COLUMBIA UNIVERSITY
IN THE CITY OF NEW YORK

Image Restoration

Lecture 7, March 23rd, 2008

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EE4830 Digital Image Processing
http://www.ee.columbia.edu/~xlx/ee4830/

thanks to G&W website, Min Wu and others for slide materials

Announcements

- Midterm results today
- HW3 due next Monday
  - question 1.4: reproduce the equivalence of the following %energyloss-vs-index graph for the USPS dataset.

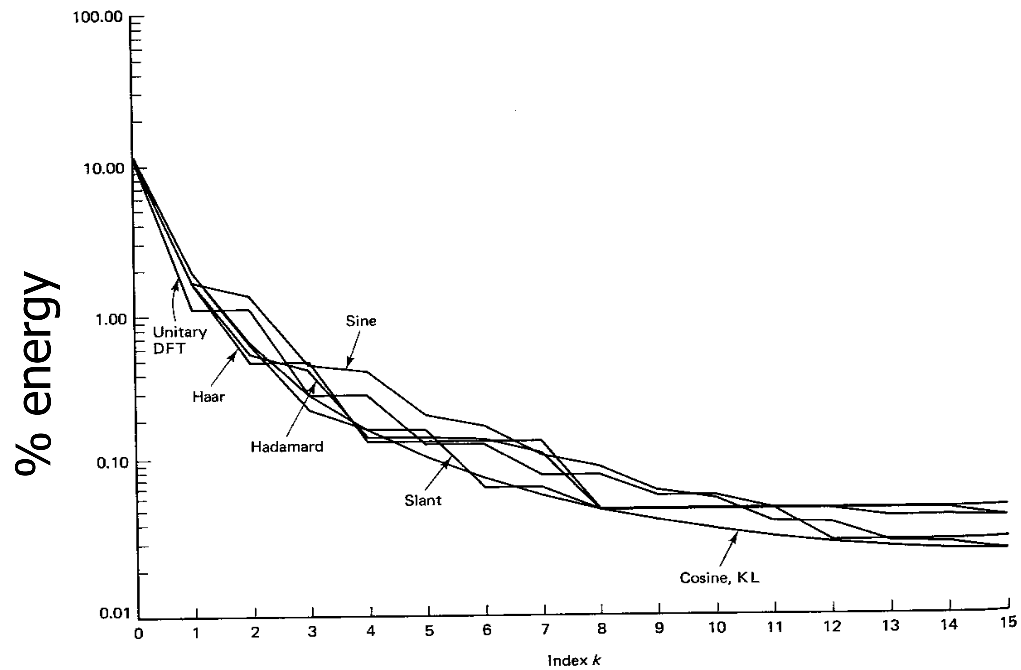
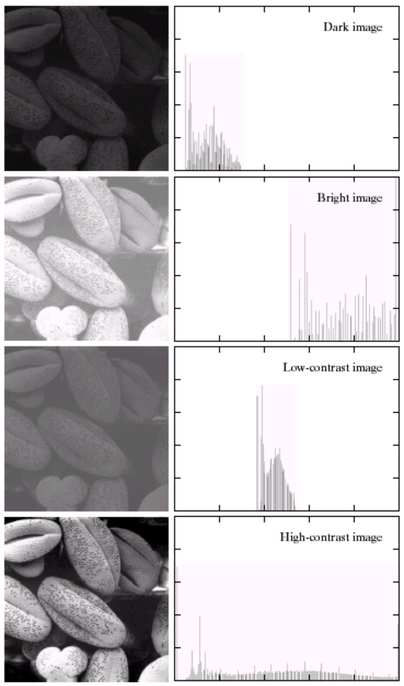


Figure 5.18 Distribution of variances of the transform coefficients (in decreasing order) of a stationary Markov sequence with $N = 16$, $\rho = 0.95$ (see Example 5.9).

we have covered …

Image sensing



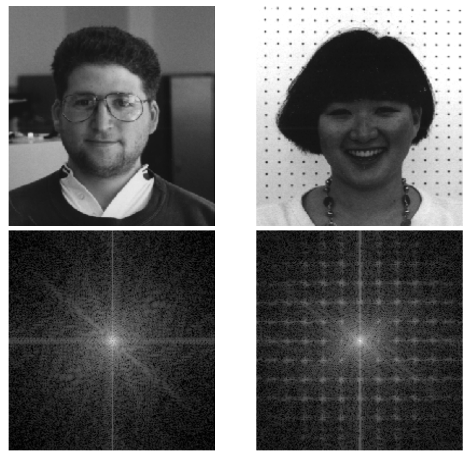


Image Transform
and Filtering

Spatial Domain
processing

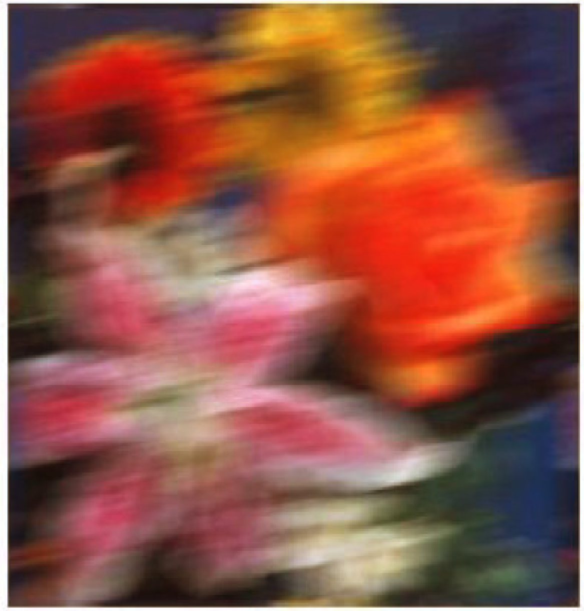
Image Restoration

outline

- What is image restoration
  - Scope, history and applications
  - A model for (linear) image degradation
- Restoration from noise
  - Different types of noise
  - Examples of restoration operations
- Restoration from linear degradation
  - Inverse and pseudo-inverse filtering
  - Wiener filters
  - Blind de-convolution
- Geometric distortion and its corrections

degraded images

Original image



Blurred image

- What caused the image to blur?
- Can we improve the image, or “undo” the effects?

Original image

Blurred image

- Image enhancement: “improve” an image subjectively.
- Image restoration: remove distortion from image in order to go back to the “original” → objective process.

image restoration

- started from the 1950s
- application domains
  - Scientific explorations
  - Legal investigations
  - Film making and archival
  - Image and video (de-)coding
  - …
  - Consumer photography

- related problem: image reconstruction in radio astronomy, radar imaging and tomography

[Banham and Katsaggelos 97]

a model for image distortion

- Image enhancement: “improve” an image subjectively.
- Image restoration: remove distortion from image, to go back to the “original” -- objective process

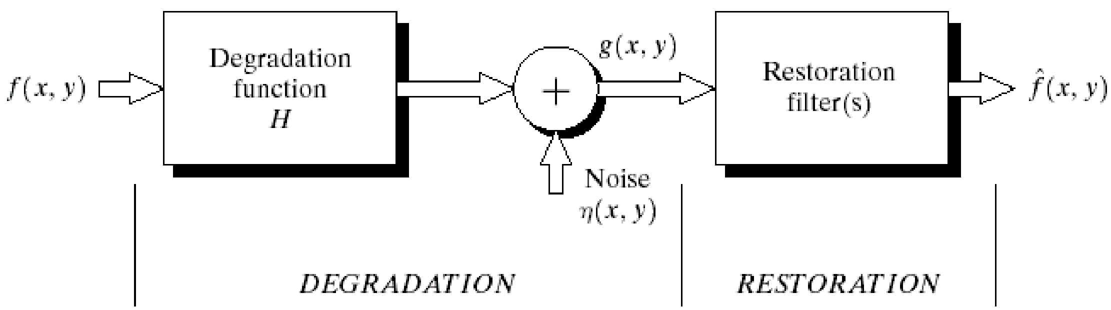


FIGURE 5.1 A model of the image degradation/ restoration process.

$$g(x,y) = H[f(x,y)] + \eta(x,y)$$

a model for image distortion

- Image restoration
  - Use a priori knowledge of the degradation
  - Modeling the degradation and apply the inverse process
  - Formulate and evaluate objective criteria of goodness

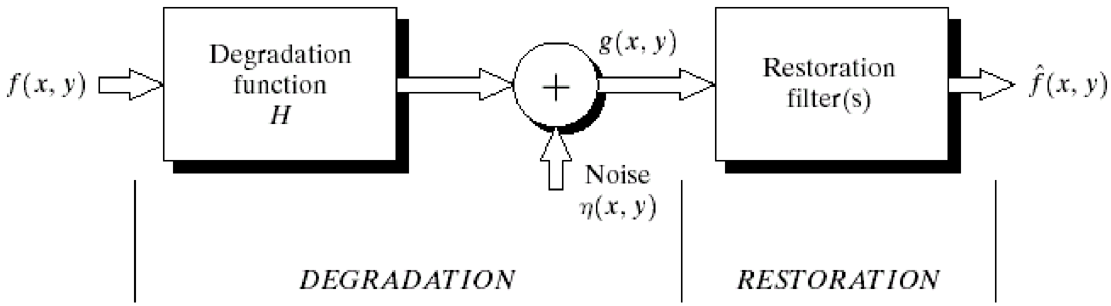


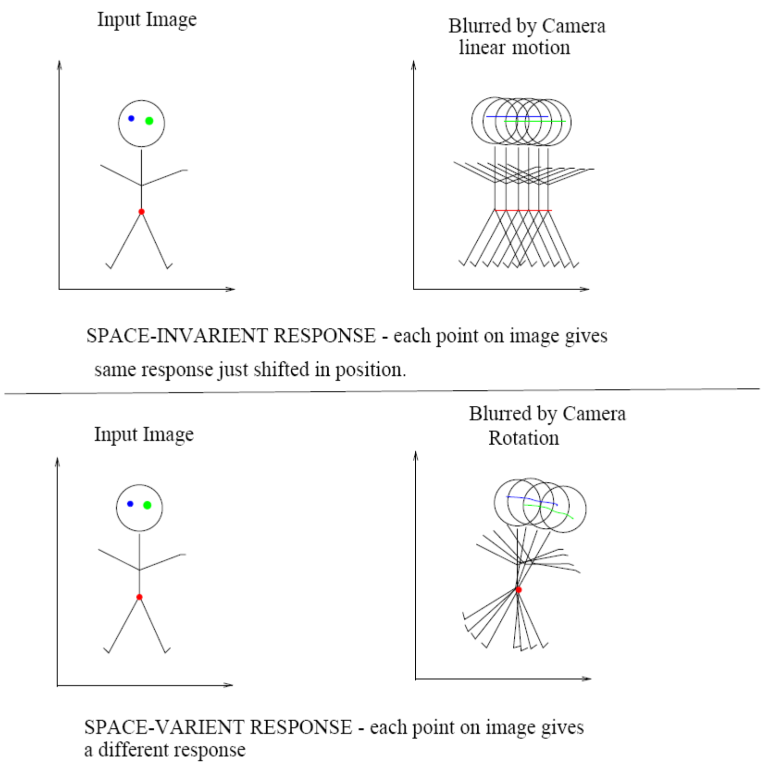
FIGURE 5.1 A model of the image degradation/ restoration process.

