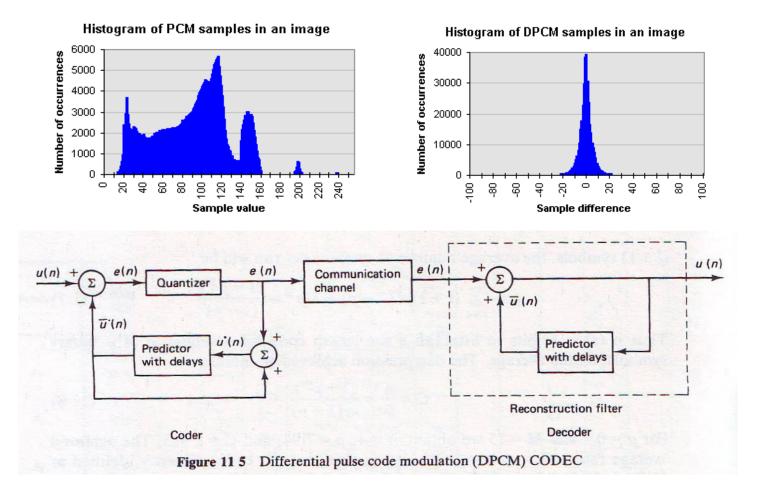
compression ratio $m = \log_2 M$ 

$$C = \frac{\mu_l}{m} = \frac{1 - p^M}{m(1 - p)}$$
 M=15  

$$\mu = 7.94$$
  
C=1.985

## **Predictive Coding**

- Signals are correlated → predict and encoding the difference lowers the bitrate
- Good prediction is the key: e.g. LPC (linear-predctive) speech coding



# outline

- image/video compression: what and why
- source coding basics
  - basic idea
  - symbol codes
  - stream codes
- compression systems and standards
  - system standards and quality measures
  - image coding and JPEG
  - video coding and MPEG
  - audio coding (mp3) vs. image coding
- summary

#### measuring image quality

- Quality measures
  - PSNR (Peak-Signal-to-Noise-Ratio) PSNR = 10
    - Why would we prefer PSNR over SNR?
  - Visual quality
    - Compression Artifacts
    - Subjective rating scale

TABLE 8.3	
-----------	--

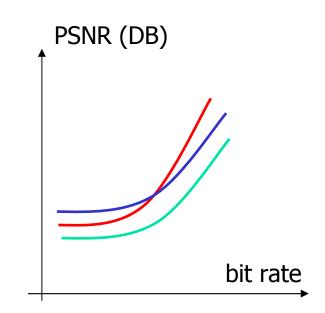
Rating scale of the Television Allocations Study Organization. (Frendendall and Behrend.)

Value	Rating	Description		
1	Excellent	An image of extremely high quality, as good as you could desire.		
2	Fine	An image of high quality, providing enjoyable viewing. Interference is not objectionable.		
3	Passable	An image of acceptable quality. Interference is not objectionable.		
4	Marginal	An image of poor quality; you wish you could improve it. Interference is somewhat objectionable.		
5	Inferior	A very poor image, but you could watch it. Objectionable interference is definitely present.		
6	Unusable	An image so bad that you could not watch it.		

$$D\log_{10}\left[\frac{255^{2}}{\frac{1}{MN}\sum_{xy}|f'(x,y)-f(x,y)|^{2}}\right]$$

### measuring coding systems

- End-to-end measures of source coding system: Rate-Distortion
- Other considerations
  - Computational complexity
  - Power consumption
  - Memory requirement
  - Delay
  - Error resilience/sensitivity
  - Subjective quality



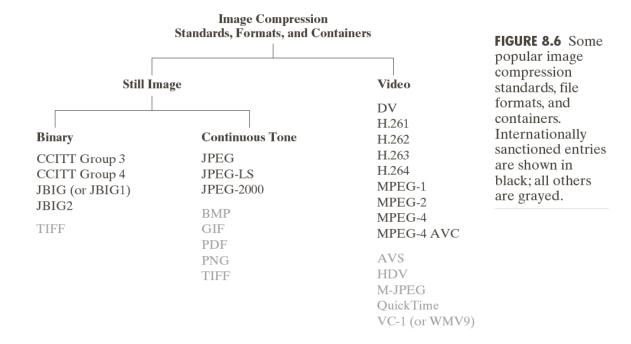
bpp: bit-per-pixel; Kbps: Kilo-bits-per-second

# Image/Video Compression Standards

- Bitstream useful only if the recipient knows the code!
- Standardization efforts are important
  - Technology and algorithm benchmark
  - System definition and development
  - Patent pool management

Defines the bitstream (decoder), not how you generate them (encoder)!

