Lecture 10 (4.14.07)

Image Representation and Description

Shahram Ebadollahi
Lecture Outline

- Image Description
  - Shape Descriptors
  - Texture & Texture Descriptors
  - SIFT
  - Motion Descriptors
  - Color Descriptors
Shape Description

- Shape Represented by its Boundary
  - Shape Numbers,
  - Fourier Descriptors,
  - Statistical Moments

- Shape Represented by its Interior
  - Topological Descriptors
  - Moment Invariants
Boundary Representation: (Freeman) Chain Code

Boundary representation = 0766666453321212
Chain Code: example

8-directional chain code \[\rightarrow 00006066666664444422222022022]\n
Starting point normalized chain code \[\rightarrow 00006066666664444422222022022]\n
Rotation normalized chain code \[\rightarrow 0006200000600006260000620626\]

First difference of chain code
Shape Number –
A boundary descriptor

Order 4

Chain code: 0 3 2 1
Difference: 3 3 3 3
Shape no.: 3 3 3 3

Order 6

Chain code: 0 0 3 2 2 1
Difference: 3 0 3 3 0 3
Shape no.: 0 3 3 0 3 3

Order 8

Chain code: 0 0 3 2 2 1 1
Difference: 3 0 3 0 3 0 3
Shape no.: 0 3 0 3 1 3 3

Order 10

Chain code: 0 0 0 3 2 2 2 2 2 1 1
Difference: 3 0 0 3 1 1 3 0 1 3 0
Shape no.: 0 0 0 3 1 0 3 3 0 1 3 0 3 1 3 0 3
Boundary descriptor – Fourier

\[ s(k) = x(k) + jy(k) \quad k = 0, 1, 2, \ldots, K - 1 \]

\[ a(u) = \sum_{k=0}^{K-1} s(k)e^{-j2\pi uk/K} \quad u = 0, 1, 2, \ldots, K - 1 \]

\[ s(k) = \frac{1}{K} \sum_{u=0}^{K-1} a(u)e^{j2\pi uk/K} \quad k = 0, 1, 2, \ldots, K - 1 \]

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Boundary</th>
<th>Fourier Descriptor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identity</td>
<td>( s(k) )</td>
<td>( a(u) )</td>
</tr>
<tr>
<td>Rotation</td>
<td>( s_r(k) = s(k)e^{j\theta} )</td>
<td>( a_r(u) = a(u)e^{j\theta} )</td>
</tr>
<tr>
<td>Translation</td>
<td>( s_t(k) = s(k) + \Delta xy )</td>
<td>( a_t(u) = a(u) + \Delta xy\delta(u) )</td>
</tr>
<tr>
<td>Scaling</td>
<td>( s_s(k) = \alpha s(k) )</td>
<td>( a_s(u) = \alpha a(u) )</td>
</tr>
<tr>
<td>Starting point</td>
<td>( s_p(k) = s(k - k_0) )</td>
<td>( a_p(u) = a(u)e^{-j2\pi k_0 u/K} )</td>
</tr>
</tbody>
</table>
Boundary Reconstruction using Fourier Descriptors

2868 descriptors

Only 8 descriptors

<table>
<thead>
<tr>
<th>100%</th>
<th>50%</th>
<th>10%</th>
<th>5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Image](4/15/2008 950% 10% 5%2.5% 1.25%100%0.63%0.28% Only 8 descriptors2868 descriptors)</td>
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</tr>
</tbody>
</table>
Boundary Representation: Signatures

- Represent 2-D boundary shape using 1-D signature signal
Boundary Representation: Signatures
Boundary Description using Statistical Moments

\[ \mu_n(v) = \sum_{i=0}^{A-1} (v_i - m)^n p(v_i) \]  
\text{n-th moment of } v

\[ m = \sum_{i=1}^{A-1} v_i p(v_i) \]
Region Descriptors - Simple

- Area
- Perimeter
- Compactness
- Circularity Ratio
- Mean/Median intensity
- Max/Min intensity
- Normalized area

\[ R_c = \frac{A}{P^2 / 4\pi} \]

Area of circle with same perimeter as the shape

\[ C : \quad 4\pi \quad 5\pi \quad 16 \]

\[ R_c : \quad 1 \quad \frac{4}{5} \approx 0.8 \quad \frac{\pi}{4} \approx 0.78 \]
Topological Region Descriptors