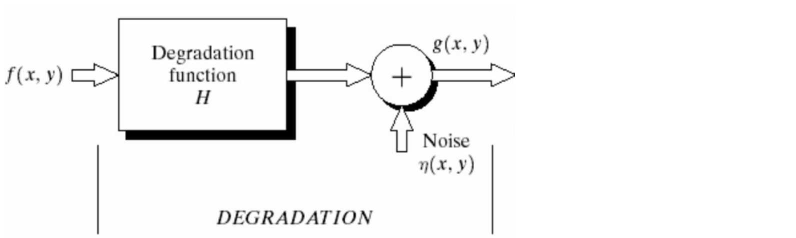
$$g(x,y) = H[f(x,y)] + \eta(x,y)$$

$\rightarrow$ design restoration filters such that $\widehat{f}(x,y)$ is as close to $f(x,y)$ as possible.

usual assumptions for the distortion model

- Noise
  - Independent of spatial location
    - Exception: periodic noise …
  - Uncorrelated with image
- Degradation function H
  - Linear
  - Position-invariant





divide-and-conquer step #1: image degraded only by noise.

common noise models

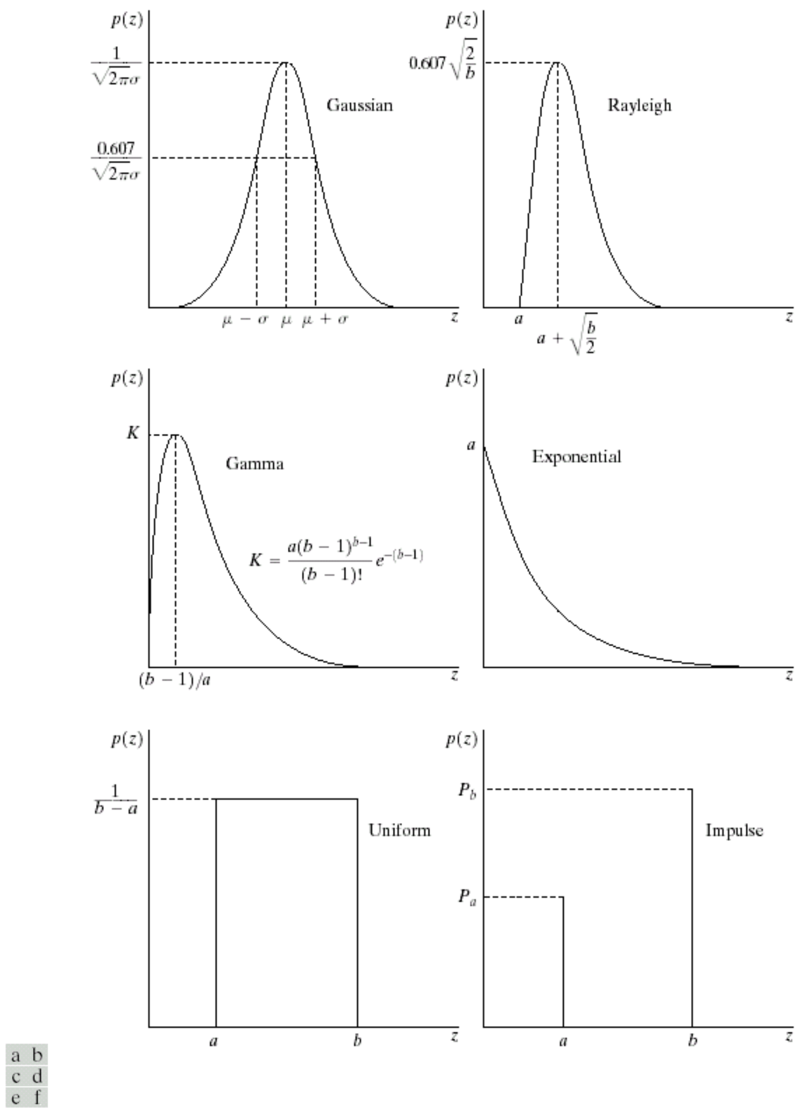


FIGURE 5.2 Some important probability density functions.

Gaussian

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\mu)^2/2\sigma^2}$$

Rayleigh

$$p(z) = \frac{2}{b}(z-a)e^{-(z-a)^2/b}, \; for \qquad z \geq a$$

Erlang, Gamma(a, b)

$$p(z) = \frac{a^b z^{b-1}}{(b-a)!} e^{-az}, \; for \qquad z \geq 0$$

Exponential

$$p(z) = ae^{-az}, \; for \qquad z \geq 0$$

→ additive noise

Salt-and-Pepper:

$$p(z) = P_a \delta(z-a) + P_b \delta(z-b)$$

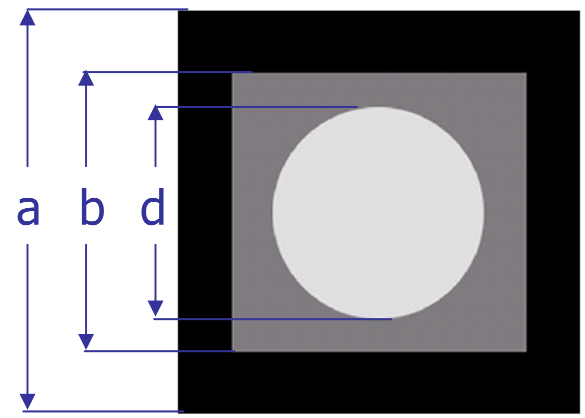
Speckle noise: $a = a_R + ja_I$

$$|g(x,y)|^2 \simeq |f(x,y)|^2 |a(x,y)|^2 + \eta(x,y)$$

a_R,a_I zero mean, independent Gaussian

→ multiplicative noise on signal magnitude

the visual effects of noise



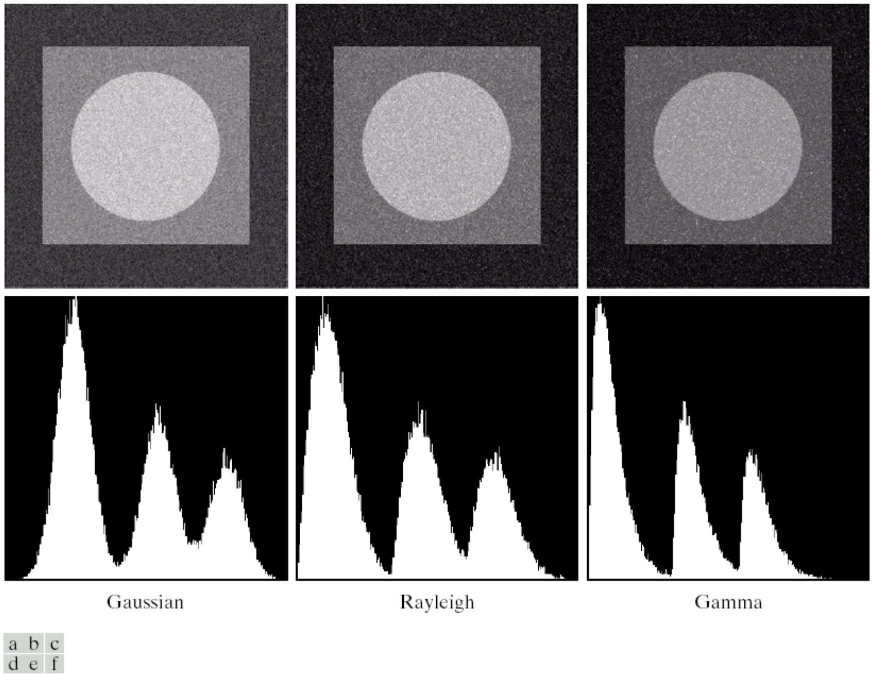


FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

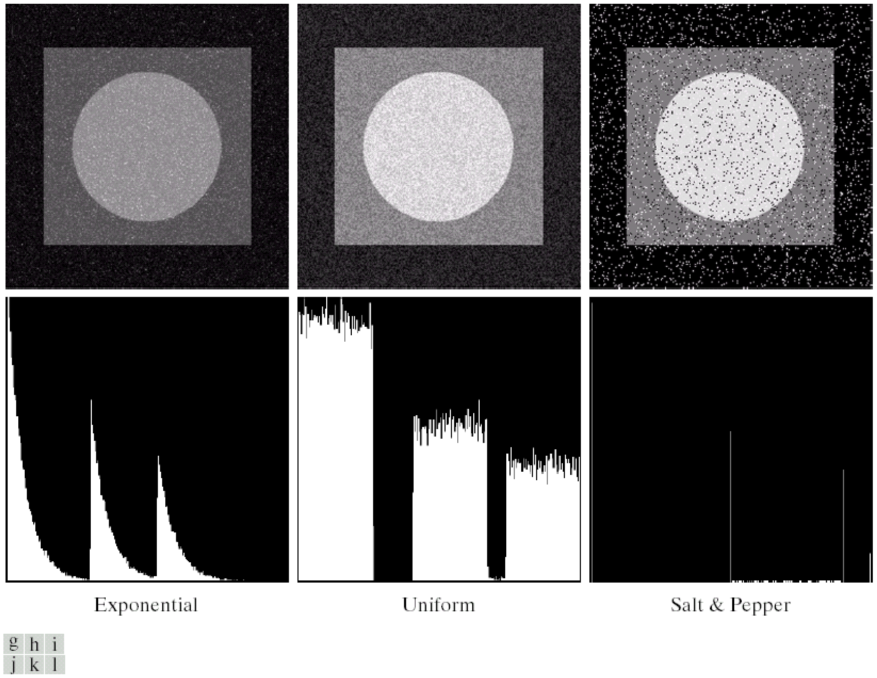


FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and impulse noise to the image in Fig. 5.3.