v•d•e	Multimedia compression formats [hid					
Video	ISO/IEC	ITU-T	Others			
compression formats	MPEG-1 · MPEG-2 · MPEG-4 ASP · H.261 · H.262 · H.263 · MPEG-4/AVC H.264		AVS • Bink • Dirac • Indeo • MJPEG • RealVideo • Theora • VC-1 • VP6 • VP7 • WMV			
Audio compression formats	ISO/IEC MPEG	ITU-T	Others			
	MPEG-1 Layer III (MP3) • MPEG-1 Layer II • AAC • HE-AAC		AC3 · Apple Lossless · ATRAC · FLAC · iLBC · Monkey's Audio · µ-law · Musepack · Nellymoser · RealAudio · SHN Speex · Vorbis · WavPack · WMA			
Image	ISO	/IEC/ITU-T	Others			
compression formats		ess JPEG · JBIG · JBIG2 · PNG · A	NG · ICER · MNG · BMP · GIF · ILBM · PCX · TGA · TIFF · HD Photo			
Media container formats	General					
	3GP · ASF · AVI · DMF · DPX · FLV · Matroska · MP4 · MXF · NUT · Ogg · Ogg Media · QuickTime · RealMedia · VOB			AIFF · AU · WAV		



current industry focus:

H.264 encoding/decoding on mobile devices, low-latency video transmission over various networks, low-power video codec ...



#### Digital TV Patent License Fees to Go to Columbia Very Soon

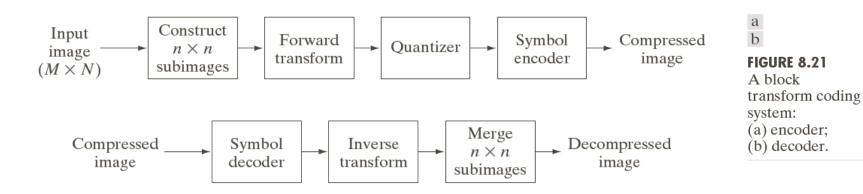
#### By Bob Nelson

igital television is on its way and Columbia, the only academic institution in the patent pool created to license the MPEG-2 digital video compression standard, expects to begin receiving license fees from the technology as early as this year.

Columbia and eight companies together hold 33 patents that now comprise MPEG-2, which allows the transmission of high-quality video and audio signals over limited bandwidth. Dimitris Anastassiou, professor of electrical engineering at Columbia's School of Engineering and Applied Science and director of the Columbia New Media Technology Center, developed one of the MPEG-2 compression technologies with one of his graduate students.

"We believe the patent pool approach offers Columbia an excellent opportunity to receive significant royalty payments over the next few years," said Jack Granowitz, executive director of the Columbia Innovation Enterprise (CIE), the University's technology licensing office. Granowitz, along with

## block-based transform coding systems

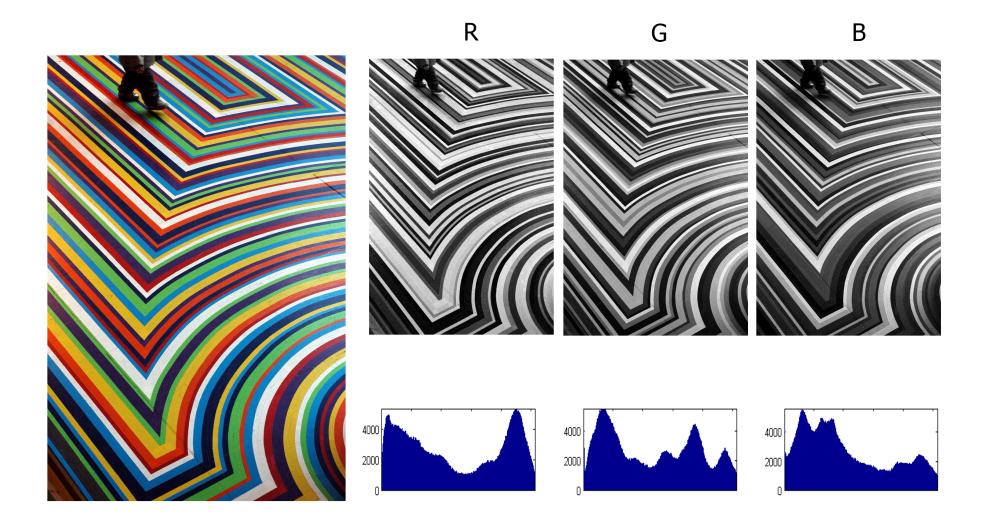


- Review: properties of unitary transforms/DCT
  - De-correlation: highly correlated input elements → quite uncorrelated output coefficients
  - Energy compaction: many common transforms tend to pack a large fraction of signal energy into just a few transform coefficients
- Symbol coding/decoding
  - predictive coding
  - run-length coding
  - Huffman codes
  - adaptive arithmetic coding ...

