Columbia University in the City of New York

EE4830 Digital Image Processing
Lecture 7

Image Restoration

March 19th, 2007

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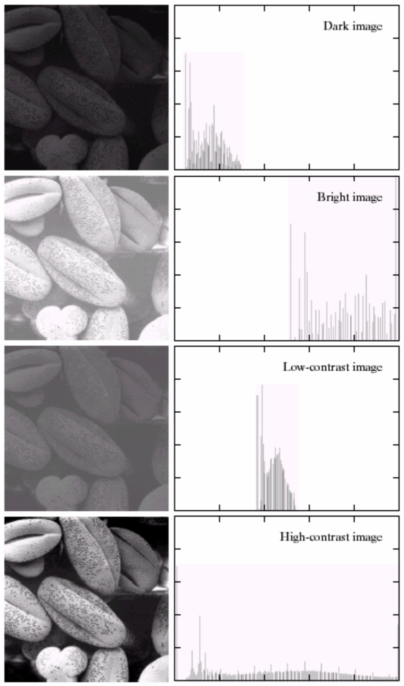
We have covered ...

Image sensing



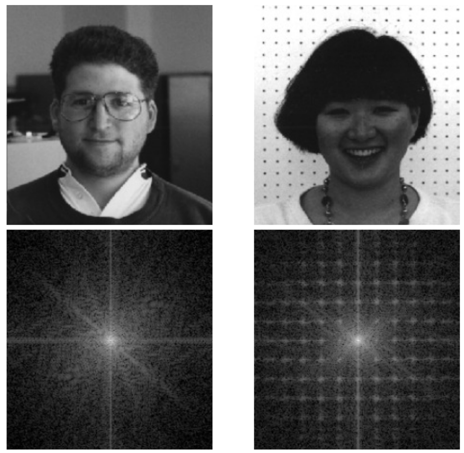


Image Transform and Filtering

Spatial Domain processing

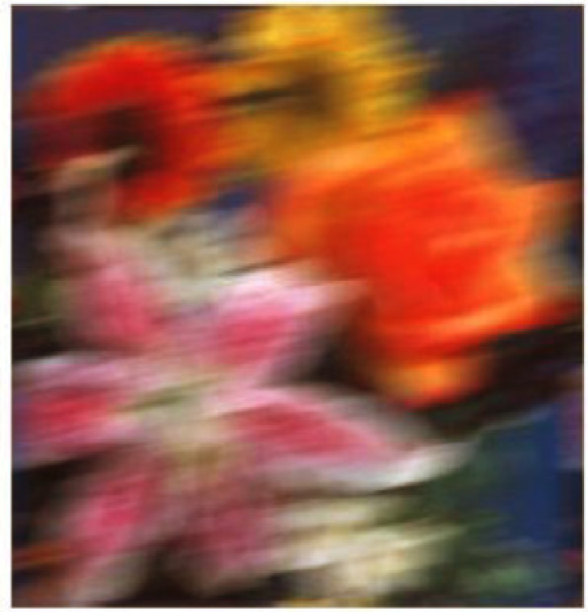
Image Restoration

Lecture Outline

- What is image restoration
  - Scope, history and applications
  - A model for (linear) image degradation
- Restoration from noise
  - Different types of noise
  - Examples of restoration operations
- Restoration from linear degradation
  - Inverse and pseudo-inverse filtering
  - Wiener filters
  - Blind de-convolution
- Geometric distortion and its corrections

Degraded Images

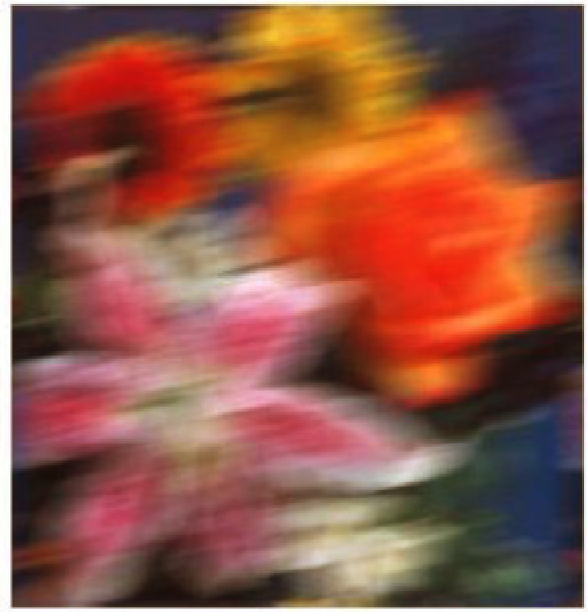
Original image



Blurred image

- What caused the image to blur?
- Can we improve the image, or “undo” the effects?

Original image



Blurred image

- Image enhancement: "improve" an image subjectively.
- Image restoration: remove distortion from image in order to go back to the "original" → objective process.

Image Restoration

- Started from the 1950s
- Application domains
  - Scientific explorations
  - Legal investigations
  - Film making and archival
  - Image and video (de-)coding
  - …
  - Consumer photography

- Related problem: image reconstruction in radio astronomy, radar imaging and tomography

See [Banham and Katsaggelos 97]

A Model for Image Distortion

- Image enhancement: "improve" an image subjectively.
- Image restoration: remove distortion from image, to go back to the "original" → objective process

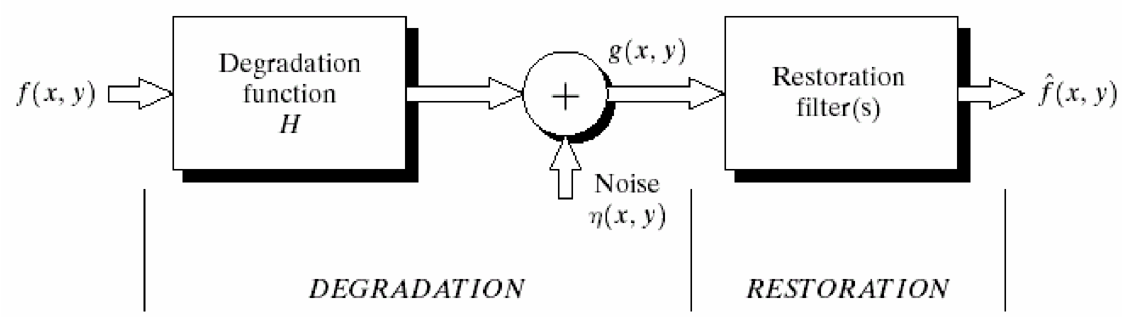


FIGURE 5.1 A model of the image degradation/restoration process.

$$g(x, y) = H[f(x, y)] + \eta(x, y)$$

A Model for Image Distortion

- Image restoration
  - Use a priori knowledge of the degradation
  - Modeling the degradation and apply the inverse process
  - Formulate and evaluate objective criteria of goodness