recovering from noise

- overall process
  Observe and estimate noise type and parameters → apply optimal (spatial) filtering (if known) → observe result, adjust filter type/parameters …

- Example noise-reduction filters [G&W 5.3]
  - Mean/median filter family
  - Adaptive filter family
  - Other filter family
    - e.g. Homomorphic filtering for multiplicative noise [G&W 4.5, Jain 8.13]

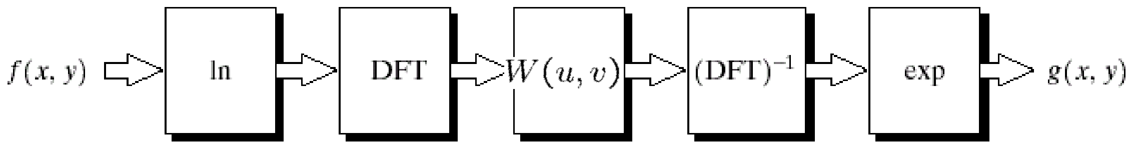


FIGURE 4.31 Homomorphic filtering approach

example: Gaussian noise

a b
c d

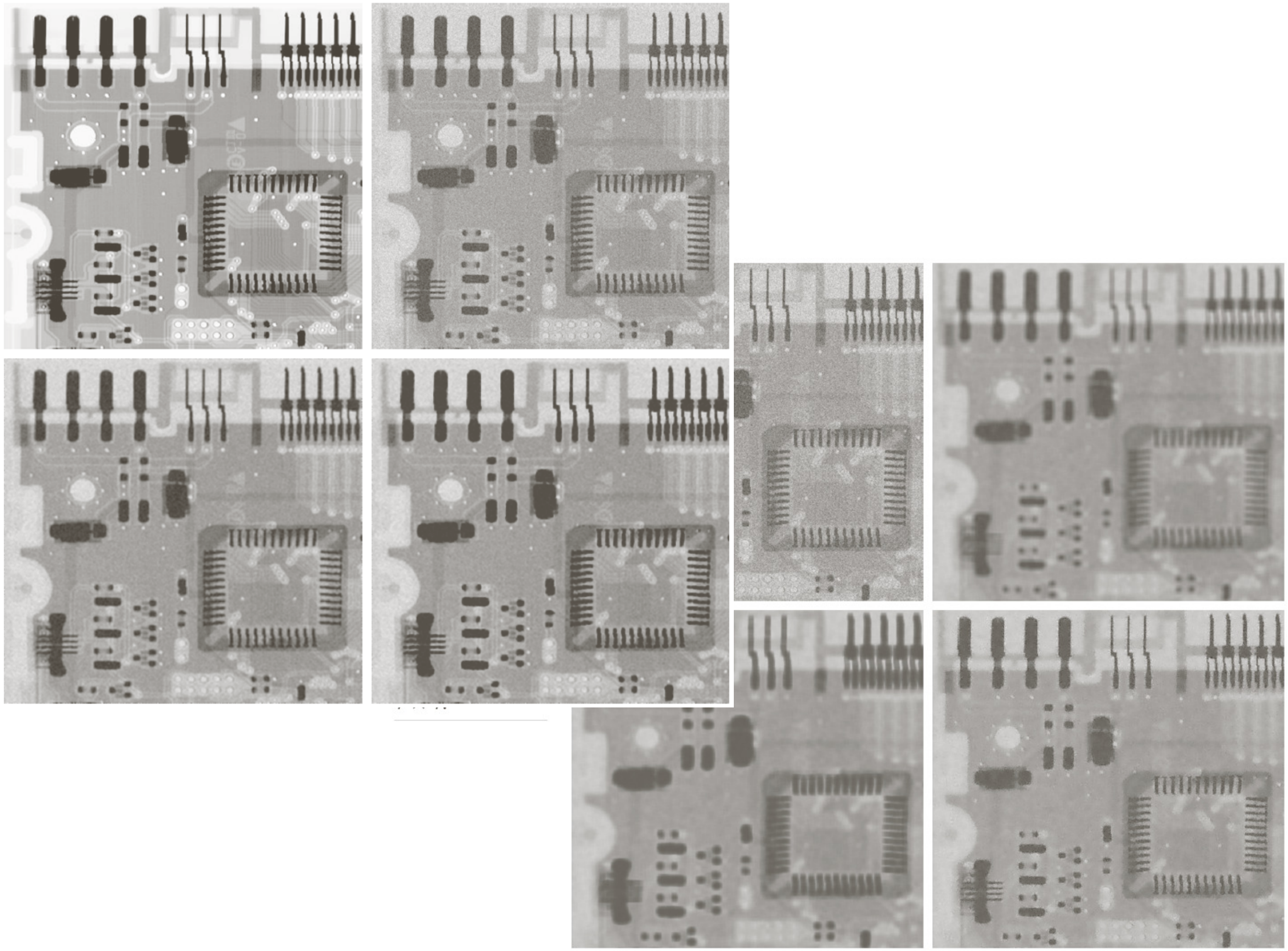
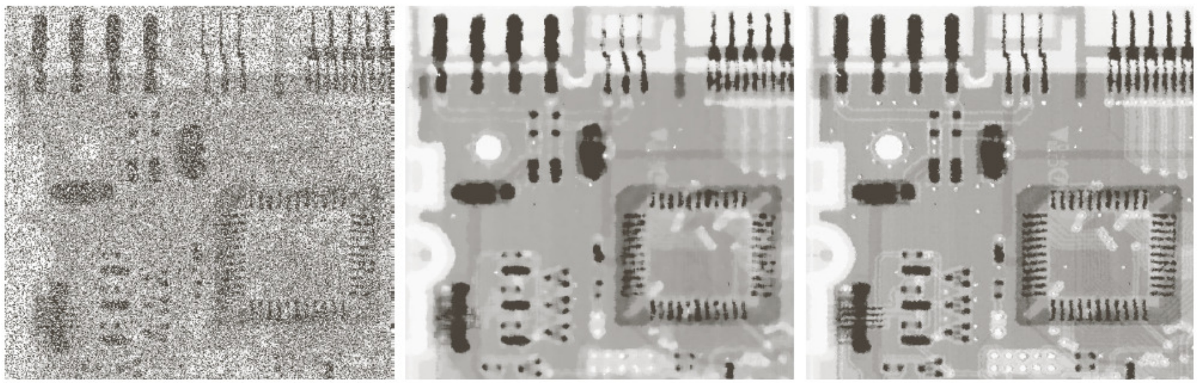


FIGURE 5.7 (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3 × 3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

example: salt-and-pepper noise



a b c

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{max} = 7$.

Recovering from Periodic Noise

[G&W 5.4]

Recall: Butterworth LPF

$$H(u,v) = \frac{1}{1 + [D(u,v)/D_0]^{2n}}$$

Butterworth bandreject filter

$$H(u,v) = \frac{1}{1 + [\frac{D(u,v)W}{D^2(u,v) - D_0^2}]^{2n}}$$

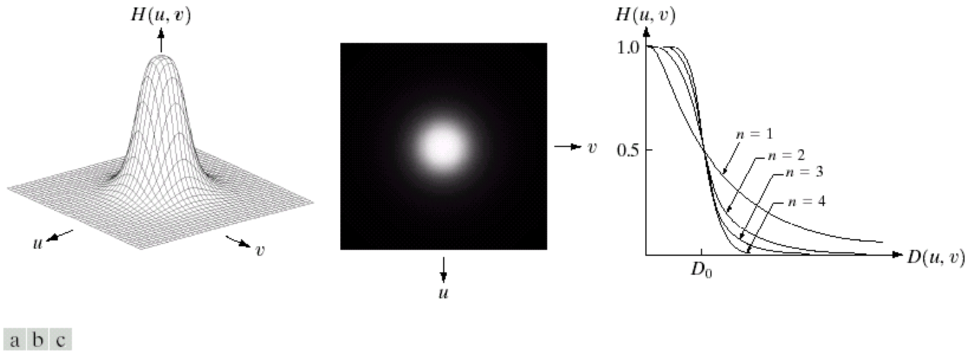


FIGURE 4.14 (a) Perspective plot of a Butterworth lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

