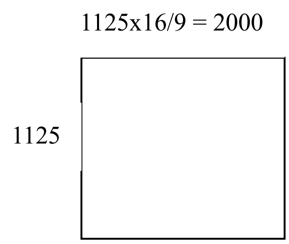
Digital Image Processing Midterm Exam Solution
Revised 03/25/2004

1.



-3

Total number of bits needed to encode a 2-hour video program

= (1125x2000) pixels/frame x 30 frames/sec x 8 bits/color x 3 colors/pixel x (2x60x60) secs

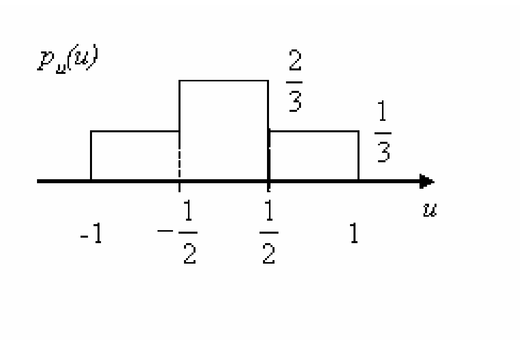
= 1.1664×10^{13} bits

-3 -3

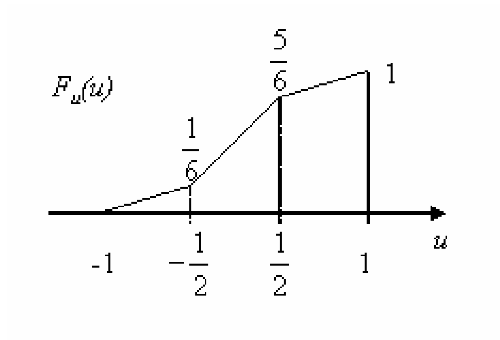
= 11.664 Tbits

2.

(a)



(a) pdf of u



(b) cdf of u

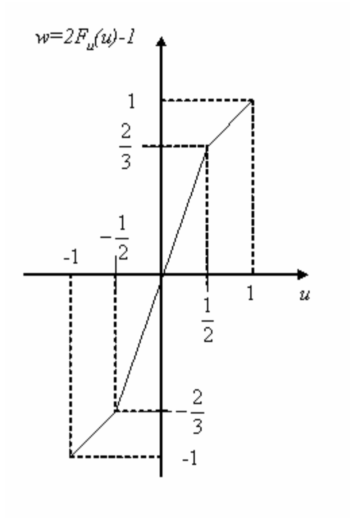
$$F_u(u) = \begin{cases} \frac{1}{3}u + \frac{1}{3} & [-1, -\frac{1}{2}) \\ \frac{1}{3}u + \frac{1}{2} & [-\frac{1}{2}, \frac{1}{2}) \\ \frac{1}{3}u + \frac{2}{3} & [\frac{1}{2}, 1] \end{cases} \quad \boxed{5}$$

$$\frac{w-(-1)}{1-(-1)} = F_u(u) \qquad -1 \le w \le 1$$

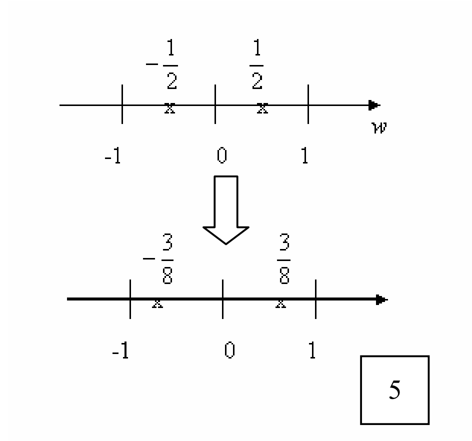
$$w = 2F_u(u) - 1 \qquad -1 \le u \le 1$$