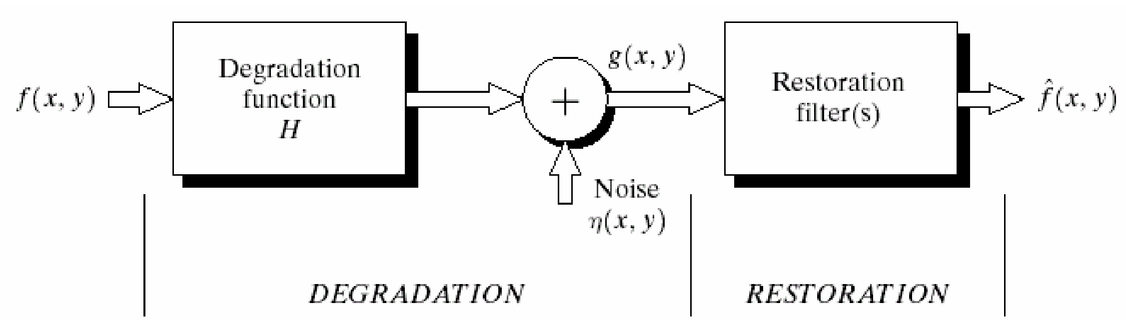


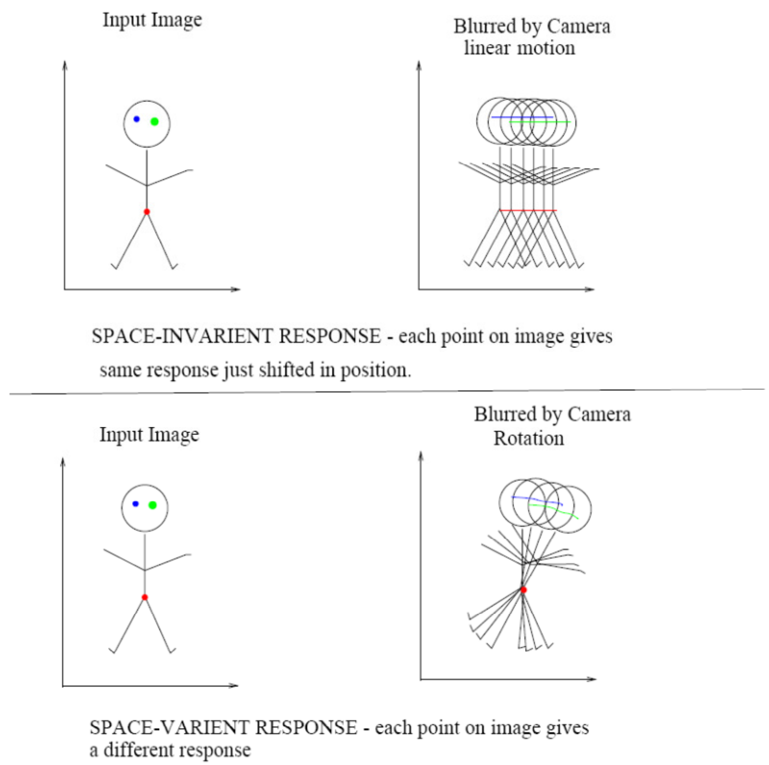
FIGURE 5.1 A model of the image degradation/restoration process.

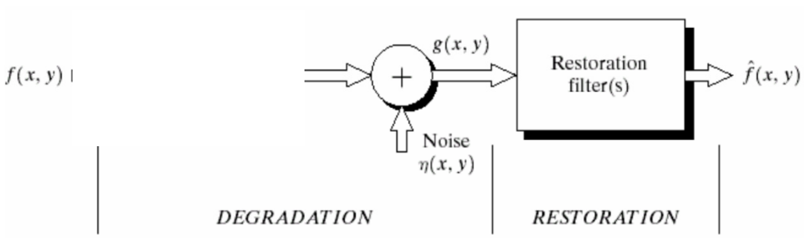
$$g(x,y) = H[f(x,y)] + \eta(x,y)$$

$\rightarrow$ design restoration filters such that $\hat{f}(x,y)$ is as close to $f(x,y)$ as possible.

Usual Assumptions for the Distortion Model

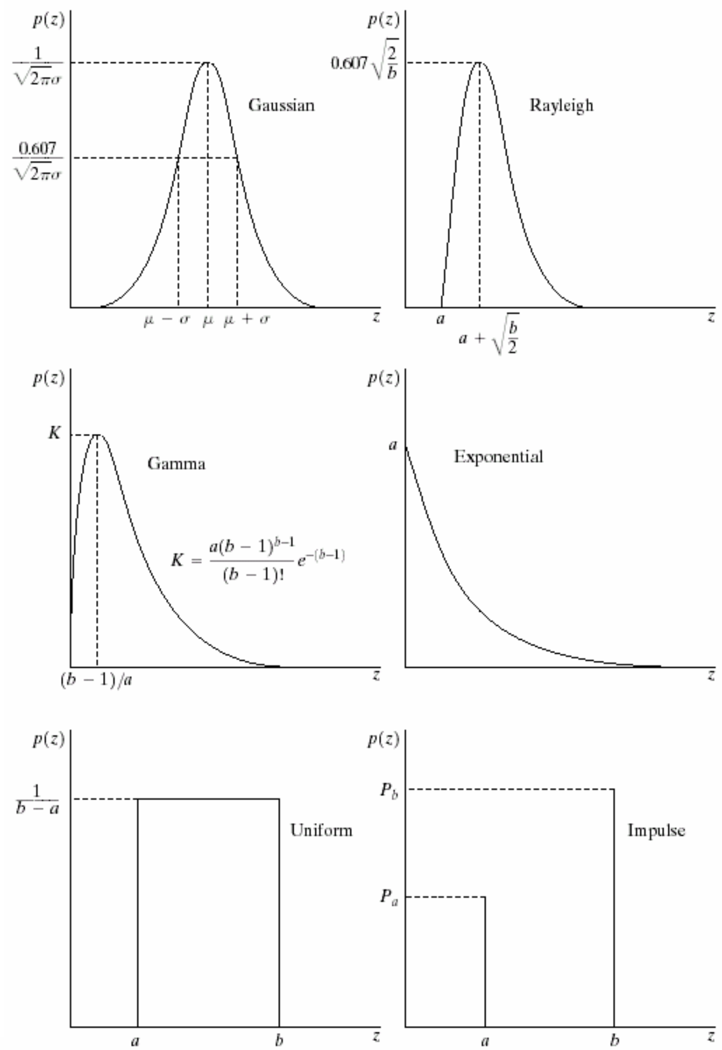
- Noise
  - Independent of spatial location
    - Exception: periodic noise …
  - Uncorrelated with image
- Degradation function
  - Linear
  - Position-invariant





Divide-and-conquer step #1: degraded only by noise.

Common Noise Models



a b
c d
e f

FIGURE 5.2 Some important probability density functions.

Gaussian

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\mu)^2/2\sigma^2}$$

Rayleigh

$$p(z) = \frac{2}{b}(z-a)e^{-(z-a)^2/b}, \textit{for} \qquad z \geq a$$

Erlang, Gamma(a,b)

$$p(z) = \frac{a^b z^{b-1}}{(b-a)!} e^{-az}, \textit{for} \qquad z \geq 0$$

Exponential

$$p(z) = ae^{-az}, \textit{for} \qquad z \geq 0$$

Salt-and-Pepper:

$$p(z) = P_a\delta(z-a) + P_b\delta(z-b)$$

→ additive noise

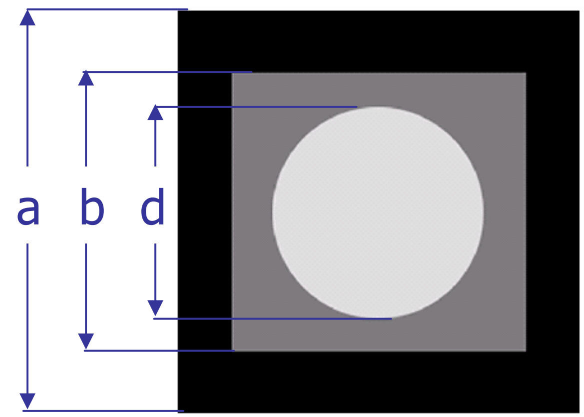
Speckle noise: $a = a_R + ja_I$

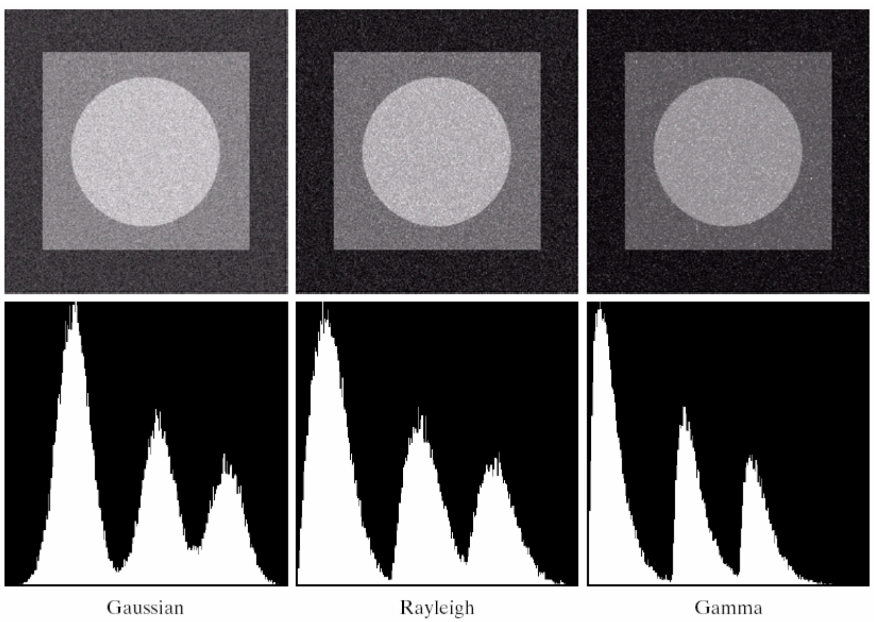
$$|g(x,y)|^2 \simeq |f(x,y)|^2 |a(x,y)|^2 + \eta(x,y)$$

a_R,a_I zero mean, independent Gaussian
→ multiplicative noise on signal magnitude