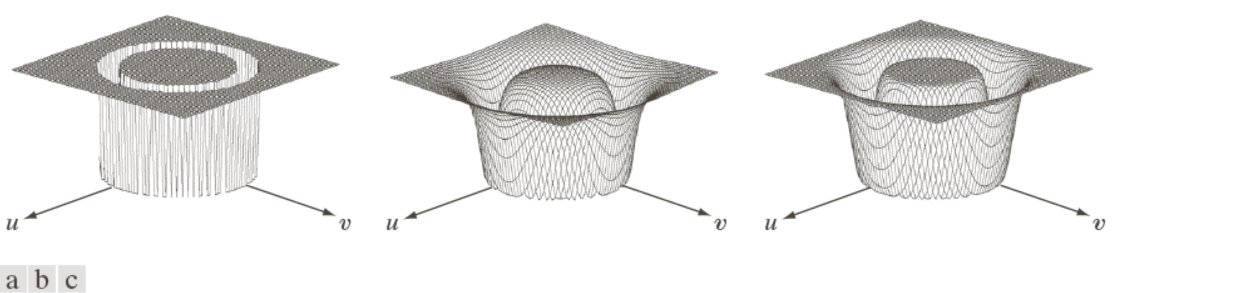
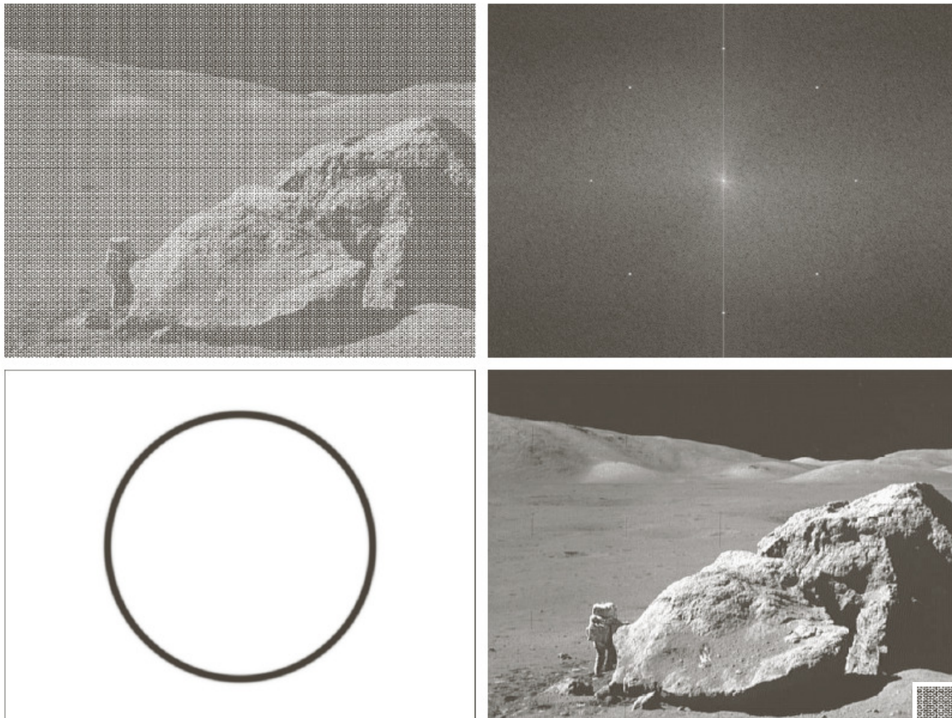


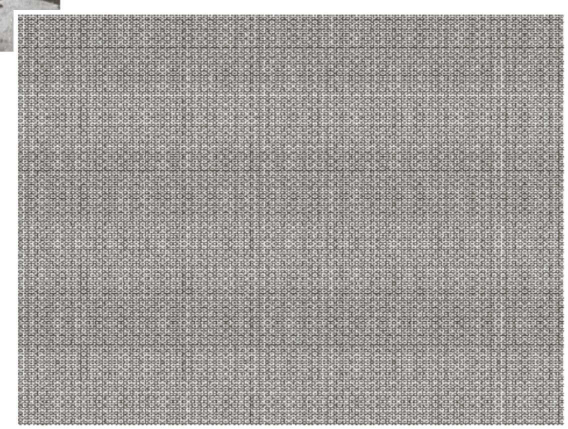
FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.

example of bandreject filter

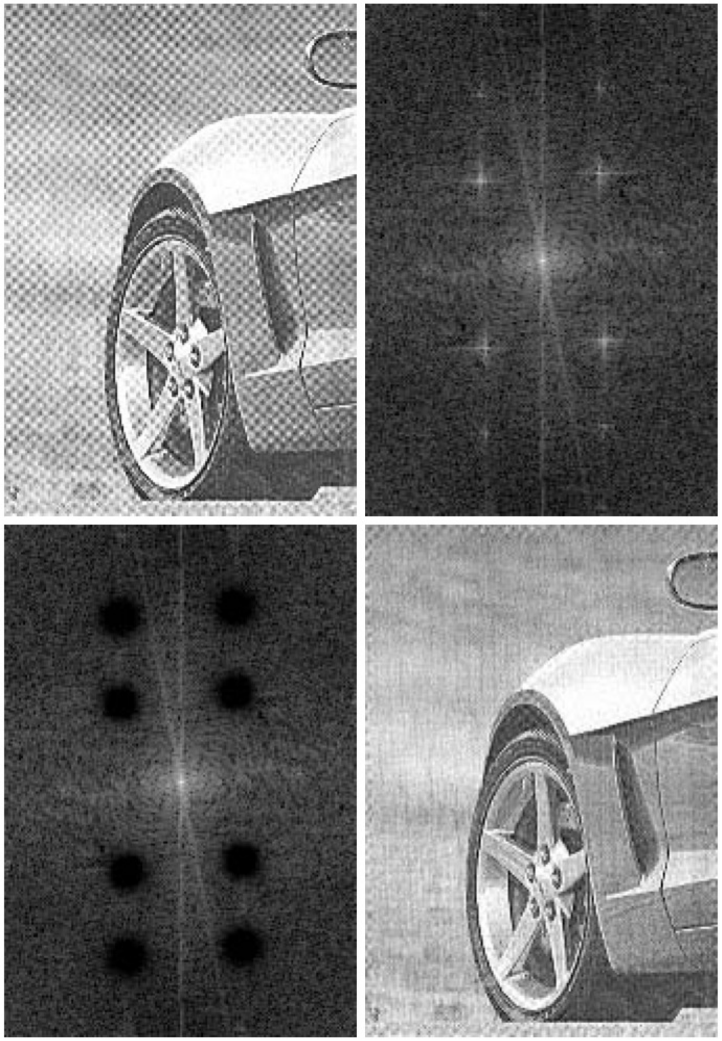
a b
c d

FIGURE 5.16 (a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)





notch filter



a b
c d

FIGURE 4.64 (a) Sampled newspaper image showing a moiré pattern. (b) Spectrum. (c) Butterworth notch reject filter multiplied by the Fourier transform. (d) Filtered image.

outline

- Scope, history and applications
- A model for (linear) image degradation
- Restoration from noise
  - Different types of noise
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  - Inverse and pseudo-inverse filtering
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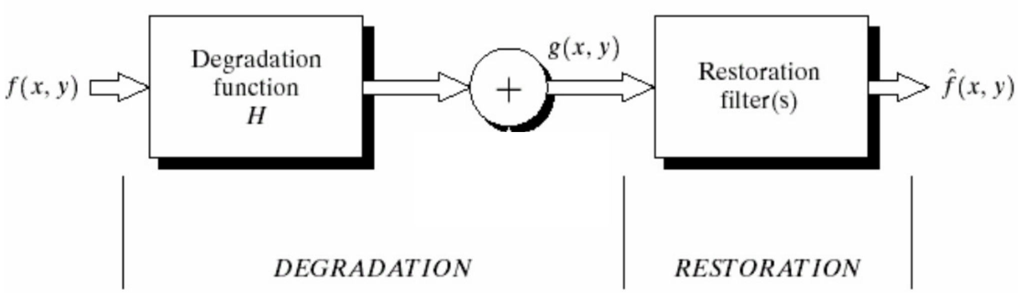
- Geometric distortion and example corrections

recover from linear degradation

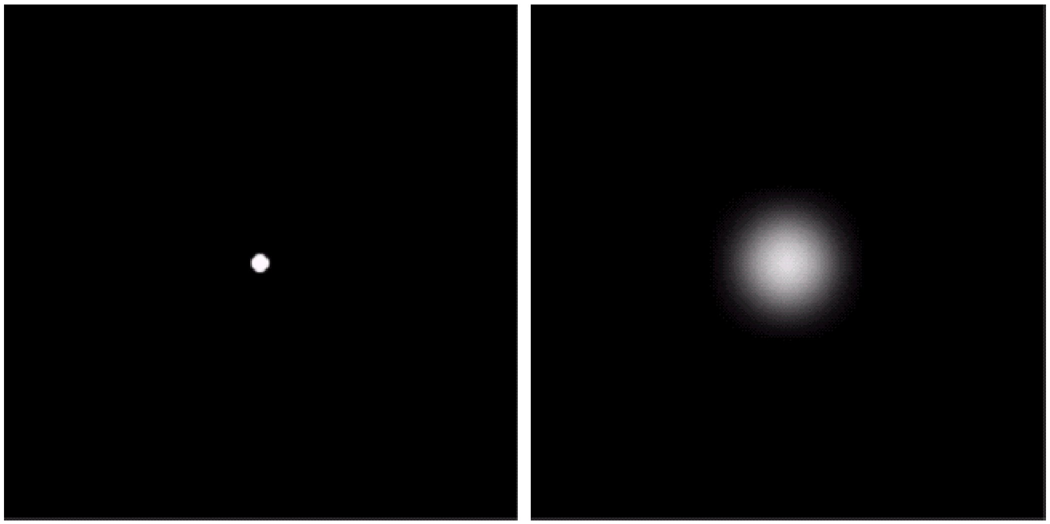
- Degradation function
  - Linear (eq 5.5-3, 5.5-4)
    - Homogeneity
    - Additivity
  - Position-invariant (in cartesian coordinates, eq 5.5-5)

→ linear filtering with H(u,v)
convolution with h(x,y) – point spread function



Divide-and-conquer step #2: linear degradation, noise negligible.

point-spread function



a b

FIGURE 5.24 Degradation estimation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.