#### JPEG compression standard (early 1990s)

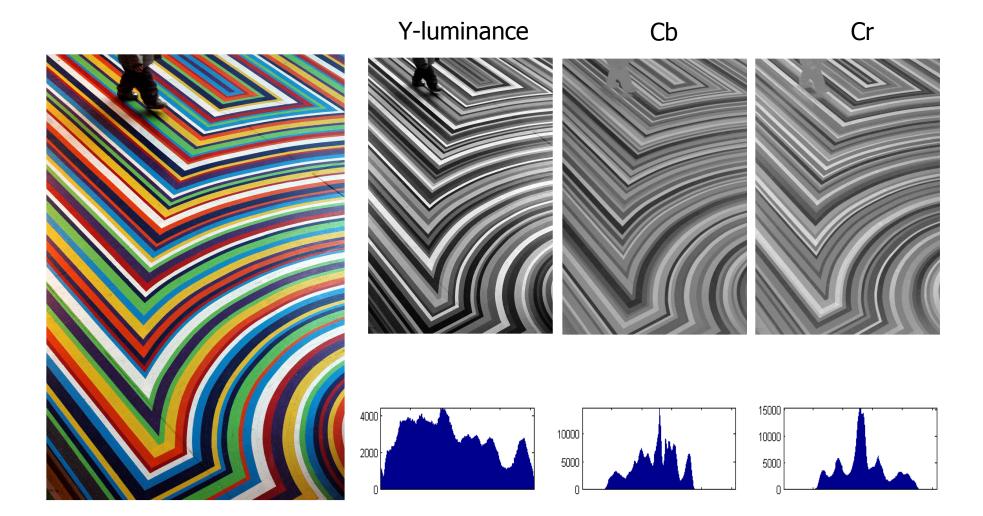
- JPEG Joint Photographic Experts Group
  - Compression standard of generic continuous-tone still image
  - Became an international standard in 1992
- Allow for lossy and lossless encoding of still images
  - Part-1 DCT-based lossy compression
    - average compression ratio 15:1
  - Part-2 Predictive-based lossless compression
- Sequential, Progressive, Hierarchical modes
  - Sequential: encoded in a single left-to-right, top-to-bottom scan
  - Progressive: encoded in multiple scans to first produce a quick, rough decoded image when the transmission time is long
  - Hierarchical: encoded at multiple resolution to allow accessing low resolution without full decompression

### color representation in JPG



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#### color representation in JPG



assign more bits to Y, less bits to Cb and Cr

# baseline JPEG algorithm

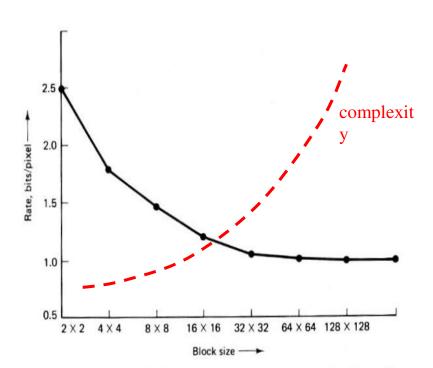
- "Baseline"
  - Simple, lossy compression
    - Subset of other DCT-based modes of JPEG standard
- A few basics
  - 8x8 block-DCT based coding
  - Shift to zero-mean by subtracting 128 → [-128, 127]
    - Allows using signed integer to represent both DC and AC coeff.
  - Color (YCbCr / YUV) and downsample
    - Color components can have lower spatial resolution than luminance
  - Interleaving color components

$$\begin{bmatrix} Y \\ C_b \\ C_r \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ -0.147 & -0.289 & 0.436 \\ 0.615 & -0.515 & -0.100 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

(Based on Wang's video book Chapt.1)

# block-based transform

- Review: why transform?
  - Compacting energy
  - Data de-correlation
- Why block based?
  - High transform computation complexity for larger blocks
    - O( m log m × m ) per block in transform for (MN/m<sup>2</sup>) blocks
  - High complexity in bit allocation
  - Block transform captures local info
- Commonly used block sizes: 8x8, 16x16, 8x4, 4x8 ...



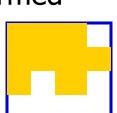
**Figure 11.16** Rate achievable by block KL transform coders for Gaussian random fields with separable covariance function,  $\rho = \rho_2 = 0.95$ , at distortion D = 0.25%.

From Jain's Fig.11.16

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# zonal coding and threshold coding

- Zonal coding
  - Only transmit a small predetermined zone of transformed coeff.
- Threshold coding
  - Transmit coeff. that are above certain thresholds
- Compare
  - Threshold coding is inherently adaptive
    - introduce smaller distortion for the same number of coded coeff.
  - Threshold coding needs overhead in specifying index of coded coeff.
    - run-length coding helps to reduce overhead





# perform quantization

- Input:
  - 8x8 DCT image X(u,v)
  - Quantization table Q(u,v)
- The quantizer output is: I(u,v)=Round[X(u,v)/Q(u,v)]
  - "round" is to the nearest integer
- JPEG default luminance table shown on the right
  - Smaller Q(u,v) means a smaller step size and hence more resolution, vice-versa
  - Q(u,v) may be scaled by a quality factor

16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

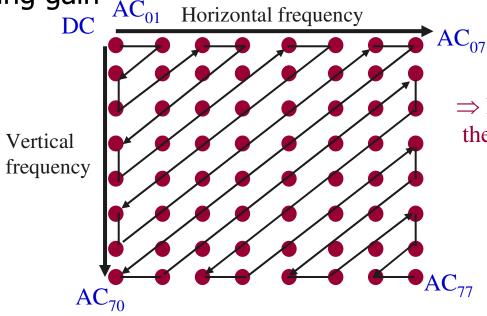
## quantization of transform coefficients

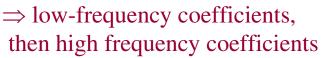
- Default quantization table
  - "Generic" over a variety of images
- Adaptive Quantization (bit allocation)
  - Different quantization step size for different coeff. bands
  - Use same quantization matrix for all blocks in one image
  - Choose quantization matrix to best suit the image
  - Different quantization matrices for luminance and color components
- Quality factor "Q"
  - Scale the quantization table
  - Medium quality  $Q = 50\% \sim no$  scaling
  - High quality  $Q = 100\% \sim unit quantization step size$
  - Poor quality ~ small Q, larger quantization step
    - visible artifacts like ringing and blockiness