Visual Effects of Noise

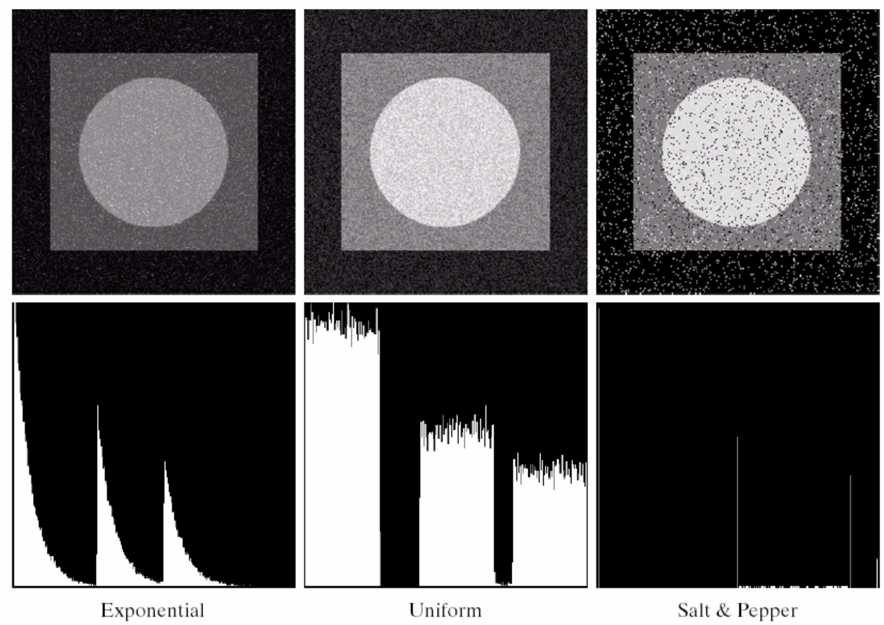
Original image shown on the right with the annotated dimensions.





a b c
d e f

FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.



g h i
j k l

FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and impulse noise to the image in Fig. 5.3.

Recovering from Noise

- Overview of noise reduction
  Observe and estimate noise type and parameters → apply optimal (spatial) filtering (if known) → observe result, adjust filter type/parameters …

- Example noise-reduction filters [G&W 5.3]
  - Mean/median filter family
  - Adaptive filter family
  - Other filter family
    - e.g. Homomorphic filtering for speckle noise [G&W 4.5, Jain 8.13]

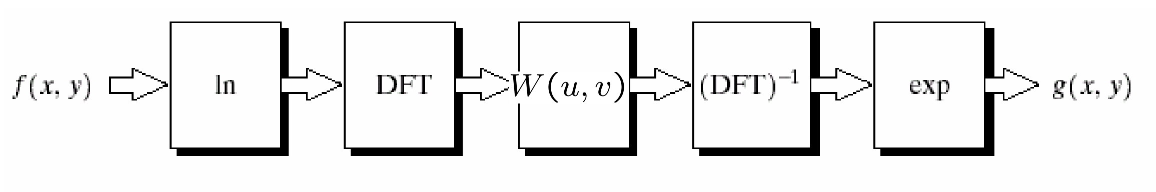


FIGURE 4.31 Homomorphic filtering approach

Recovering from Periodic Noise

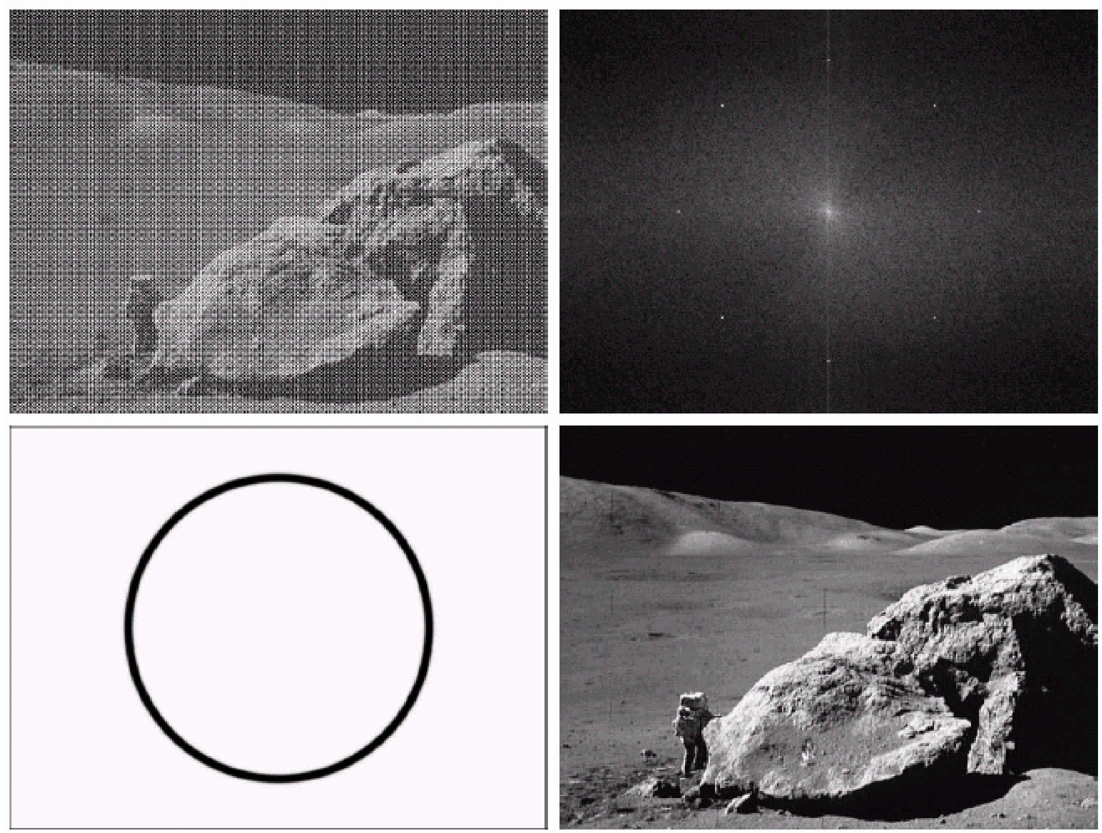
[G&W 5.4]

Recall: Butterworth LPF

$$H(u,v) = \frac{1}{1 + [D(u,v)/D_0]^{2n}}$$

Butterworth bandreject filter

$$H(u,v) = \frac{1}{1 + [\frac{D(u,v)W}{D^2(u,v) - D_0^2}]^{2n}}$$



a b
c d

FIGURE 5.16 (a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)

Lecture Outline

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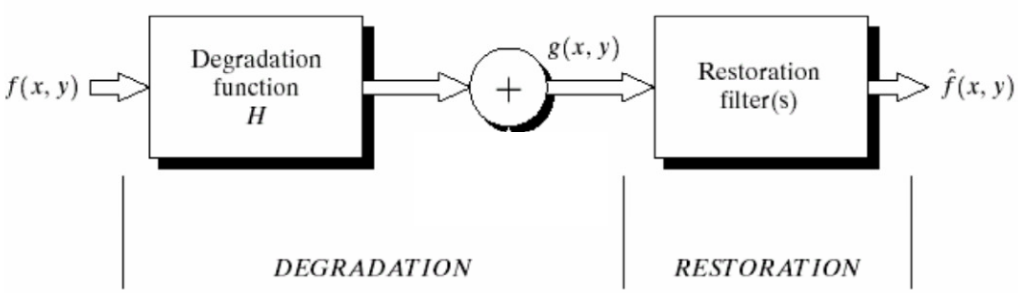
  - Wiener filters
  - Blind de-convolution
- Geometric distortion and example corrections