point-spread functions

Spatial domain

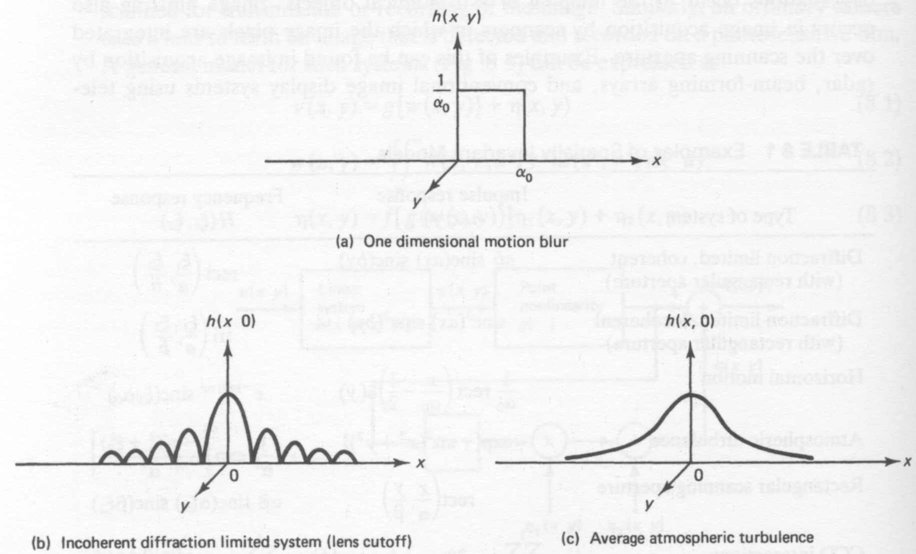


Figure 8.5 Examples of spatially invariant PSFs

Frequency domain

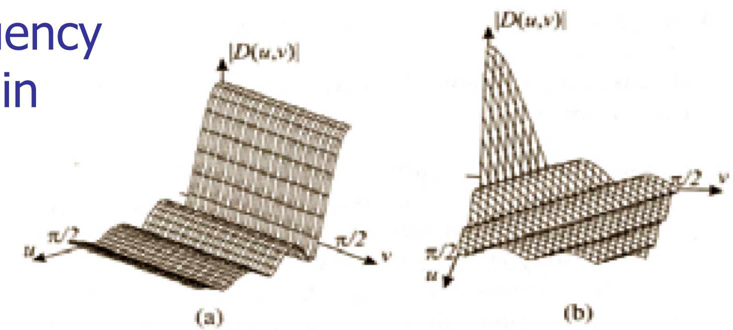


FIGURE 2 PSF of motion blur in the Fourier domain, showing $|D(u, v)|$, for (a) $L = 7.5$ and $\phi = 0$; (b) $L = 7.5$ and $\phi = \pi/4$

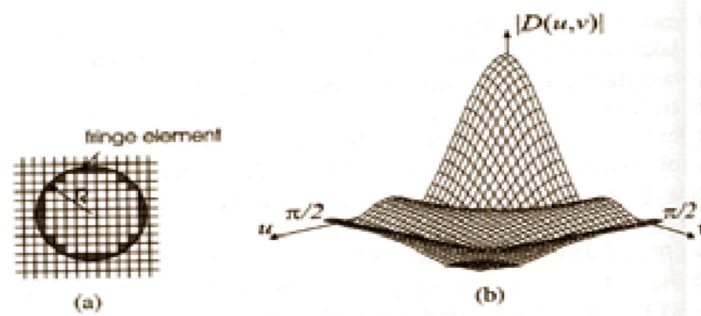
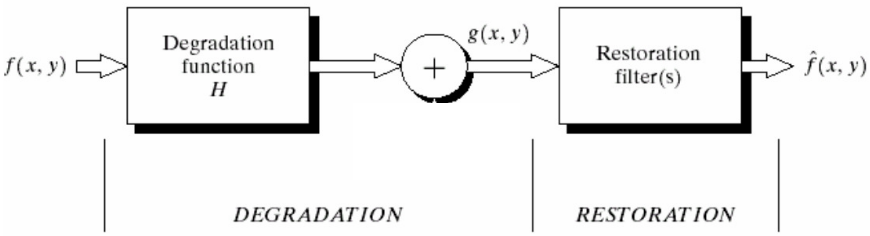


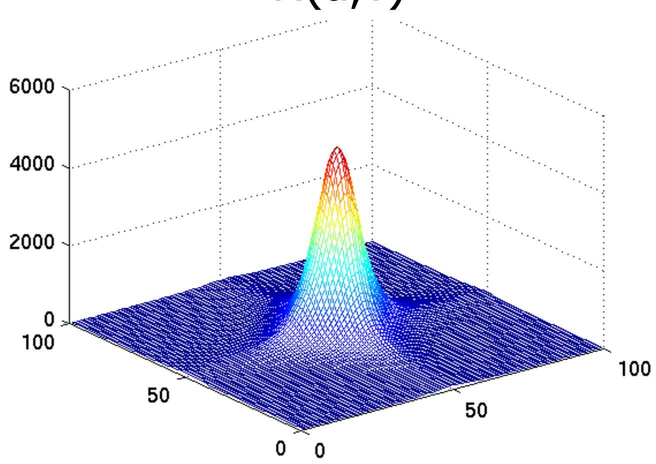
FIGURE 3 (a) Fringe elements of discrete out-of-focus blur that are calculated by integration; (b) PSF in the Fourier domain, showing $|D(u, v)|$, for $R = 2.5$.

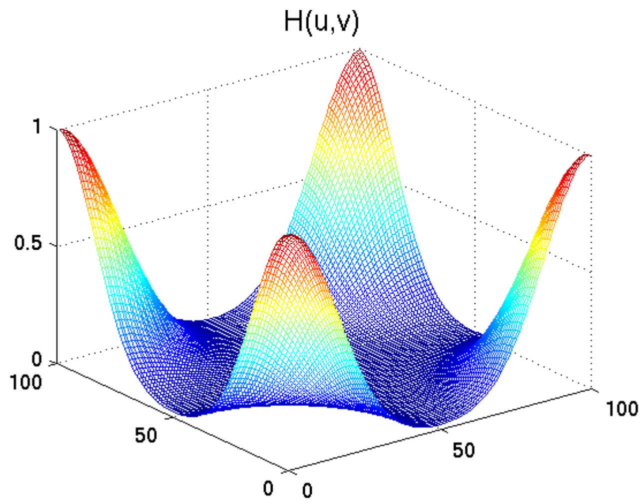
inverse filter

- assume h is known: low-pass filter H(u,v)



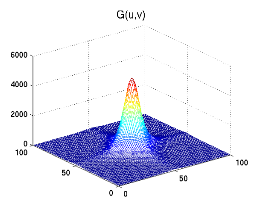
- inverse filter $\hat{H}(u,v) = 1/H(u,v)$
- recovered image $\hat{F}(u,v) = G(u,v)\hat{H}(u,v)$

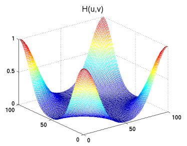




[EE381K, UTexas]

inverse filtering example





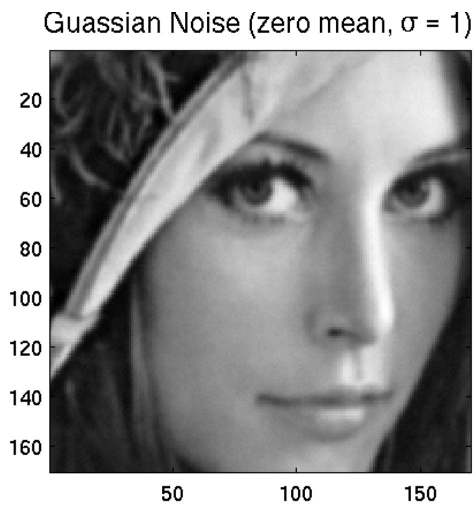
loss of
information

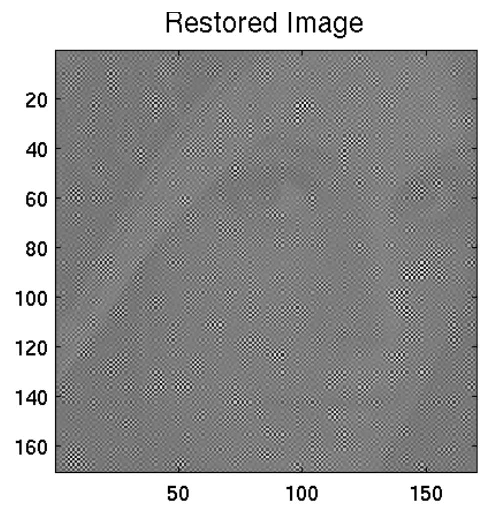
inverse filtering under noise

- in reality, we often have
  - H(u,v) = 0, for some u, v. e.g. motion blur
  - noise N(u,v) $\neq$ 0

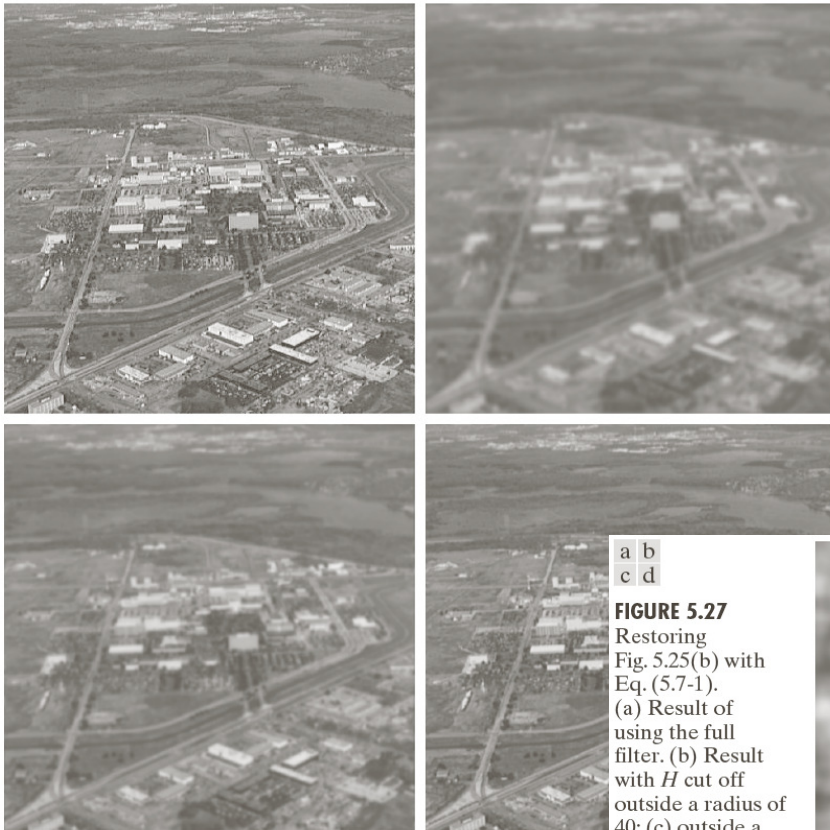
$$\hat{H}(u,v) = 1/H(u,v)$$
$$\hat{F}(u,v) = G(u,v)\hat{H}(u,v)$$

$$G(u,v) = F(u,v)H(u,v) + N(u,v)$$
$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{\hat{H}(u,v)}$$