• **Topological properties:** Properties of image preserved under rubber-sheet distortions

\[ H: \# \text{ holes in the image} \]
\[ C: \# \text{ connected components} \]
\[ E = C - H: \text{ Euler Number} \]

\[ V - Q + F = C - H = E \]
Geometric Moment Invariants

\[ m_{pq} = \int \int x^p y^q f(x, y) \, dx \, dy \]  
\( (p+q) \)-th 2D geometric moment

\[ m_{pq} = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} x^p y^q f(x, y) \]  
Projection of \( f(x,y) \) onto monomial \( x^p y^q \)

- Why use moments?
  - Geometric moments of different orders represent spatial characteristics of the image intensity distribution

\[ m_{00} \]  
Total intensity of image. For binary image \( \rightarrow \) area

\[ x_0 = \frac{m_{10}}{m_{00}} \]  
Intensity centroid

\[ y_0 = \frac{m_{01}}{m_{00}} \]  
binary image \( \rightarrow \) geometrical center
Central Moments

\[ \mu_{pq} = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} (x - x_0)^p (y - y_0)^q f(x, y) \]

[Translation invariance]

\[ \mu_{00} = m_{00} \]
\[ \mu_{10} = \mu_{01} = 0 \]
\[ \mu_{02}, \mu_{20} \quad \text{Variance about the centroid} \]
\[ \mu_{11} \quad \text{covariance} \]

Scaled Central Moment

\[ \lambda_{pq} = \mu'_{pq} / (\mu'_{00})^{(p+q+2)/2} \]
Scale and translation invariant
\[ \mu'_{pq} = \frac{\mu_{pq}}{\alpha^{p+q+2}} \]

Normalized Un-Scaled Central Moment

\[ \eta_{pq} = \mu_{pq} / (\mu_{00})^{(p+q+2)/2} \]
Moment Invariants
(translation, scale, mirroring, rotation)

\[
\phi_1 = \eta_{20} + \eta_{02}
\]

\[
\phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2
\]

\[
\phi_3 = (\eta_{30} - \eta_{12})^2 + (\eta_{21} - \eta_{03})^2
\]

\[
\phi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2
\]

\[
\phi_5 = \cdots
\]

\[
\phi_6 = \cdots
\]

\[
\phi_7 = \cdots
\]

<table>
<thead>
<tr>
<th>Moment Invariant</th>
<th>Original Image</th>
<th>Translated</th>
<th>Half Size</th>
<th>Mirrored</th>
<th>Rotated 45°</th>
<th>Rotated 90°</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_1)</td>
<td>2.8662</td>
<td>2.8662</td>
<td>2.8664</td>
<td>2.8662</td>
<td>2.8661</td>
<td>2.8662</td>
</tr>
<tr>
<td>(\phi_2)</td>
<td>7.1265</td>
<td>7.1265</td>
<td>7.1257</td>
<td>7.1265</td>
<td>7.1266</td>
<td>7.1265</td>
</tr>
<tr>
<td>(\phi_3)</td>
<td>10.4109</td>
<td>10.4109</td>
<td>10.4047</td>
<td>10.4109</td>
<td>10.4115</td>
<td>10.4109</td>
</tr>
<tr>
<td>(\phi_4)</td>
<td>10.3742</td>
<td>10.3742</td>
<td>10.3719</td>
<td>10.3742</td>
<td>10.3742</td>
<td>10.3742</td>
</tr>
</tbody>
</table>

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Affine Transform & Affine Moment Invariants

\[ x' = T_x(x, y) \]
\[ y' = T_y(x, y) \]

\[ x' = \sum_{r=0}^{m} \sum_{k=0}^{m-r} a_{rk} x^r y^k \]
\[ y' = \sum_{r=0}^{m} \sum_{k=0}^{m-r} b_{rk} x^r y^k \]

In practice: bilinear transform

4 pairs of corresponding points needed to find coefficients

\[ x' = a_0 + a_1 x + a_2 y + a_3 xy \]
\[ y' = b_0 + b_1 x + b_2 y + b_3 xy \]

In practice: affine transform

3 pairs of corresponding points needed to find coefficients

\[ x' = a_0 + a_1 x + a_2 y \]
\[ y' = b_0 + b_1 x + b_2 y \]