# encode an image block

- Basic tools
  - Run-length coding
  - Predictive coding (esp. for DC coefficient)
  - Entropy coding (Huffman, etc.)
- Scan order
  - zig-zag scan for block-DCT to better achieve run-length coding gain ACon Horizontal frequency





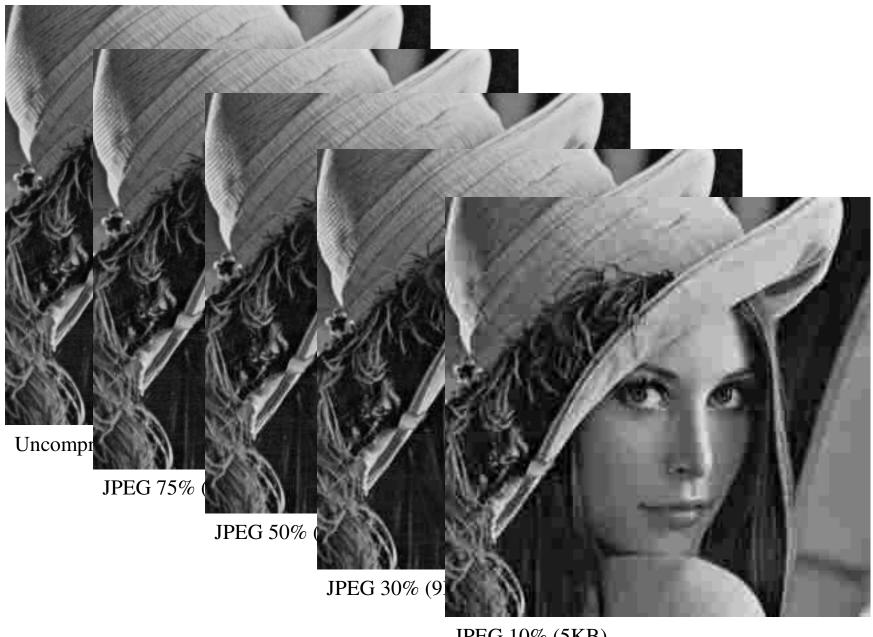
# encoding a block in JPEG

- Differentially encode DC (and quantize)
  - (SIZE, AMPLITUDE), with amplitude range in [-2048, 2047]
- AC coefficients in one block
  - Zig-zag scan after quantization for better run-length
    - save bits in coding consecutive zeros
  - Represent each AC run-length using entropy coding
    - use shorter codes for more likely AC run-length symbols
    - Symbol-1: ( RUNLENGTH, SIZE )  $\rightarrow$  Huffman coded
    - Symbol-2: AMPLITUDE → Variable length coded

```
RUNLENGTH \in [0,15]
```

# of consecutive zero-valued AC coefficients preceding the nonzero AC coefficient  $\in [0,15]$ 

```
AMPLITUDE \in in range of [-1023, 1024]
```



JPEG 10% (5KB)

# JPEG Compression (Q=96, 75 & 25) <sup>50</sup>



644 KB

239 KB

55 KB

# JPEG 2000

- Better image quality/coding efficiency, esp. low bit-rate compression performance
  - DWT
  - Bit-plane coding (EBCOT)
  - Flexible block sizes
  - ...
- More functionality
  - Support larger images
  - Progressive transmission by quality, resolution, component, or spatial locality
  - Lossy and Lossless compression
  - Random access to the bitstream
  - Region of Interest coding
  - Robustness to bit errors

### Video ?= Motion Pictures

- Capturing video
  - Frame by frame => image sequence
  - Image sequence: A 3-D signal
    - 2 spatial dimensions & time dimension
    - continuous I(x, y, t) => discrete  $I(m, n, t_k)$
- Encode digital video
  - Simplest way ~ compress each frame image individually
    - e.g., "motion-JPEG"
    - only spatial redundancy is explored and reduced
  - How about temporal redundancy? Is differential coding good?
    - Pixel-by-pixel difference could still be large due to motion
- ➔ Need better prediction

### hybrid video coding system

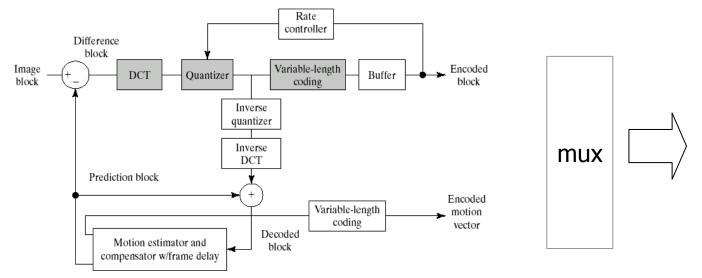
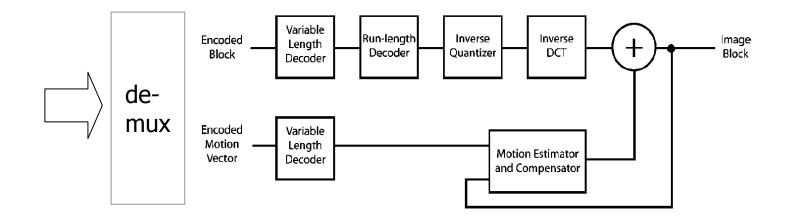
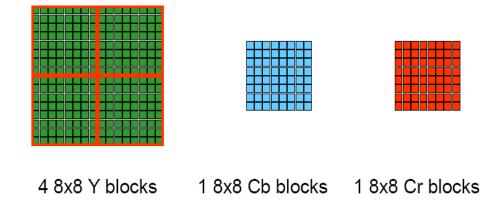