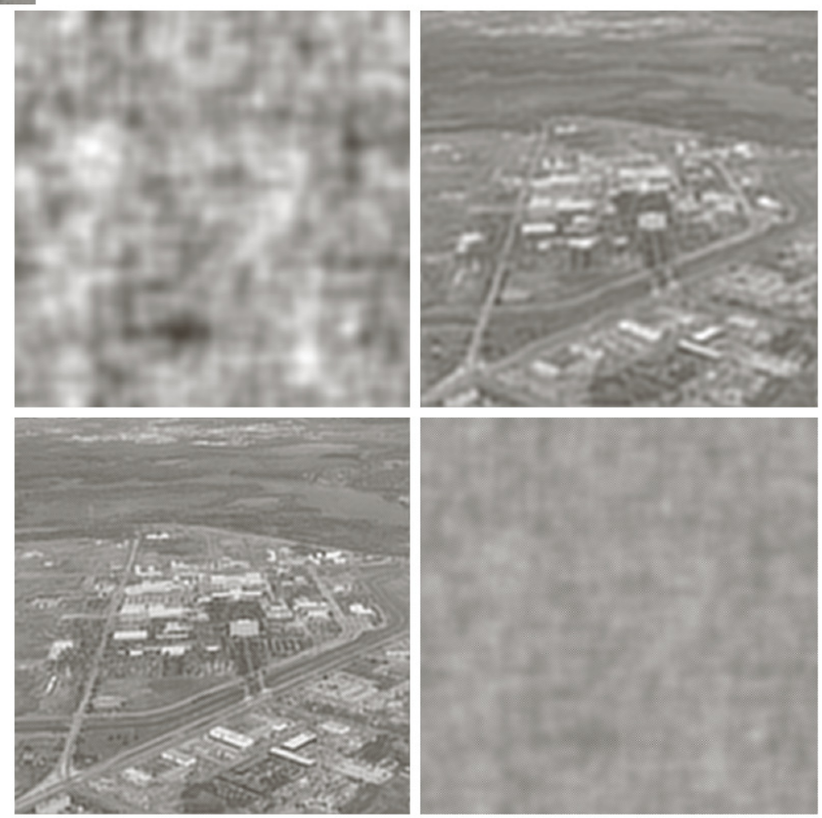


[EE381K, UTexas]



a b
c d

FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)



a b
c d

FIGURE 5.27 Restoring Fig. 5.25(b) with Eq. (5.7-1). (a) Result of using the full filter. (b) Result with H cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.

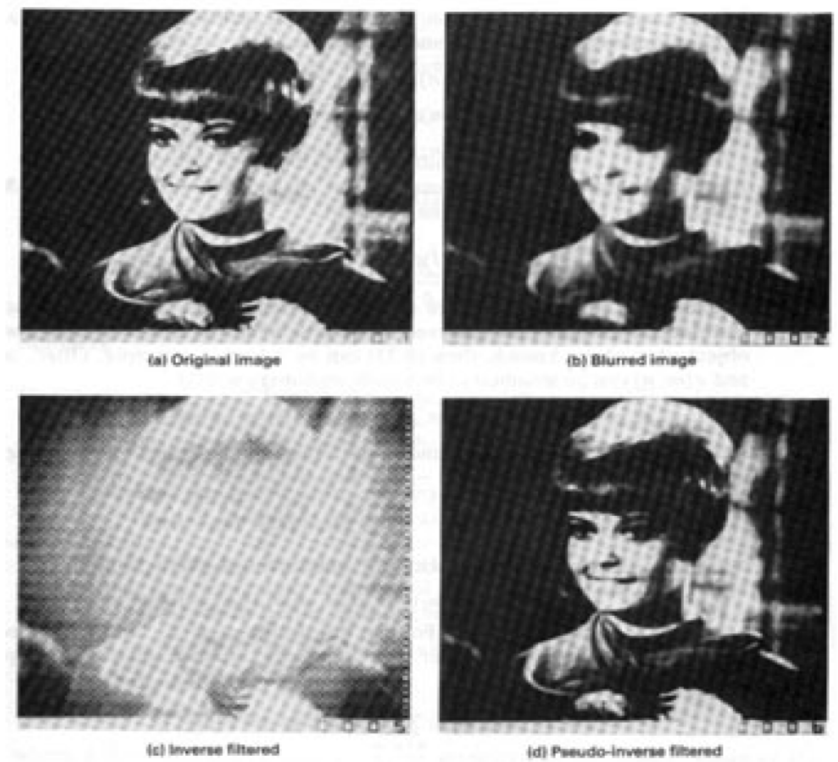
remedy 1:
inverse filter with cut-off

$$\hat{H}(u,v) = \begin{cases} 1/H(u,v), & |D(u,v)| \le \varepsilon \\ 0, & |D(u,v)| > \varepsilon \end{cases}$$

pseudo-inverse filtering

cut-off based on fiter frequency

$$\hat{H}(u,v) = \begin{cases} 1/H(u,v), & |H(u,v)| \geq \epsilon \\ 0, & |H(u,v)| < \epsilon \end{cases}$$



[Jain, Fig 8.10]

back to the original problem

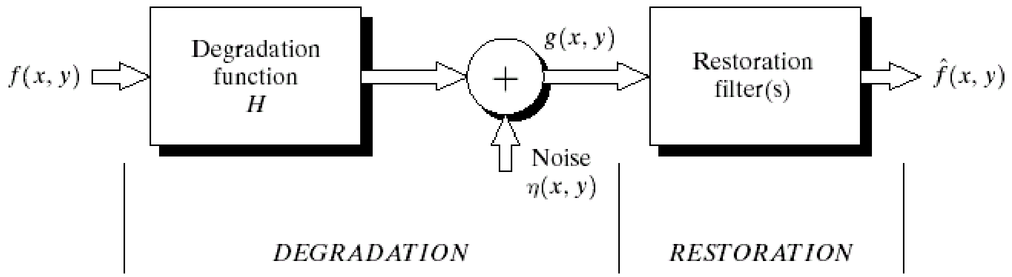


FIGURE 5.1 A model of the image degradation/restoration process.

Inverse filter with cut-off:

$$\hat{H}(u,v) = \begin{cases} 1/H(u,v), & |D(u,v)| \leq \varepsilon \\ 0, & |D(u,v)| > \varepsilon \end{cases}$$

Pseudo-inverse filter:

$$\hat{H}(u,v) = \begin{cases} 1/H(u,v), & |H(u,v)| \geq \epsilon \\ 0, & |H(u,v)| < \epsilon \end{cases}$$

- Can the filter take values between 1/H(u,v) and zero?
- Can we model noise directly?

Wiener filter

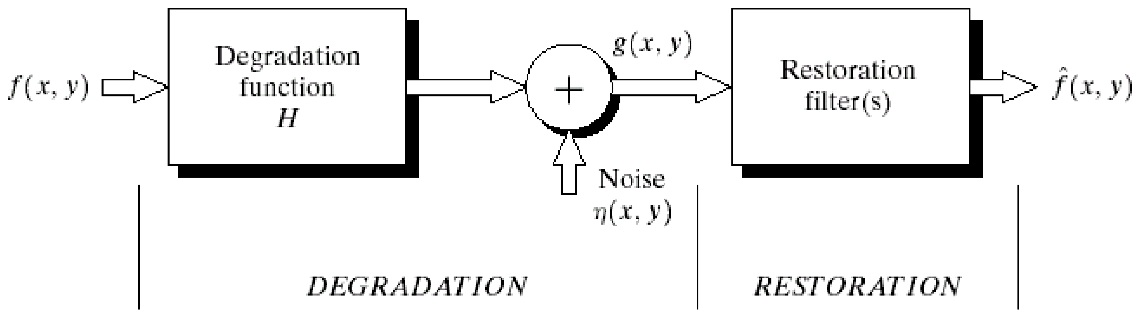


FIGURE 5.1 A model of the image degradation/ restoration process.

$W(u, v)$

- goal: restoration with minimum mean-square error (MSE)

$$\min_W \quad e^2 = E\{(f - \widehat{f})^2\}$$

- optimal solution (nonlinear):

$$\widehat{f}(x, y) = E\{f(x, y)|g(m, n), \forall(m, n)\}$$

- restrict to linear space-invariant filter

$$\widehat{f}(x, y) = w(x, y) * g(x, y)$$

- find "optimal" linear filter W(u,v) with min. MSE …

Wiener filter

- goal: restoration with minimum mean-square error (MSE)

$$\min_{W} \quad e^2 = E\{(f-\hat{f})^2\}$$

$$\hat{f}(x,y) = w(x,y) * g(x,y)$$

- find "optimal" linear filter W(u,v) with min. MSE

→ orthogonal condition $E\{g(f-\hat{f})\} = 0$

→ correlation function $R_{fg}(x,y) = w(x,y) * R_{gg}(x,y)$

→ wide-sense-stationary (WSS) signals

$$R_{fg}(x_1,y_1,x_2,y_2) = E\{f(x_1,y_1)g(x_2,y_2)\} \xrightarrow{WSS} R_{fg}(x_1-x_2,y_1-y_2)$$

→ Fourier Transform: from correlation to spectrum

$$S_{fg}(u,v) = \mathcal{F}\{R_{fg}(x,y)\},\ \ S_{gg}(u,v) = \mathcal{F}\{R_{gg}(x,y)\}$$

⇨ $$W(u,v) = \frac{S_{fg}(u,v)}{S_{gg}(u,v)} = \frac{H^*(u,v)S_{ff}(u,v)}{|H(u,v)|^2 S_{ff}(u,v) + S_{\eta\eta}(u,v)}$$

S_{ff} and $S_{\eta\eta}$ are the power spectra of the signal and noise, respectively

observations about Wiener filter

$$W(u,v) = \frac{H^*(u,v)S_{ff}(u,v)}{|H(u,v)|^2 S_{ff}(u,v) + S_{\eta\eta}(u,v)}$$
$$= \frac{1}{H(u,v) + \frac{S_{\eta\eta}}{H^*(u,v)S_{ff}}}$$