Rotation:

\[ x' = x \cos \phi + y \sin \phi \]
\[ y' = -x \sin \phi + y \cos \phi \]

Scale:

\[ x' = ax \]
\[ y' = by \]

Skew:

\[ x' = x + y \tan \phi \]
\[ y' = y \]
Elliptical Shape Descriptors

Principal moment of inertia:

\[ I_1 = \frac{\mu_{20} + \mu_{02} + [ (\mu_{20} - \mu_{02})^2 + 4\mu_{11}^2 ]^{1/2}}{2} \]

\[ I_2 = \frac{\mu_{20} + \mu_{02} - [ (\mu_{20} - \mu_{02})^2 + 4\mu_{11}^2 ]^{1/2}}{2} \]

\( (I_1 + I_2) / m_{00} \) spreadness

\( (I_2 - I_1) / (I_1 + I_2) \) elongation

Image ellipse characterizes fundamental shape features and also 2D position and orientation

\[ \theta = 0.5 \tan^{-1} \left( \frac{2\mu_{11}}{\mu_{20} - \mu_{02}} \right) \]

\[ a = 2(I_1 / \mu_{00})^{1/2} \quad b = 2(I_2 / \mu_{00})^{1/2} \]
Texture - Definition
Texture – Quantification Methods

- Statistical: compute local features at each point in image and derive a set of statistics from the distribution of local features
  - 1\textsuperscript{st}, 2\textsuperscript{nd}, and higher-order statistics based on how many points are used to define local features

- Structural: texture is considered to be composed of “texture elements”. Properties of the “texture elements” and their spatial placement rules characterizes the texture
  - Original texture can be reconstructed from its structural description
Statistical Texture Analysis
1\textsuperscript{st} order statistics

\[
\text{image } f \rightarrow h_f \text{ histogram}
\]

- Obtain statistics of the histogram:

\[
\text{Mean: } \sum_{i=0}^{L-1} ih(i) \quad \text{: average intensity}
\]

\[
\text{Variance: } \sum_{i=0}^{L-1} (i - \mu)^2 h(i) \quad \text{: measure of intensity contrast}
\]

\[
\text{Skewness: } \sum_{i=0}^{L-1} (i - \mu)^3 h(i)
\]

\[
\text{Entropy: } -\sum_{i=0}^{L-1} h(i) \log h(i) \quad \text{Measure of variability of intensity}
\]
<table>
<thead>
<tr>
<th>Texture</th>
<th>Mean</th>
<th>Standard deviation</th>
<th>$R$ (normalized)</th>
<th>Third moment</th>
<th>Uniformity</th>
<th>Entropy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth</td>
<td>82.64</td>
<td>11.79</td>
<td>0.002</td>
<td>$-0.105$</td>
<td>0.026</td>
<td>5.434</td>
</tr>
<tr>
<td>Coarse</td>
<td>143.56</td>
<td>74.63</td>
<td>0.079</td>
<td>$-0.151$</td>
<td>0.005</td>
<td>7.783</td>
</tr>
<tr>
<td>Regular</td>
<td>99.72</td>
<td>33.73</td>
<td>0.017</td>
<td>0.750</td>
<td>0.013</td>
<td>6.674</td>
</tr>
</tbody>
</table>
Statistical Texture Analysis
1st order statistics

image

histogram

statistics

Skewness = 2.08
Entropy = 0.88

Skewness = 2.44
Entropy = 0.77

Skewness = -0.092
Entropy = 0.97
Statistical Texture Analysis

2\textsuperscript{nd} order statistics: Co-occurrence

\[ f(m_2,n_2) = j \]

\[ P_{(d,\theta)}(i, j) \approx \Pr[f(m_1,n_1) = i, f(m_2,n_2) = j] \]

\[ f(m_1,n_1) = i \]

- Joint gray-level histogram of pairs of pixels
- 2D histogram
Statistical Texture Analysis

$2^{\text{nd}}$ order statistics: Co-occurrence

\[
P_{(d,\theta=0^\circ)}(i, j) = \left| \left\{ ((k, l), (m, n)) \in (M \times N) \times (M \times N) : \right. \right.
\]
\[
k - m = 0, \left| l - n \right| = d, f(k, l) = i, f(m, n) = j \left. \right\} \right|
\]

\[
P_{(d,\theta=45^\circ)}(i, j) = \left| \left\{ ((k, l), (m, n)) \in (M \times N) \times (M \times N) : \right. \right.
\]
\[
(k - m = d, l - n = -d) \lor (k = m = -d, l - n = d), f(k, l) = i, f(m, n) = j \left. \right\} \right|
\]

\[
P_{(d,\theta=90^\circ)}(i, j)
\]

\[
P_{(d,\theta=135^\circ)}(i, j)
\]

\[
\left\{ \cdots \right\} \text{ is set cardinality}
\]
Statistical Texture Analysis