Recover from Degradation

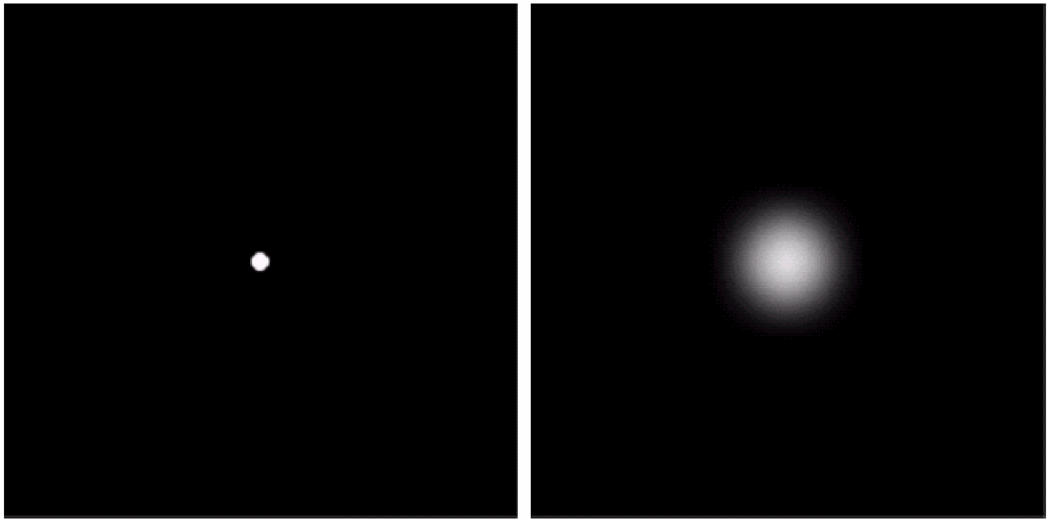
- Degradation function
  - Linear (eq 5.5-3, 5.5-4)
    - Homogeneity
    - Additivity
  - Position-invariant (in cartesian coordinates, eq 5.5-5)

→ linear filtering with H(u,v)
convolution with h(x,y) – point spread function



Divide-and-conquer step #2: linear degradation, noise negligible.

Point Spread Functions



a b

FIGURE 5.24 Degradation estimation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.

Point Spread Functions

Spatial domain

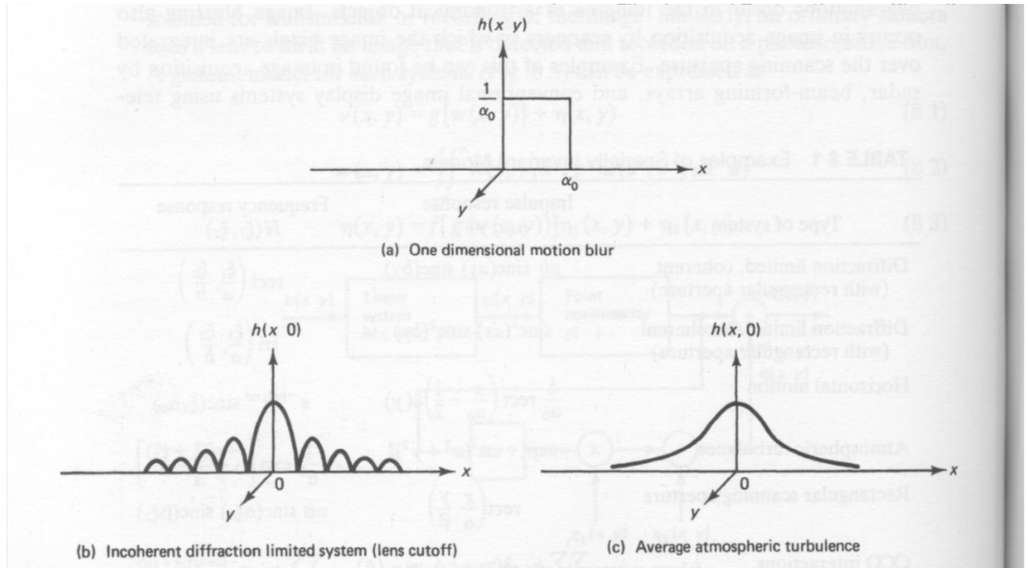


Figure 8.5 Examples of spatially invariant PSFs

Frequency domain

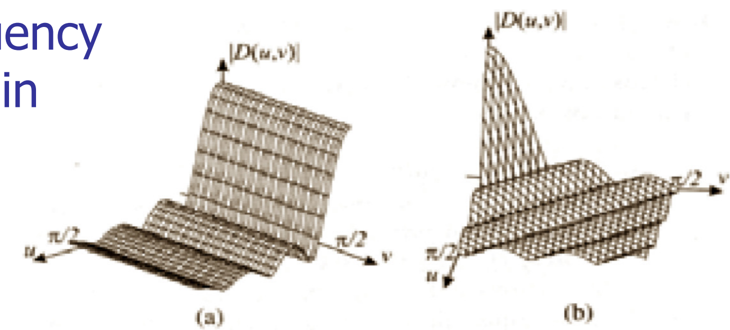


FIGURE 2 PSF of motion blur in the Fourier domain, showing $|D(u, v)|$, for (a) $L = 7.5$ and $\phi = 0$; (b) $L = 7.5$ and $\phi = \pi/4$

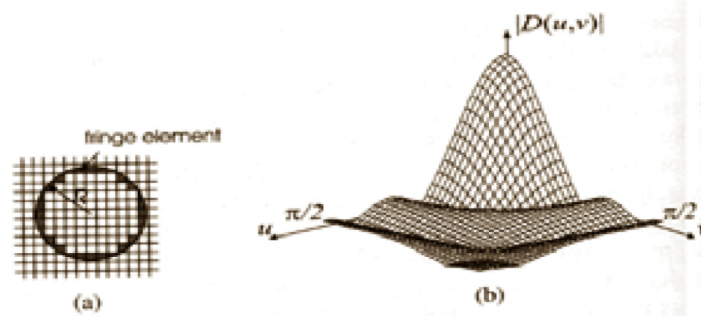
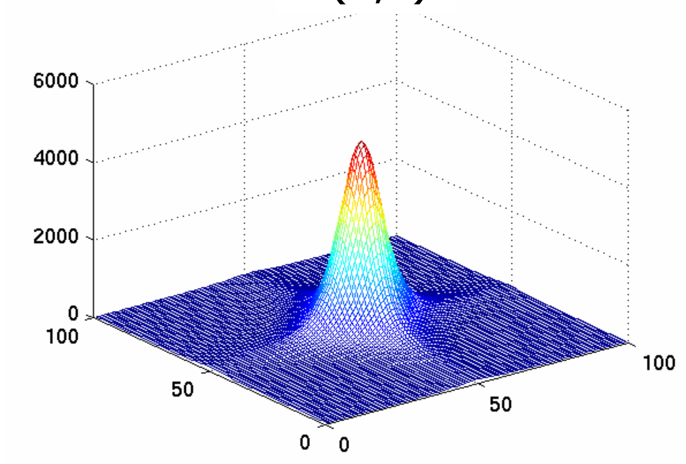


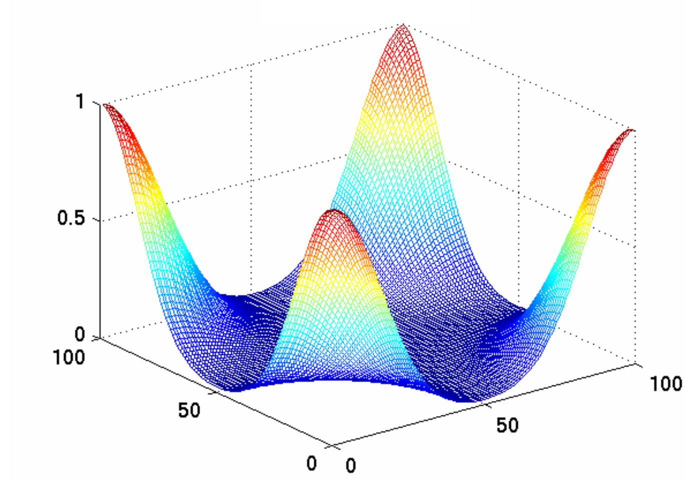
FIGURE 3 (a) Fringe elements of discrete out-of-focus blur that are calculated by integration; (b) PSF in the Fourier domain, showing $|D(u, v)|$, for $R = 2.5$.