FIGURE 8.47 A basic DPCM/DCT encoder for motion compensated video compression.



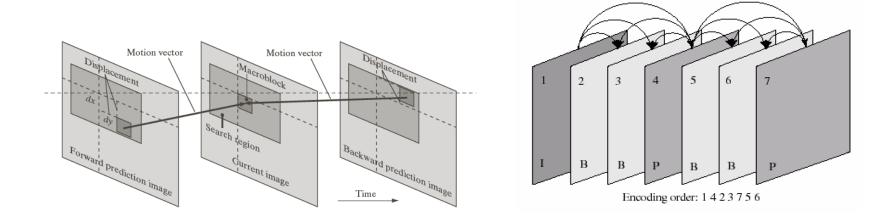
### Block partition in video coding systems

- Work on each macroblock (MB) (16x16 pixels) independently for reduced complexity
  - Motion compensation done at the MB level
  - DCT coding at the block level (8x8 pixels)



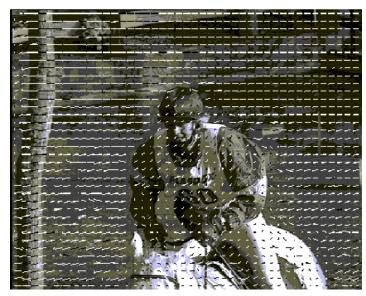
#### representing motion

- Predict a new frame from a previous frame and only code the prediction error --- *Inter* prediction on "B" and "P" frames
- Predict a current block from previously coded blocks in the same frame --- *Intra* prediction (introduced in the latest standard H.264)
- Prediction errors have smaller energy than the original pixel values and can be coded with fewer bits
  - DCT on the prediction errors
- Those regions that cannot be predicted well will be coded directly using DCT --- Intra coding without intra-prediction





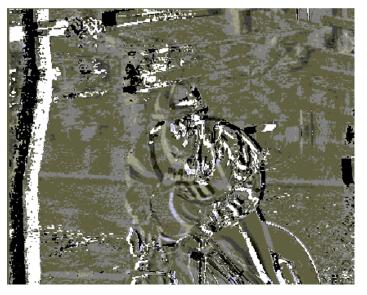
"Horse ride"



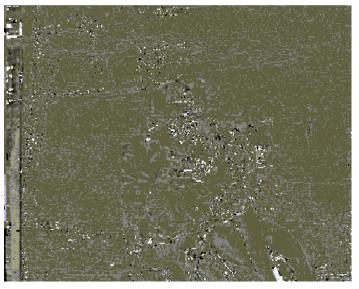
Motion estimation

#### (From Princeton EE330 S'01 by B.Liu)

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Pixel-wise difference w/o motion compensation

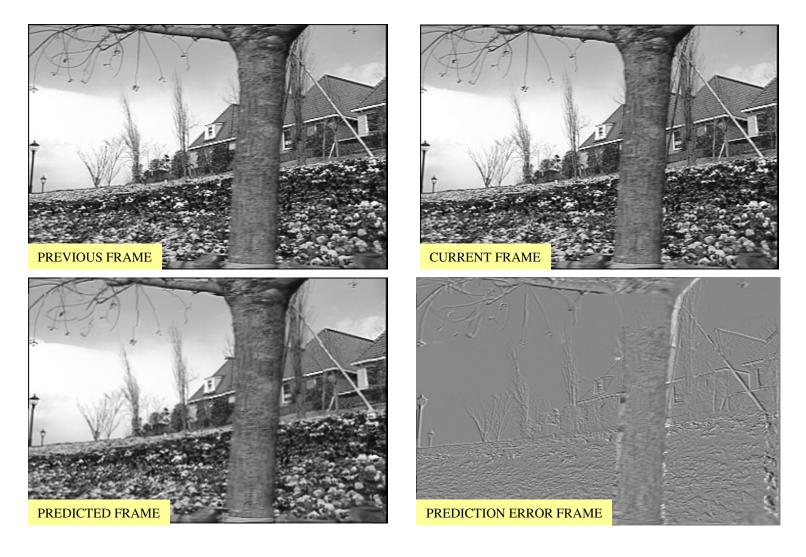


Residue after motion compensation

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#### motion compensation

#### Help reduce temporal redundancy of video



# motion estimation

- Help understanding the content of image sequence
- Help reduce temporal redundancy of video
  - For compression
- Stabilizing video by detecting and removing small, noisy global motions
  - For building stabilizer in camcorder
- A hard problem in general!