- If no noise, $S_{\eta\eta} \rightarrow 0$ $$W(u,v)|_{S_{\eta\eta}\rightarrow 0} = \begin{cases} \frac{1}{H(u,v)}, & H(u,v) \neq 0 \\ 0, & H(u,v) = 0 \end{cases}$$

  → Pseudo inverse filter

- If no blur, H(u,v)=1 (Wiener smoothing filter)

$$W(u,v)|_{H=1} = \frac{1}{1 + S_{\eta\eta}(u,v)/S_{ff}(u,v)} = \frac{SNR(u,v)}{SNR(u,v)+1}$$

  → More suppression on noisier frequency bands

1-D Wiener Filter Shape

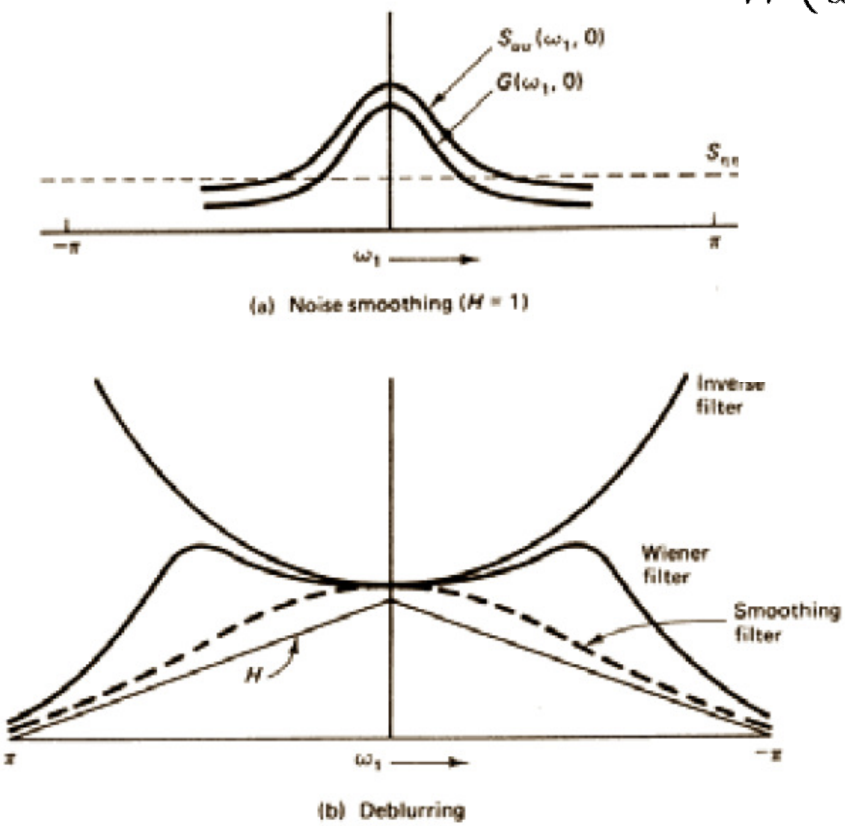


Figure 8.11 Wiener filter characteristics.

[Jain, Fig 8.11]

Wiener Filter implementation

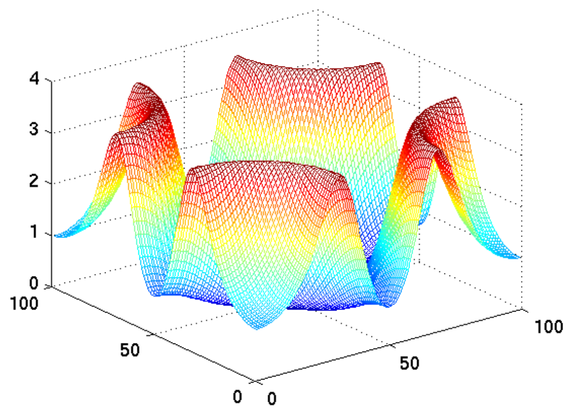
$$
\begin{aligned}
W(u,v) &= \frac{H^*(u,v)S_{ff}(u,v)}{|H(u,v)|^2 S_{ff}(u,v) + S_{\eta\eta}(u,v)} \\
&= \frac{H^*(u,v)}{|H(u,v)|^2 + \frac{S_{\eta\eta}}{S_{ff}}} \\
&= \frac{H^*(u,v)}{|H(u,v)|^2 + K}
\end{aligned}
$$

Where K is a constant (w.r.t. u and v) chosen according to our knowledge of the noise level.

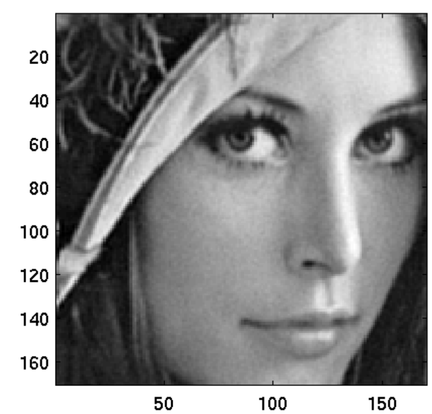
Wiener Filter example

$$W(u,v) = \frac{H^*(u,v)}{|H(u,v)|^2 + K}$$

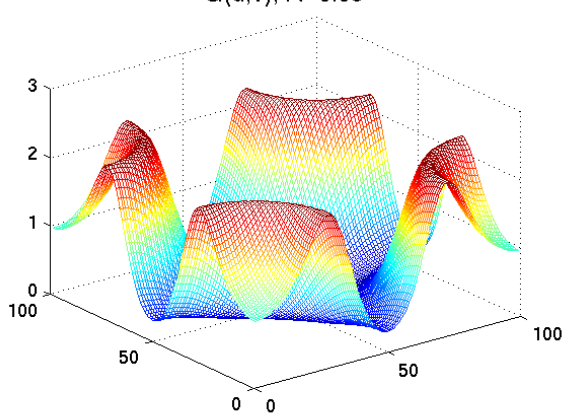
G(u,v), K=0.02



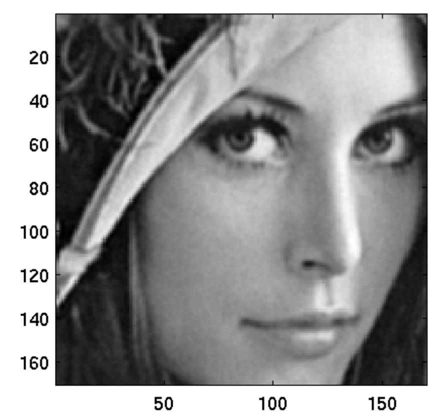
K=0.02



G(u,v), K=0.05



K=0.05



[EE381K, UTexas]

Wiener filter example

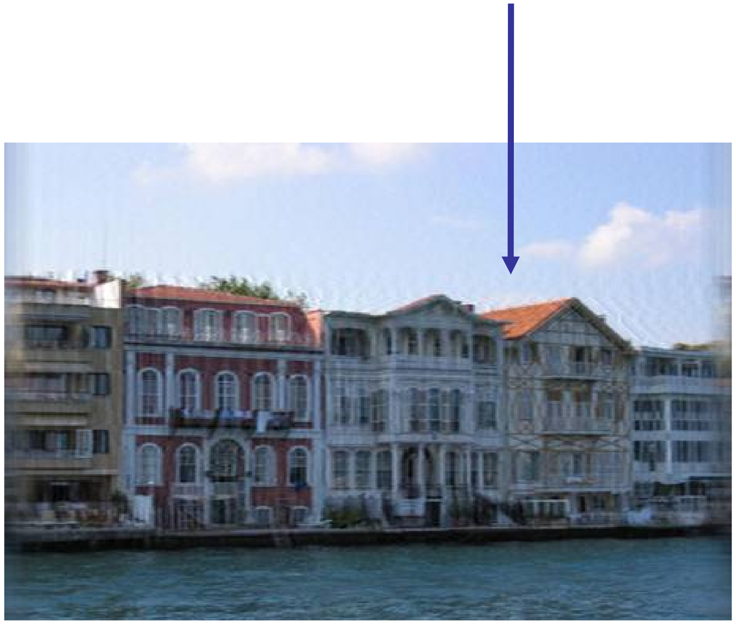
a b c

FIGURE 5.28 Comparison of inverse- and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.

- Wiener filter is more robust to noise, and preserves high-frequency details.

Wiener filter example

Ringing effect visible, too many high frequency components?



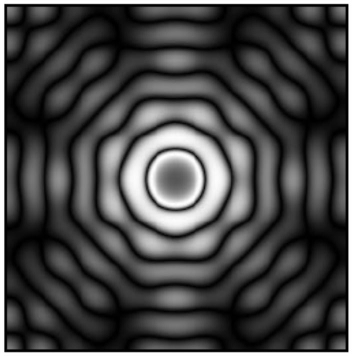
(a) Blurry image (b) restored w. regularized pseudo inverse
(c) restored with wiener filter

[UMD EE631]

Wiener filter: when does it not work?