2\textsuperscript{nd} order statistics: Co-occurrence (example)

image

Co-occurrence matrix

\[
\begin{bmatrix}
0 & 0 & 1 & 1 \\
0 & 0 & 1 & 1 \\
0 & 2 & 2 & 2 \\
2 & 2 & 3 & 3 \\
\end{bmatrix}
\]
Statistical Texture Analysis
2\textsuperscript{nd} order statistics: Co-occurrence (statistics)

Angular 2\textsuperscript{nd} moment (energy):
(measure of image homogeneity)

\[
\sum_{i=1}^{L} \sum_{j=1}^{L} P_{(d,\theta)}^2(i, j)
\]

Maximum Probability:

\[
\max_{i,j} P_{(d,\theta)}(i, j)
\]

Entropy:

\[
-\sum_{i=1}^{L} \sum_{j=1}^{L} P_{(d,\theta)}(i, j) \log P_{(d,\theta)}(i, j)
\]

Contrast:
(measure of local variations)

\[
\sum_{i=1}^{L} \sum_{j=1}^{L} |i - j|^{\kappa} P_{(d,\theta)}^4(i, j)
\]

Correlation:
(measure of image linearity)

\[
\frac{\sum_{i=1}^{L} \sum_{j=1}^{L} [ijP_{(d,\theta)}(i, j)] - \mu_x \mu_y}{\sigma_x \sigma_y}
\]

\[
\mu_x = \sum_{i=1}^{L} i \sum_{j=1}^{L} P_{(d,\theta)}(i, j)
\]

\[
\sigma_x = \sum_{i=1}^{L} (i - \mu_x)^2 \sum_{j=1}^{L} P_{(d,\theta)}(i, j)
\]

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<table>
<thead>
<tr>
<th>Normalized Co-occurrence Matrix</th>
<th>Max Probability</th>
<th>Correlation</th>
<th>Contrast</th>
<th>Uniformity</th>
<th>Homogeneity</th>
<th>Entropy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_1/n_1$</td>
<td>0.00006</td>
<td>-0.0005</td>
<td>10838</td>
<td>0.00002</td>
<td>0.0366</td>
<td>15.75</td>
</tr>
<tr>
<td>$G_2/n_2$</td>
<td>0.01500</td>
<td>0.9650</td>
<td>570</td>
<td>0.01230</td>
<td>0.0824</td>
<td>6.43</td>
</tr>
<tr>
<td>$G_3/n_3$</td>
<td>0.06860</td>
<td>0.8798</td>
<td>1356</td>
<td>0.00480</td>
<td>0.2048</td>
<td>13.58</td>
</tr>
</tbody>
</table>
Statistical Texture Analysis

2\textsuperscript{nd} order statistics: Difference Statistics

\[ P_{(d, \theta)}(k) = \sum_{i, j \in \{1, \ldots, L\}} P_{(d, \theta)}(i, j) \quad \text{is a subset of co-occurrence matrix} \]

Angular 2\textsuperscript{nd} moment (energy):

\[ \sum_{k=0}^{L-1} P_{(d, \theta)}^{2}(k) \]

Mean:

\[ \sum_{k=0}^{L-1} kP_{(d, \theta)}(k) \]

Entropy:

\[ -\sum_{k=0}^{L-1} P_{(d, \theta)}(k) \log P_{(d, \theta)}(k) \]

Contrast:

\[ \sum_{k=0}^{L-1} k^2 P_{(d, \theta)}(k) \]
Statistical Texture Analysis

2nd order statistics: Autocorrelation

\[ C_{ff}(p, q) = \frac{MN}{(M - p)(N - q)} \sum_{k=1}^{M-p} \sum_{l=1}^{N-q} f(k, l) f(k + p, l + q) \]

\[ \sum_{k=1}^{M} \sum_{l=1}^{N} f^2(k, l) \]