## block-matching with exhaustive search <sup>59</sup>

- Assume block-based translation motion model
- Search every possibility over a specified range for the best matching block
  - MAD (mean absolute difference) often used for simplicity

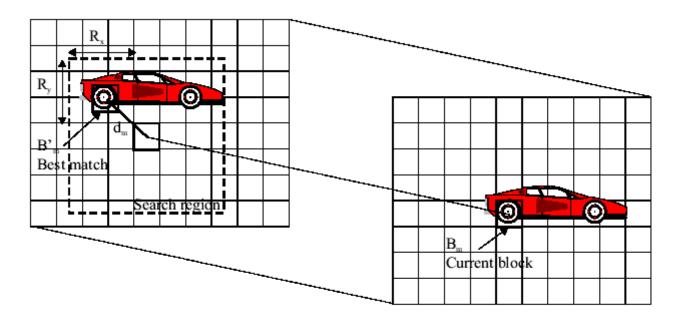


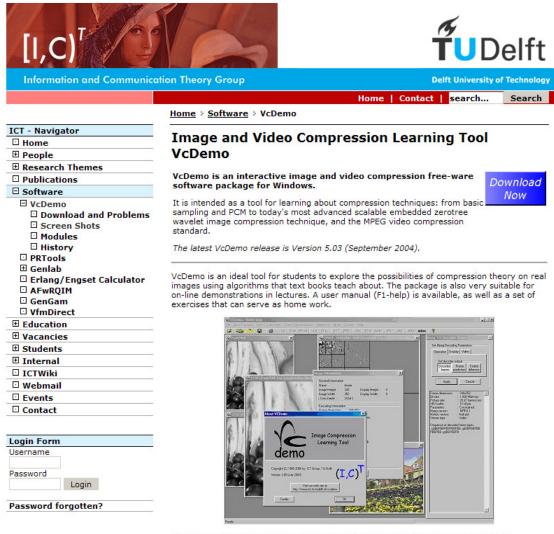
Figure 6.6. The search procedure of the exhaustive block matching algorithm.

From Wang's Preprint Fig.6.6

### audio coding versus image coding

	MP3 (wideband audio coding)	JPEG	
Data Unit	Frame	Block	
Transform	MDCT	DCT	
Quantization	Fixed Quantization matrix base on psychoacoustic masking	Baseline quantization matrix + adaptive rate control	
Entropy coding	Huffman code	Huffman code, run-length, differential	

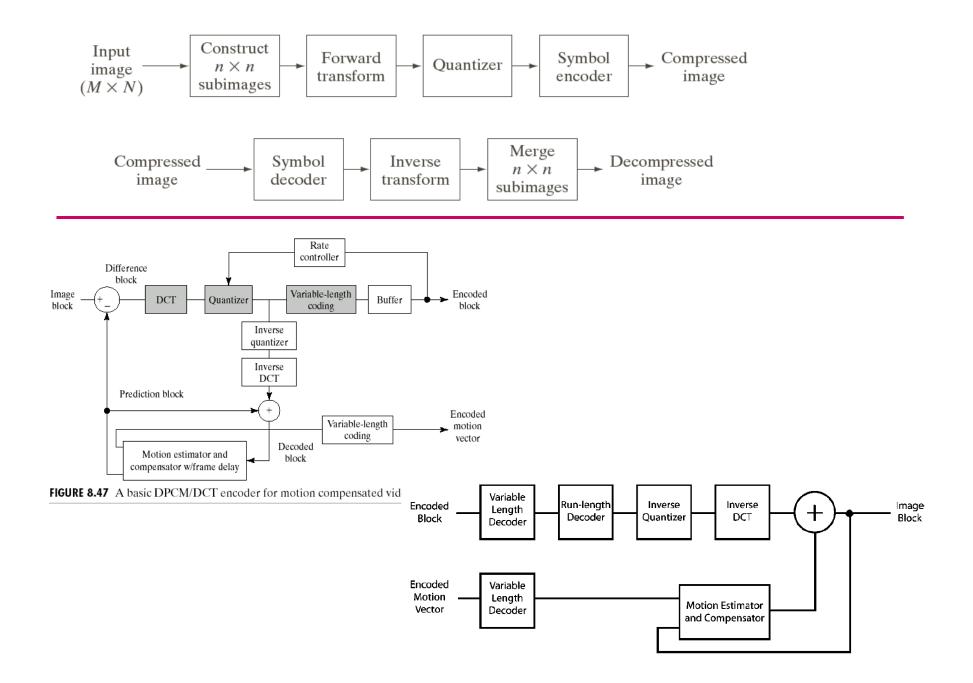
#### VC demo



The screen shot above gives an impression of the user interface. VcDemo is entirely menu-driven. No programming is necessary, but the user has control over the crucial compression parameters. Example of such parameters are bit rate, prediction structure in DPCM, the block-size in DCT compression, and the Group-of-Picture structure in MPEG.