How much de-blurring is just enough?

image 'blurr1'



wiener filter

restored license plate

[Image Analysis Course, TU-Delft]

improve Wiener filters

- geometric mean filters

$$W(u,v) = \left[\frac{H*(u,v)}{|H(u,v)^2|}\right]^{\alpha} \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \beta\frac{S_{\eta\eta}(u,v)}{S_{ff}(u,v)}}\right]^{1-\alpha}$$

- Constrained Least Squares
  - Wiener filter emphasizes high-frequency components, while images tend to be smooth

$$\min_{f} \; |g - H\widehat{f}|^2 + \alpha|C\widehat{f}|^2$$

$\widehat{f}$: the estimate for undegraded image

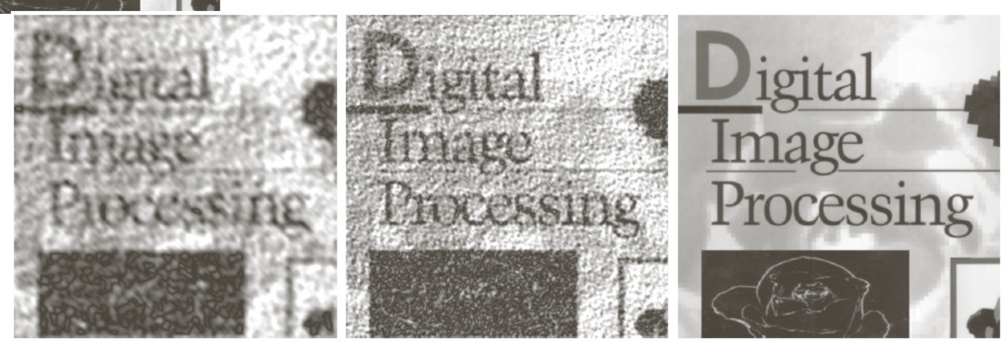
$C\widehat{f}$: a high-passed version of $\widehat{f}$

degraded inverse-filtered Wiener-filtered

motion blur + noise

noise*10^{-1}

noise*10^{-5}



a b c

FIGURE 5.30 Results of constrained least squares filtering. Compare (a), (b), and (c) with the Wiener filtering results in Figs. 5.29(c), (f), and (i), respectively.

geometric distortions

- Modify the spatial relationships between pixels in an image
- a. k. a. “rubber-sheet” transformations

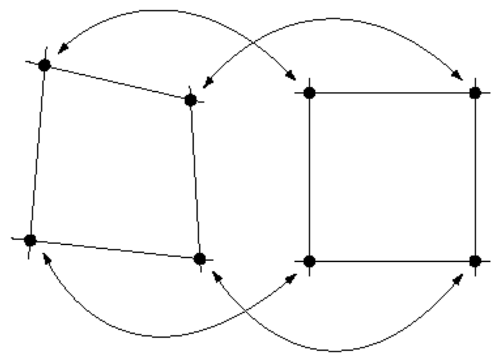


FIGURE 5.32 Corresponding tiepoints in two image segments.

- Two basic steps
  - Spatial transformation
  - Gray-level interpolation

$$x' = r(x, y)$$
$$y' = s(x, y)$$

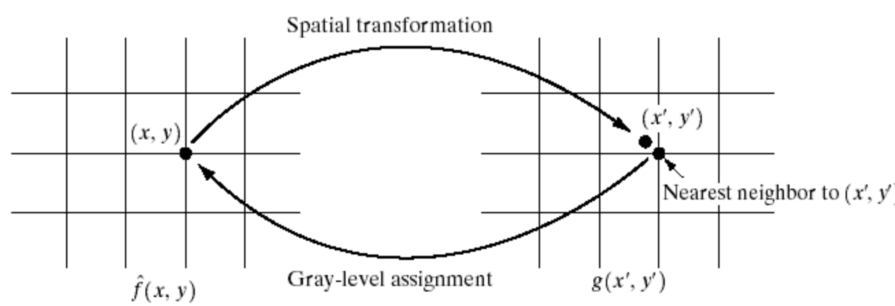
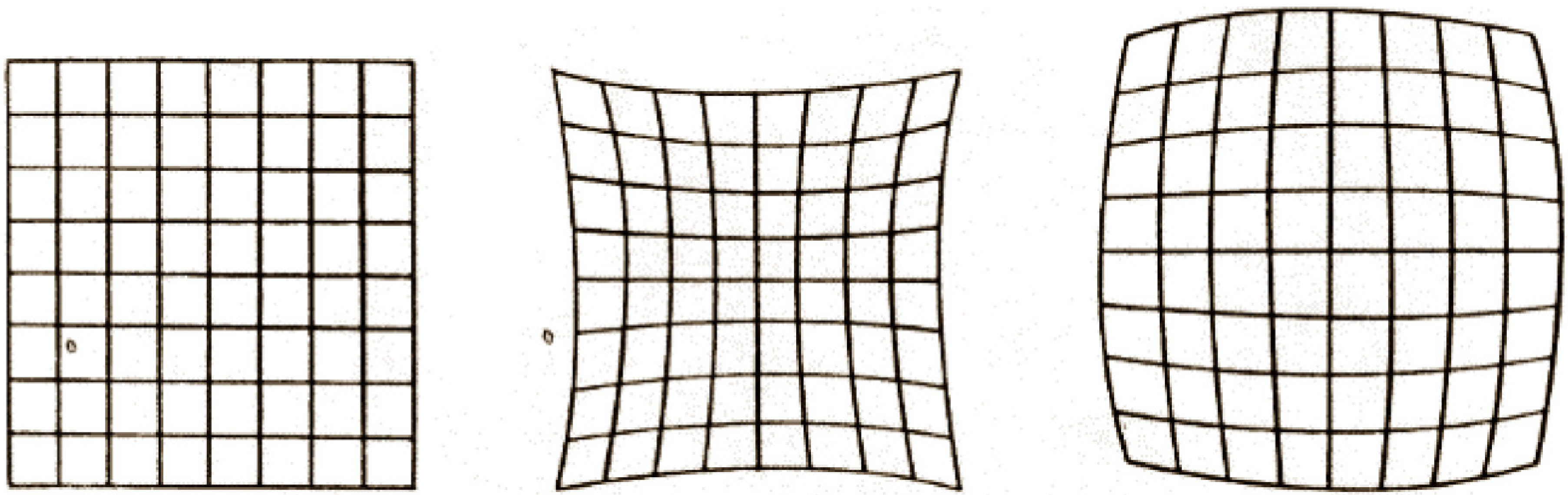


FIGURE 5.33 Gray-level interpolation based on the nearest neighbor concept.

geometric/spatial distortion examples



(a) Original (b) Pincushion distortion (c) Barrel distortion

FIGURE 14.2-1. Example of geometric distortion.

recovery from geometric distortion

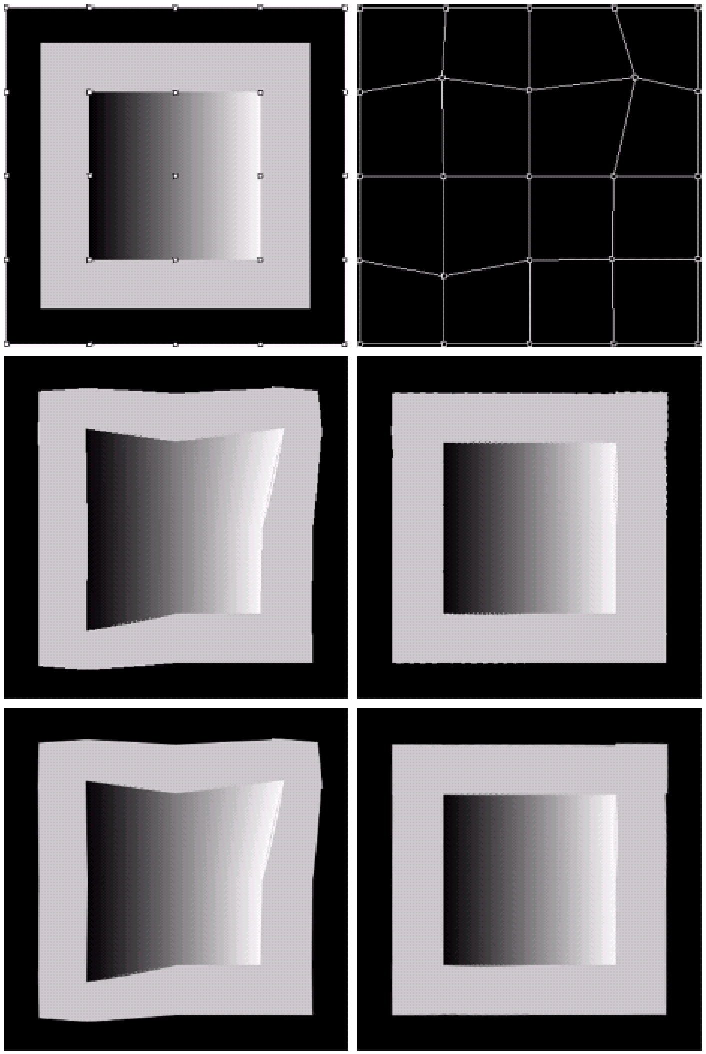


FIGURE 5.34 (a) Image showing tiepoints. (b) Tiepoints after geometric distortion. (c) Geometrically distorted image, using nearest neighbor interpolation. (d) Restored result. (e) Image distorted using bilinear interpolation. (f) Restored image.

recovery from geometric distortion

(a)

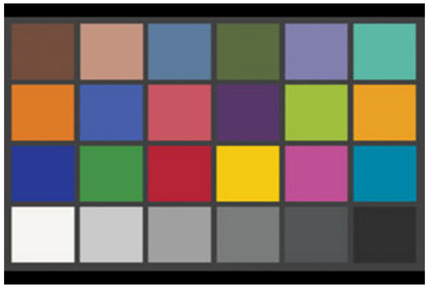
(b)

Fig. 5. (c) Image produced by a Computar 2.5mm lens and a Computar $1/3''$ CCD board camera. (b) Distortion parameters recovered via the minimization of ξ_3 are used to map (a) to perspective image. Notice that straight lines in the scene, such as door edges, map to straight lines in the undistorted images.

Rahul Swaminathan, Shree K. Nayar: Nonmetric Calibration of Wide-Angle Lenses and Polycameras. IEEE Trans. Pattern Anal. Mach. Intell. 22(10): 1172-1178 (2000)

estimating distortions

- calibrate
- use flat/edge areas
- … ongoing work



http://photo.net/learn/dark_noise/

[Tong et. al. ICME2004]

summary

- a image degradation model
- restoration from noise
- restoration from linear degradation
  - Inverse and pseudo-inverse filters, Wiener filter, constrained least squares
- geometric distortions

- readings
  - G&W Chapter 5.1 – 5.10, Jain 8.1-8.4 (at courseworks)
  - M. R. Banham and A. K. Katsaggelos "Digital Image Restoration", *IEEE Signal Processing Magazine,* vol. 14, no. 2, Mar. 1997, pp. 24-41.

who said distortion is a bad thing?

blur …

noise …

geometric …