Large texture elements \( \Rightarrow \) autocorrelation decreases slowly with increasing distance

Small texture elements \( \Rightarrow \) autocorrelation decreases rapidly with increasing distance

Periodic texture elements \( \Rightarrow \) periodic increase & decrease in autocorrelation value
Statistical Texture Analysis

2\textsuperscript{nd} order statistics: Fourier Power Spectrum

\[ f(x, y) \leftrightarrow F(u, v) \]

Power Spectrum

\[ P(u, v) = |F(u, v)|^2 \]

Note:

\[ C_{ff} = F^{-1}\{ |F(u, v)|^2 \} \]

Indicator for size of dominant texture element or texture coarseness

\[ P(r) = 2 \sum_{\theta=0}^{\pi} P(r, \theta) \]

\[ P(\theta) = \sum_{r=0}^{L/2} P(r, \theta) \]

Indicator for the directionality of the texture
Law’s Texture Energy Measures

\[ L_3 = [1,2,1] \quad E_3 = [-1,0,1] \quad S_3 = [-1,2,-1] \]

\[ L_5 = [1,4,6,4,1] \quad E_5 = [-1,-2,0,2,1] \quad S_5 = [-1,0,2,0,-1] \quad R_5 = [1,-4,6,-4,1] \quad W_5 = [-1,2,0,-2,-1] \]

\[ L_5^T \times S_5 = \begin{bmatrix}
-1 & 0 & 2 & 0 & -1 \\
-4 & 0 & 8 & 0 & -4 \\
-6 & 0 & 12 & 0 & -6 \\
-4 & 0 & 8 & 0 & -4 \\
-1 & 0 & 2 & 0 & -1 \\
\end{bmatrix} \]

- Convolute different Law’s masks with image
- Compute energy statistics
Difference image:

\[ d(i, j) = \begin{cases} 0 & \text{if } |f_1(i, j) - f_2(i, j)| \leq \varepsilon \\ 1 & \text{otherwise} \end{cases} \]

No motion direction information!

\[ d_{cum}(i, j) = \sum_{k=1}^{n} a_k |f_1(i, j) - f_k(i, j)| \]

Tells us how often the image gray level was different from gray-level of reference image

**FIGURE 10.49** ADIs of a rectangular object moving in a southeasterly direction. (a) Absolute ADI, (b) Positive ADI, (c) Negative ADI.
Motion Field

- A velocity vector is assigned to each pixel in the image
- Velocities due to relative motion between camera and the 3D scene
- Image change due to motion during a time interval dt
- Velocity field that represents 3-dimensional motion of object points across 2-dimensional image
Optical flow

• Motion of brightness patterns in image sequence

• Assumptions for computing optical flow:
  • Observed brightness of any object point is constant over time
  • Nearby points in the image plane move in a similar manner

\[
f(x + dx, y + dy, t + dt) = f(x, y, t) + \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial t} dt + O(\partial^2)
\]

\[
= f(x, y, t) + f_x dx + f_y dy + f_t dt + O(\partial^2)
\]

\[
f(x + dx, y + dy, t + dt) = f(x, y, t) \Rightarrow - f_t = f_x \frac{dx}{dt} + f_y \frac{dy}{dt}
\]

\[
c = (\frac{dx}{dt}, \frac{dy}{dt}) = (u, v)
\]

Gray-level difference at same location over time is equivalent to product of spatial gray-level difference and velocity

\[
- f_t = f_x u + f_y v = \nabla f \cdot c
\]
Optical Flow Constraints

\[-f_t = f_x u + f_y v\]

- no spatial change in brightness, induce no temporal change in brightness \(\rightarrow\) no discernible motion

- motion perpendicular to local gradient induce no temporal change in brightness \(\rightarrow\) no discernible motion

- motion in direction of local gradient, induce temporal change in brightness \(\rightarrow\) discernible motion

- only motion in direction of local gradient induces temporal change in brightness and discernible motion
Optical flow $\neq$ Motion Field

$MF \neq 0$

$OF = 0$

$MF = 0$

$OF \neq 0$
Which descriptors?
Image Feature Evaluation

1. Prototype Performance
   - Classify (Segment) image using different features
   - Evaluate which feature is optimal (minimum classification error)

2. Figure of Merit
   - Establish functional distance measurements between set of image features (large distance → low classification error)
   - Bhattacharyya distance