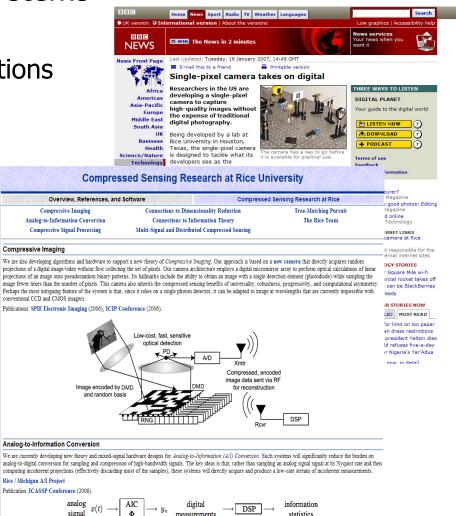
#### **Revisiting Our Questions**

- what are behind jpeg/mpeg/mp4 ... formats?
- what are the "good/fine/super fine" in my Canon Powershot?
- why/when do I want to use raw/jpeg format in my Nikon D80?
- why doesn't "zipping" jpeg files help?
- Do JPEG/MPEG compression algorithms remove perceptual redundancy, symbol redundancy, and spatial/temporal redundancy, how?
- what are the best ways to do compression?
- are we doing our best? (yes/no/maybe)



## **Recent Activities in Image Compression**

- Build better, more versatile systems
  - High-definition IPTV
  - Wireless and embedded applications
  - P2P video delivery
- In search for better basis
  - Curvelets, contourlets, ...
- "compressed sensing"



# Summary

- The image/video compression problem
- Source coding
  - entropy, source coding theorem, criteria for good codes, huffman coding, stream codes and code for symbol sequences
- Image/video compression systems
  - transform coding system for images
  - hybrid coding system for video
- Readings
  - G&W 8.1-8.2 (exclude 8.2.2)
  - McKay book chapter 1, 5 http://www.inference.phy.cam.ac.uk/mackay/itila/



 Next time: reconstruction in medical imaging and multimedia indexing and retrieval

material sources: David McKay's book, Min Wu (UMD), Yao Wang (poly tech), ...

#### i love compression

ADD TO DADD ADD FAVES

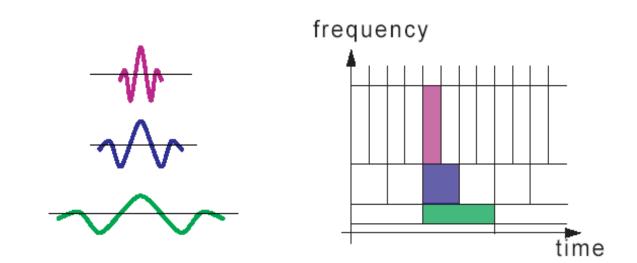


compression sacks that is. definitely one of my best travel/backpacking purchases http://www.flickr.com/photos/jmhouse/2250089958/

# jpeg 2000 supplemental slides

# Wavelets

- A wavelet is a square integrable function whose translates and dilates form an orthonormal basis for Hilbert space L<sub>2</sub>(R<sup>N</sup>).
- Theory
  - Algebra, Geometry
  - Analysis (mainly studying functions and operators)
    - Fourier, Harmonic, Wavelets

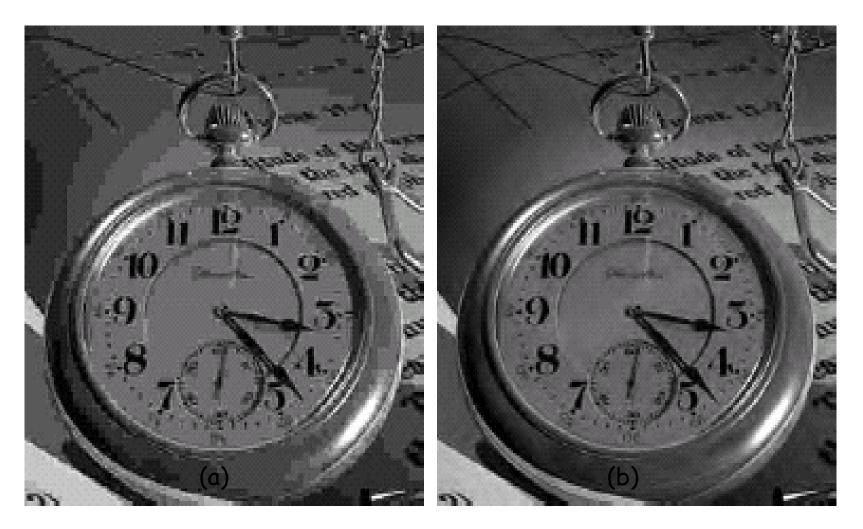


## JPEG-2000 V.S. JPEG



Compression at 0.25 b/p by means of (a) JPEG (b) JPEG-2000

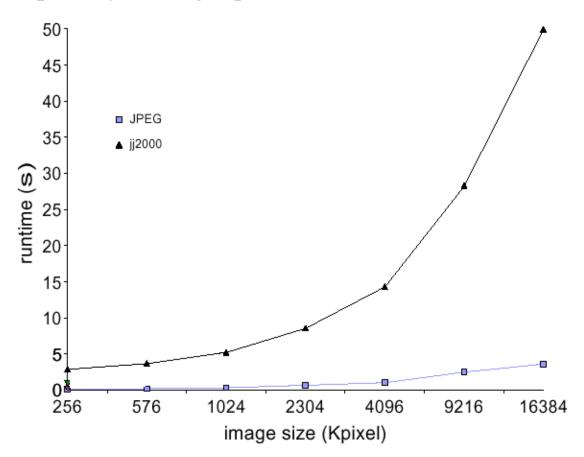
# JPEG-2000 V.S. JPEG



Compression at 0.2 b/p by means of (a) JPEG (b) JPEG-2000

#### The trade-off:

JPEG2000 has a much Higher computational complexity than JPEG, especially for larger pictures.