Inverse Filter

- Assume h is known: low-pass filter H(u,v)
- Inverse filter $\widehat{H}(u,v) = 1/H(u,v)$
- Recovered Image $\widehat{F}(u,v) = G(u,v)\widehat{H}(u,v)$

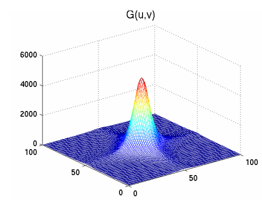
H(u,v)

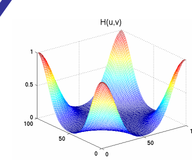




[EE381K, UTexas]

Inverse Filtering Example





loss of
information

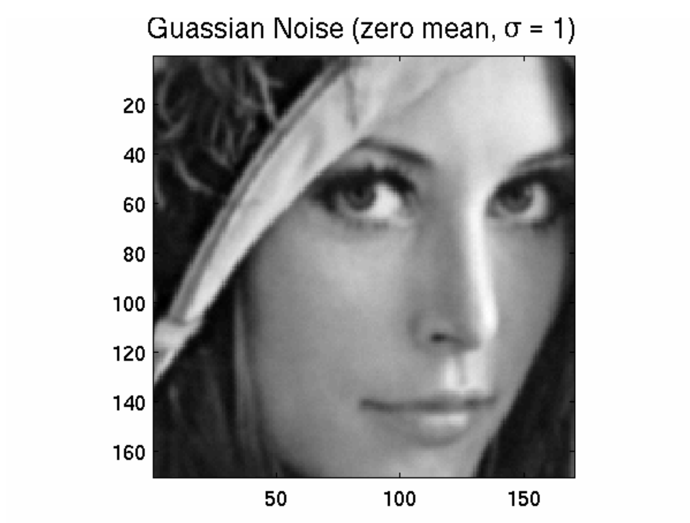
Inverse Filtering under Noise

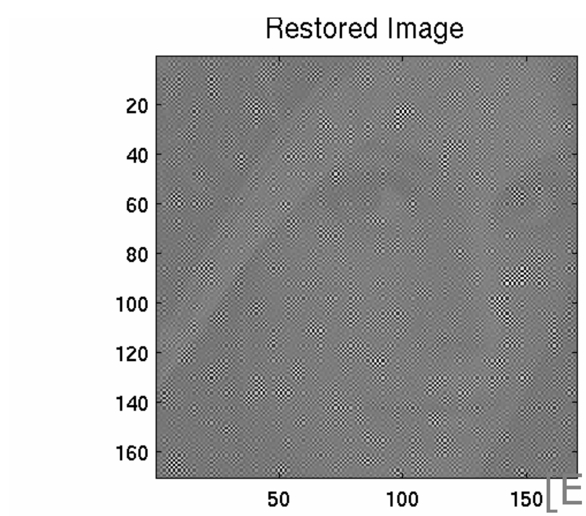
- H(u,v) = 0, for some u, v
- In the noisy case:

$$\hat{H}(u,v) = 1/H(u,v)$$
$$\hat{F}(u,v) = G(u,v)\hat{H}(u,v)$$

$\Rightarrow$

$$G(u,v) = F(u,v)H(u,v) + N(u,v)$$
$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

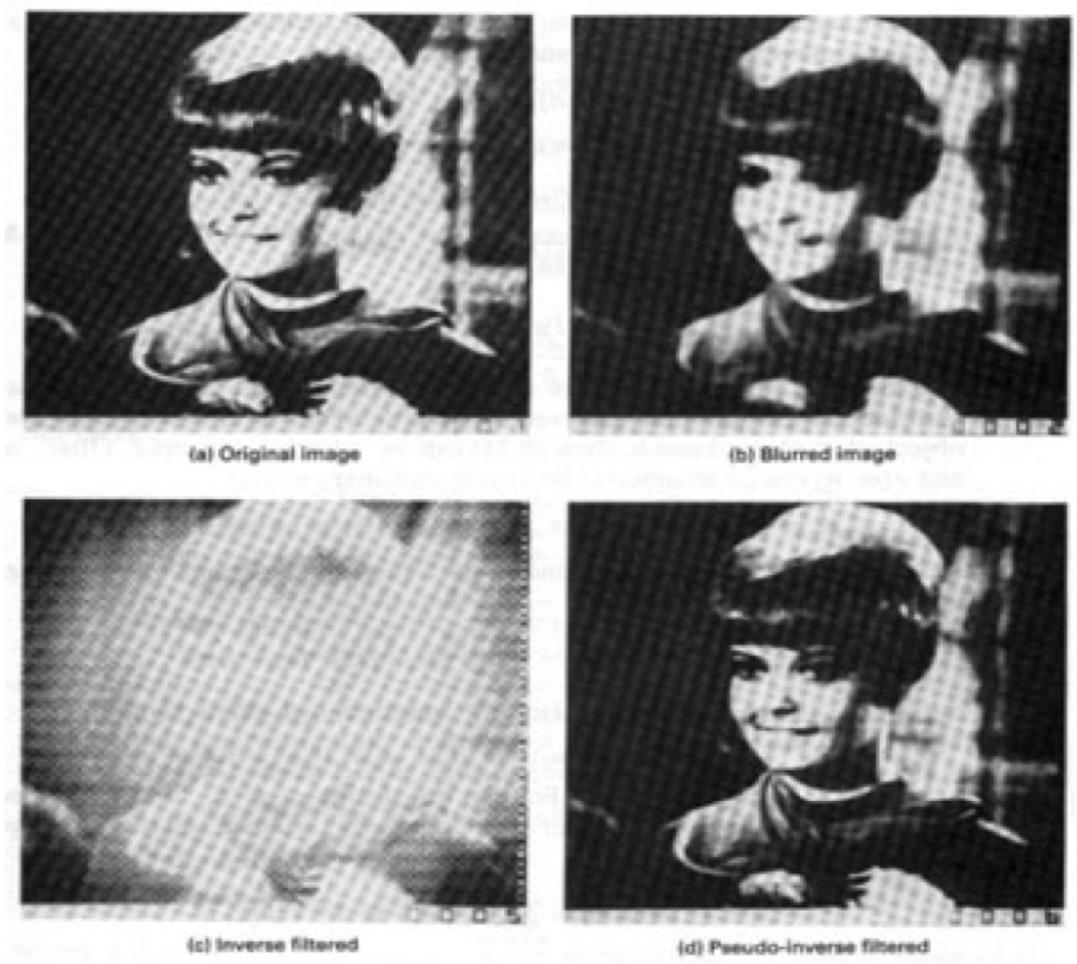




[EE381K, UTexas]

Pseudo-inverse Filtering

$$\hat{H}(u,v) = \begin{cases} 1/H(u,v), & H(u,v) \geq \epsilon \\ 0, & H(u,v) < \epsilon \end{cases}$$


(a) Original image (b) Blurred image
(c) Inverse filtered (d) Pseudo-inverse filtered

[Jain, Fig 8.10]

Back to the Original Problem

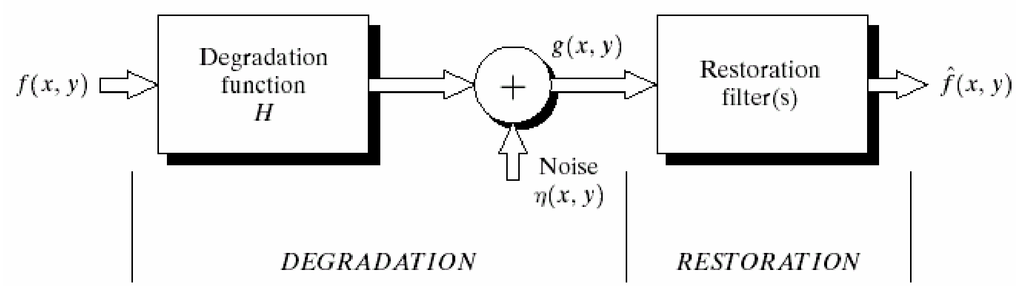


FIGURE 5.1 A model of the image degradation/restoration process.

Pseudo-inverse filter:

$$\hat{H}(u,v) = \begin{cases} 1/H(u,v), & H(u,v) \geq \epsilon \\ 0, & H(u,v) < \epsilon \end{cases}$$

- Can the filter take values between 1/H(u,v) and zero?
- Can we model noise directly?

Wiener Filter

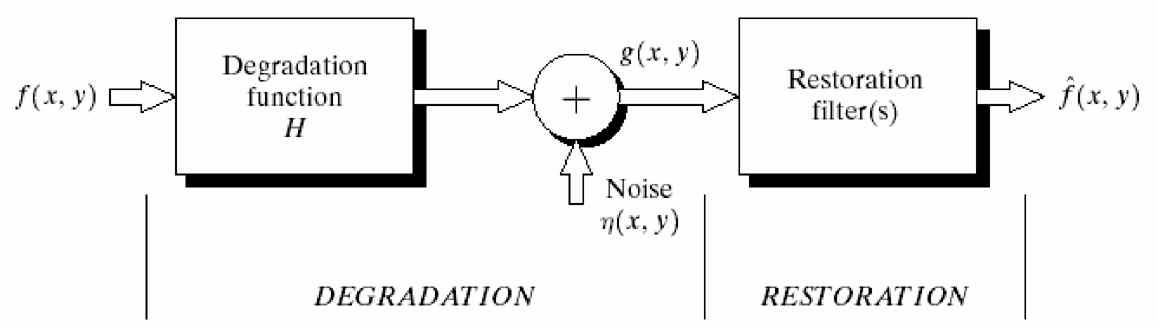


FIGURE 5.1 A model of the image degradation/restoration process.

$W(u,v)$

- Find "optimal" linear filter W(u,v) such that the Mean Square Error between $f(x,y)$ and $\hat{f}(u,v)$ is minimized

$$\min_W \quad e^2 = E\{(f-\hat{f})^2\}$$

(1) orthogonal condition $\quad E\{g(f-\hat{f})\} = 0$

(2) correlation function $\quad R_{fg}(x,y) = W(x,y) \otimes R_{gg}(x,y)$

$$\Rightarrow \quad W(u,v) = \frac{S_{fg}(u,v)}{S_{gg}(u,v)} = \frac{H^*(u,v)S_{ff}(u,v)}{|H(u,v)|^2 S_{ff}(u,v) + S_{\eta\eta}(u,v)}$$

S_{ff} and $S_{\eta\eta}$ are the power spectral densities of the signal and noise, respectively

Observations about Wiener Filter

$$W(u,v) = \frac{H^*(u,v)S_{ff}(u,v)}{|H(u,v)|^2 S_{ff}(u,v) + S_{\eta\eta}(u,v)}$$
$$= \frac{1}{H(u,v) + \frac{S_{\eta\eta}}{H^*(u,v)S_{ff}}}$$

- If no noise, $S_{\eta\eta} \rightarrow 0$ $W(u,v)|_{S_{\eta\eta}\rightarrow 0} = \begin{cases} \frac{1}{H(u,v)} & , if H(u,v) \neq 0 \\ 0 & , if H(u,v) = 0 \end{cases}$

  → Pseudo inverse filter

- If no blur, H(u,v)=1 (Wiener smoothing filter)

$$W(u,v)|_{H=1} = \frac{1}{1 + S_{\eta\eta}(u,v)/S_{ff}(u,v)}$$

  → More suppression on noisier frequency bands

1-D Wiener Filter Shape

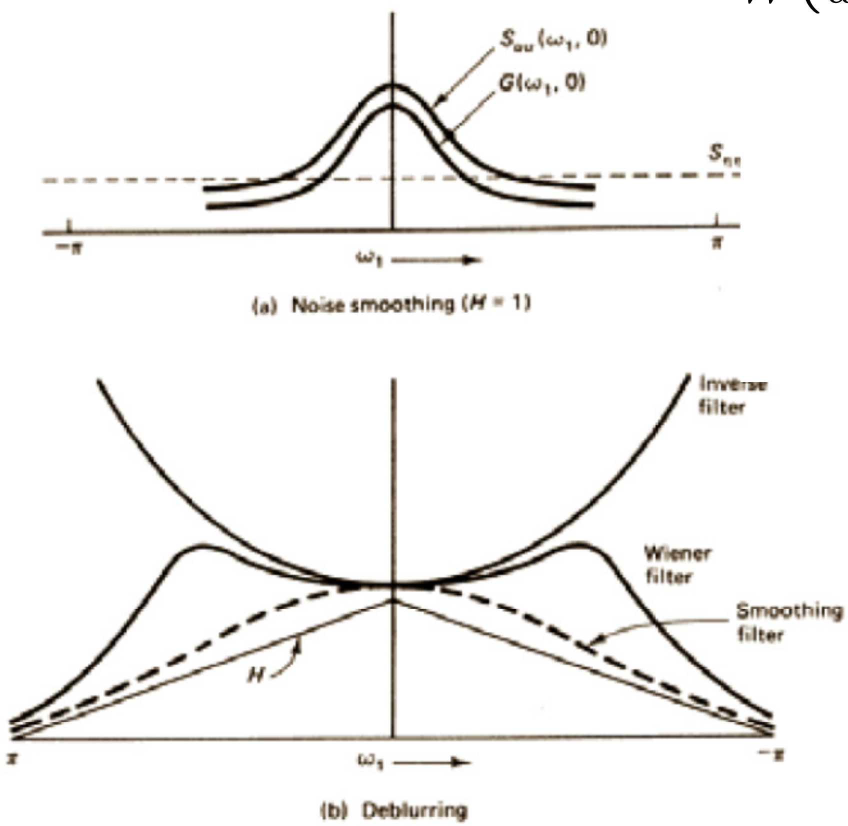


Figure 8.11 Wiener filter characteristics.

[Jain, Fig 8.11]

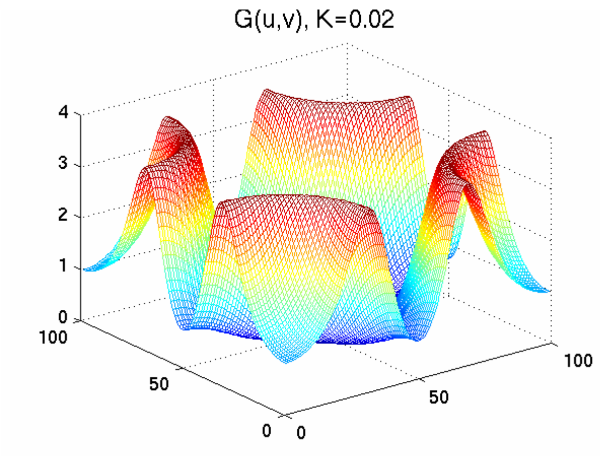
Wiener Filter implementation

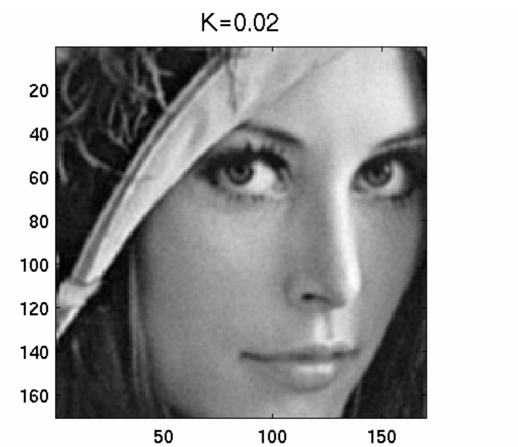
$$
\begin{aligned}
W(u,v) &= \frac{H^*(u,v)S_{ff}(u,v)}{|H(u,v)|^2 S_{ff}(u,v) + S_{\eta\eta}(u,v)} \\
&= \frac{H^*(u,v)}{|H(u,v)|^2 + \frac{S_{\eta\eta}}{S_{ff}}} \\
&= \frac{H^*(u,v)}{|H(u,v)|^2 + K}
\end{aligned}
$$

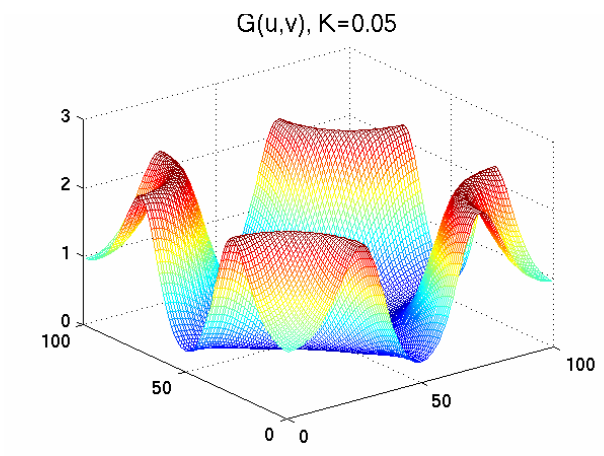
Where K is a constant chosen according to our knowledge of the noise level.

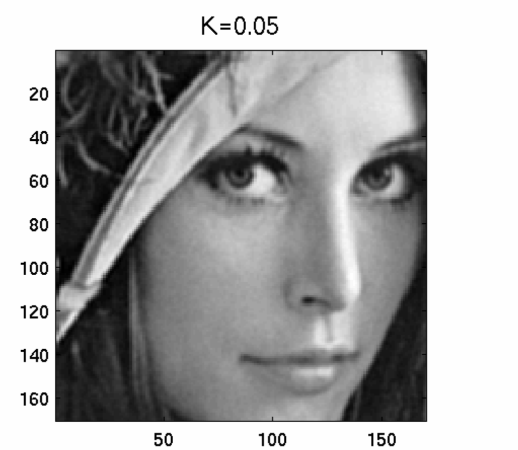
Wiener Filter Example

$$W(u,v) = \frac{H^*(u,v)}{|H(u,v)|^2 + K}$$









[EE381K, UTexas]

Wiener Filter as a LMS Filter

a) Noisy image (*SNR*=20 *dB*) *rms* = 25.7

b) Wiener filter *rms* = 20.2

c) Gauss filter (σ = 1.0) *rms* = 21.1

d) Kuwahara filter (5 × 5) *rms* = 22.4

e) Median filter (3 × 3) *rms* = 22.6

f) Morph. smoothing (3 × 3) *rms* = 26.2

Figure 49: Noise suppression using various filtering techniques.