# motion estimation supplemental slides <sup>72</sup>

#### Fractional Accuracy Search for Block Matching

- For motion accuracy of 1/K pixel
  - Upsample (interpolate) reference frame by a factor of K
  - Search for the best matching block in the upsampled reference frame
- Half-pel accuracy  $\sim$  K=2
  - Significant accuracy improvement over integer-pel (esp. for low-resolution)
  - Complexity increase

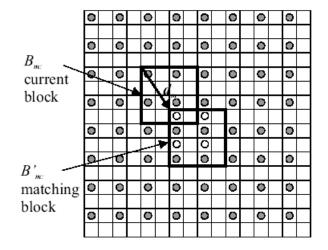


Figure 6.7. Half-pel accuracy block matching. Filled circles are samples existing in the original tracked frame, open circles are samples to be interpolated for calculating the matching error, for a candidate MV  $\mathbf{d}_m = (-1, -1.5)$ . Instead of calculating these samples on-demand for each candidate MV, a better approach is to pre-interpolate the entire tracked frame.

(From Wang's Preprint Fig.6.7)

### **Complexity of Exhaustive Block-Matching**

- Assumptions
  - Block size NxN and image size S=M1xM2
  - Search step size is 1 pixel ~ "integer-pel accuracy"
  - Search range +/–R pixels both horizontally and vertically
- Computation complexity
  - # Candidate matching blocks = (2R+1)<sup>2</sup>
  - # Operations for computing MAD for one block ~ O(N<sup>2</sup>)
  - # Operations for MV estimation per block ~ O((2R+1)<sup>2</sup> N<sup>2</sup>)
  - # Blocks = S /  $N^2$
  - Total # operations for entire frame ~ O((2R+1)<sup>2</sup> S)
    - i.e., overall computation load is independent of block size!
- E.g., M=512, N=16, R=16, 30fps
  - => On the order of 8.55 x  $10^9$  operations per second!
  - Was difficult for real time estimation, but possible with parallel hardware

# Exhaustive Search: Cons and Pros

- Pros
  - Guaranteed optimality within search range and motion model
  - Cons
    - Can only search among finitely many candidates
      - What if the motion is "fractional"?
    - High computation complexity
      - On the order of [search-range-size \* image-size] for 1-pixel step size
- → How to improve accuracy?
  - Include blocks at fractional translation as candidates
     require interpolation
- ➔ How to improve speed?
  - Try to exclude unlikely candidates

# Fast Algorithms for Block Matching

- Basic ideas
  - Matching errors near the best match are generally smaller than far away
  - Skip candidates that are unlikely to give good match

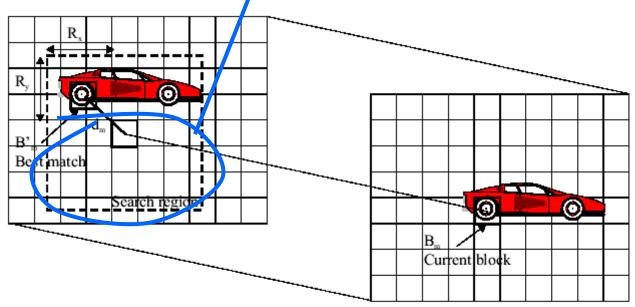


Figure 6.6. The search procedure of the exhaustive block matching algorithm.

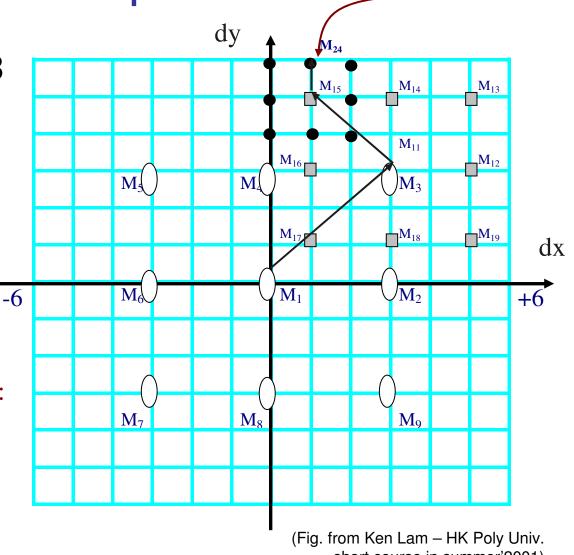
(From Wang's Preprint Fig.6.6)

motion vector  $\{dx, dy\} = \{1, 6\}'$ 

# Fast Algorithm: 3-Step Search

- Search candidates at 8 neighbor positions
- Step-size cut down by 2 after each iteration
  - Start with step size approx. half of max. search range

Total number of computations:  $9 + 8 \times 2 = 25$  (3-step)  $(2R+1)^2 = 169$  (full search)



short course in summer'2001)