[Young et. al., Fundamentals of Image Processing, TU-Delft]

Wiener Filter Example

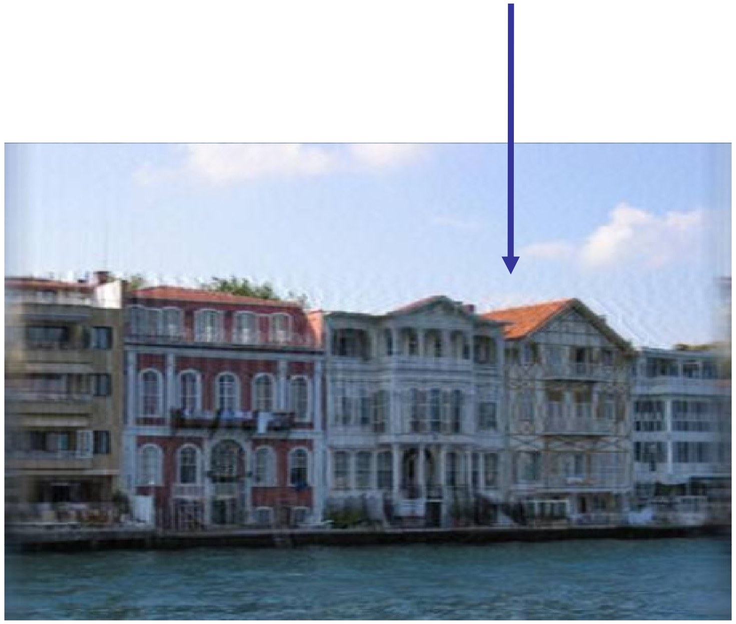
a b c

FIGURE 5.28 Comparison of inverse- and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.

- Wiener filter is robust to noise, and preserves high-frequency details.

Wiener Filter Example

Ringing effect visible, too many high frequency components?

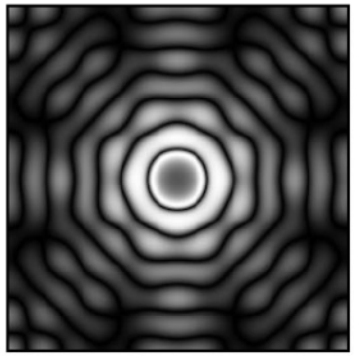


(a) Blurry image (b) restored w. regularized pseudo inverse
(c) restored with wiener filter

[UMD EE631]

Wiener Filter

image 'blurr1'



wiener filter

restored license plate

How much de-blurring is just enough?

[Image Analysis Course, TU-Delft]

Improve Wiener Filter

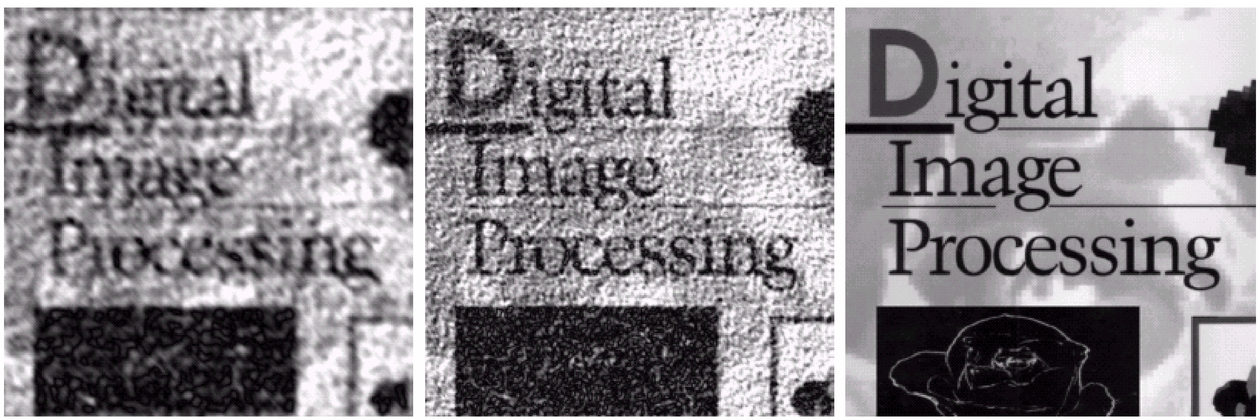
- Constrained Least Squares

  Wiener filter emphasizes high-frequency components, while images tend to be smooth

$$\min_{f} \; |g - H\hat{f}|^2 + \alpha|C\hat{f}|^2$$

$\hat{f}$: the estimate for undegraded image

$C\hat{f}$: a high-passed version of $\hat{f}$



a b c

FIGURE 5.30 Results of constrained least squares filtering. Compare (a), (b), and (c) with the Wiener filtering results in Figs. 5.29(c), (f), and (i), respectively.

Improve Wiener Filter (1)

- Constrained Least Squares

  Wiener filter emphasizes high-frequency components, while images tend to be smooth

  $$\min \ |g - H\hat{f}|^2 + \alpha |C\hat{f}|^2$$

  where $C\hat{f}$ is a high-passed version of $\hat{f}$

- Blind deconvolution

  Wiener filter assumes both the image and noise spectrum are know (or can be easily estimated), in practice this becomes trial-and-error since noise and signal parameters are often hard to obtain.

  $$\log|H|^2 = \log(S_{gg} - S_{\eta\eta}) - \log S_{ff}$$

  $$S_{\eta\eta} \approx 0 \quad \Longrightarrow \quad \log|H| \approx \frac{1}{M}\sum_{k=1}^{M}[log|G_k| - log|F_k|]$$

Maximum-Likelihood (ML) Estimation

- h(x,y) H(u,v) unknown
- Assume parametric models for the blur function, original image, and/or noise
- Parameter set θ is estimated by

$$\theta_{ml} = \arg\{\max_{\theta} p(y \mid \theta)\}$$

- Solution is difficult in general
- Expectation-Maximization algorithm
  - Guess an initial set of parameters θ
  - Restore image via Wiener filtering using θ
  - Use restored image to estimate refined parameters θ
  - ... iterate until local optimum

To explore more: D. Kundur and D. Hatzinakos, "Blind Image Deconvolution," *IEEE Signal Processing Magazine,* vol. 13, no. 3, May 1996, pp. 43-64.

Geometric Distortions

- Modify the spatial relationships between pixels in an image
- a. k. a. "rubber-sheet" transformations

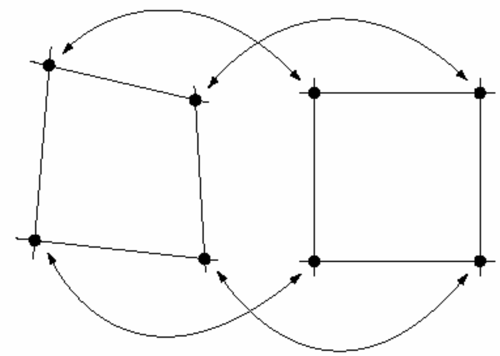


FIGURE 5.32 Corresponding tiepoints in two image segments.

- Two basic steps
  - Spatial transformation
  - Gray-level interpolation

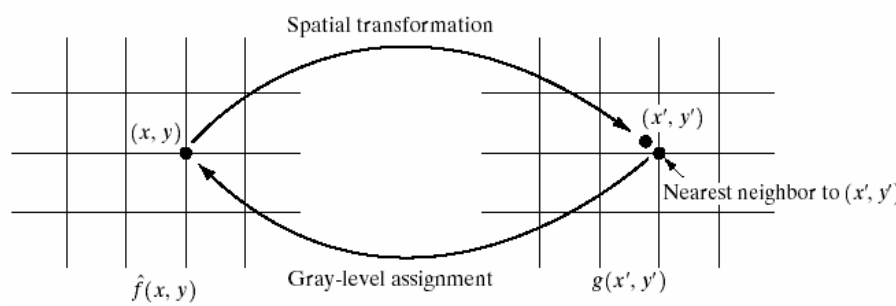
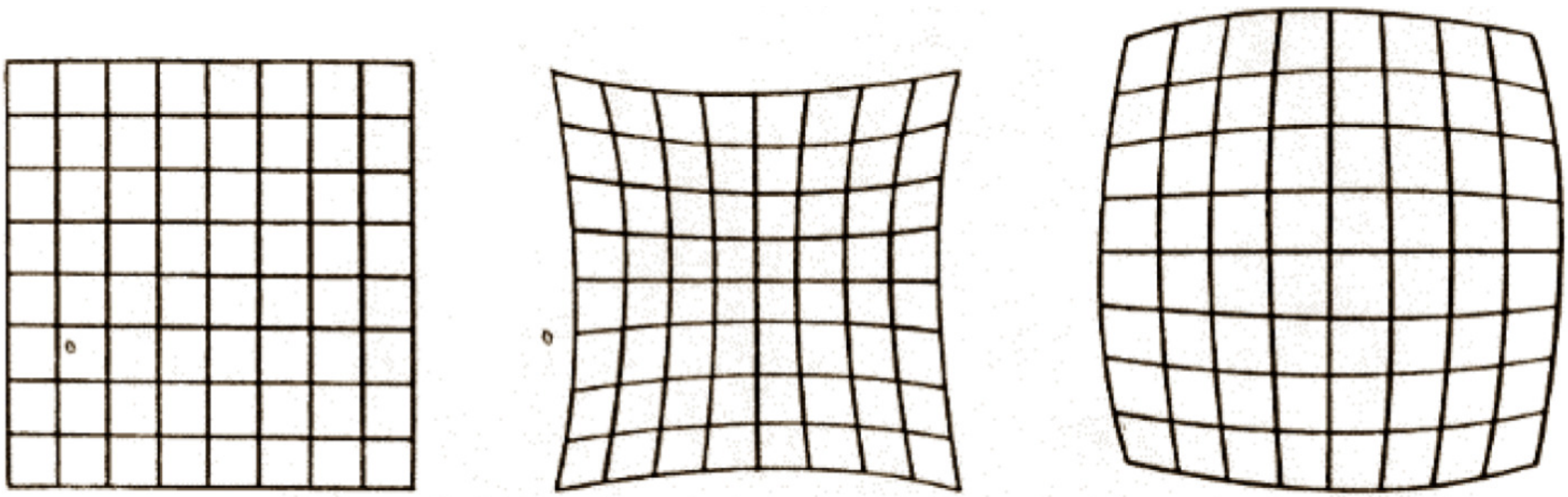


FIGURE 5.33 Gray-level interpolation based on the nearest neighbor concept.

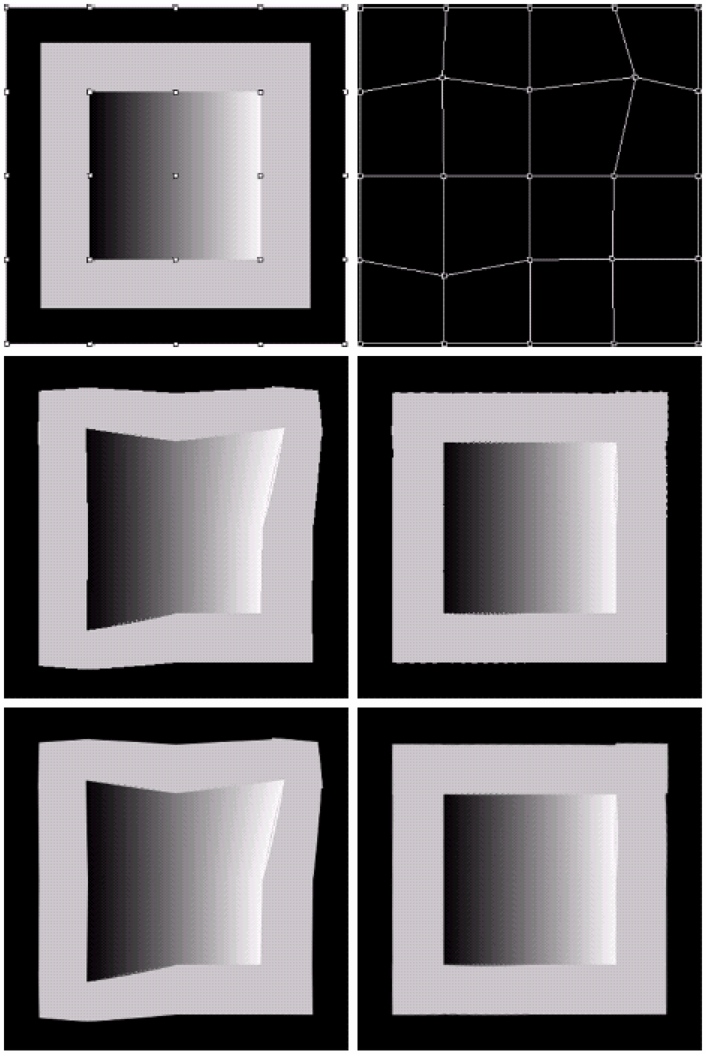
Spatial Distortion Examples



(a) Original (b) Pincushion distortion (c) Barrel distortion

FIGURE 14.2-1. Example of geometric distortion.

Recovery from Geometric Distortion



a b
c d
e f

FIGURE 5.34 (a) Image showing tiepoints. (b) Tiepoints after geometric distortion. (c) Geometrically distorted image, using nearest neighbor interpolation. (d) Restored result. (e) Image distorted using bilinear interpolation. (f) Restored image.

Recovery from Geometric Distortion

(a) (b)

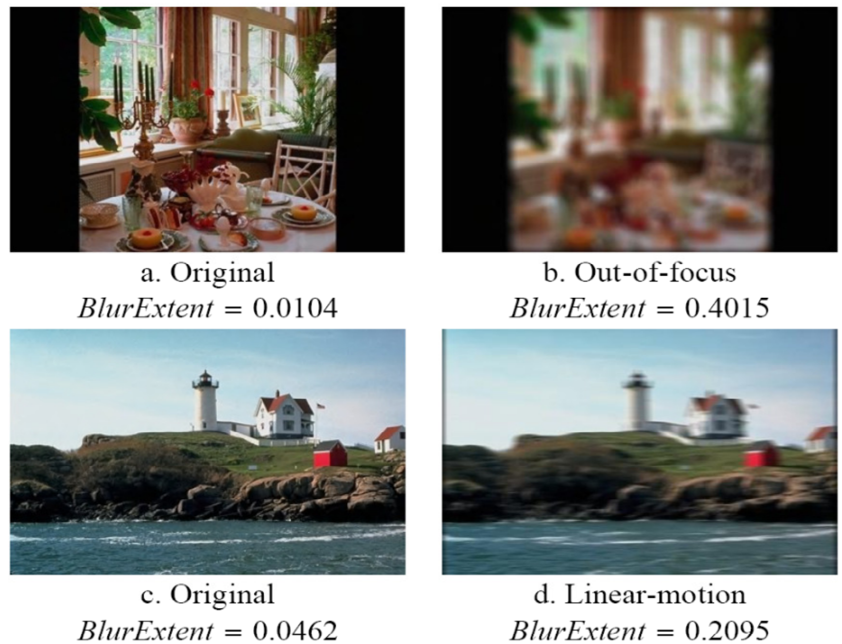
Fig. 5. (c) Image produced by a Computar 2.5mm lens and a Computar $1/3''$ CCD board camera. (b) Distortion parameters recovered via the minimization of ξ_3 are used to map (a) to perspective image. Notice that straight lines in the scene, such as door edges, map to straight lines in the undistorted images.

Rahul Swaminathan, Shree K. Nayar: Nonmetric Calibration of Wide-Angle Lenses and Polycameras. IEEE Trans. Pattern Anal. Mach. Intell. 22(10): 1172-1178 (2000)

Epilogue: Estimating Distortion

- Calibrate
- Use flat/edge areas
- … ongoing work

http://photo.net/learn/dark_noise/



a. Original *BlurExtent* = 0.0104

b. Out-of-focus *BlurExtent* = 0.4015

c. Original *BlurExtent* = 0.0462

d. Linear-motion *BlurExtent* = 0.2095

[Tong et. al. ICME2004]

Summary

- Image degradation model
- Restoration from noise
- Restoration from linear degradation
  - Inverse and pseudo-inverse filters, Wiener filter, blind deconvolution
- Geometric distortions

- Readings
  - G&W Chapter 5, Jain 8.1-8.3 (at courseworks)
  - M. R. Banham and A. K. Katsaggelos "Digital Image Restoration, " *IEEE Signal Processing Magazine,* vol. 14, no. 2, Mar. 1997, pp. 24-41.

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