$$\Rightarrow w = \begin{cases} \frac{2}{3}u - \frac{1}{3} & u \in [-1, -\frac{1}{2}) \\ \frac{4}{3}u & u \in [-\frac{1}{2}, \frac{1}{2}) \\ \frac{2}{3}u + \frac{1}{3} & u \in [\frac{1}{2}, 1] \end{cases} \quad \boxed{5}$$

$$\text{and} \quad u = \begin{cases} \frac{3}{2}w + 1 & w \in [-1, -\frac{2}{3}) \\ \frac{3}{4}w & w \in [-\frac{2}{3}, \frac{2}{3}) \\ \frac{3}{2}w - \frac{1}{2} & w \in [\frac{2}{3}, 1] \end{cases}$$



(c) Transform from *u* to *w*



(d) Quantization levels for *u* and *w*

The 2-level quantizations for both *u* and *w* are shown in (d).

(b)

Mean of *u* $\mu = 0$

Variance of *u*

$$\sigma^2 = \int (x-\mu)^2 p_u(x)dx$$

$$= \int_{-1}^{-\frac{1}{2}} x^2 \frac{1}{3}dx + \int_{-\frac{1}{2}}^{\frac{1}{2}} x^2 \frac{2}{3}dx + \int_{\frac{1}{2}}^{1} x^2 \frac{1}{3}dx$$

$$= \frac{1}{9}x^3 \Big|_{-1}^{-\frac{1}{2}} + \frac{2}{9}x^3 \Big|_{-\frac{1}{2}}^{\frac{1}{2}} + \frac{1}{9}x^3 \Big|_{\frac{1}{2}}^{1}$$

$$= \frac{1}{4}$$

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so $\sigma = \frac{1}{2}$

Optimal 2-level MSE quantizer for Gaussian with $\mu = 0$ and $\sigma = 1$ is

$r_k = \pm 0.7979$

and $r_k' = \sigma r_k + \mu$

so, in *u*,

$r_k' = \frac{1}{2} r_k = \pm 0.3989$

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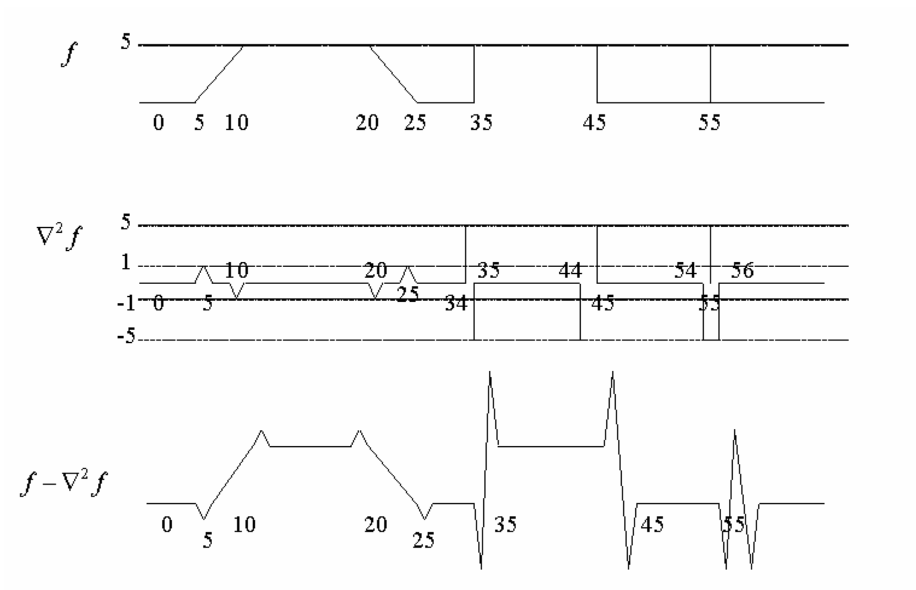
3.

(a)

$$g = f - \nabla^2 f\ ,\ \ \nabla^2 = [1 \quad -2 \quad 1]$$

so $g = [-1 \quad 3 \quad -1]$

(b)



(c)

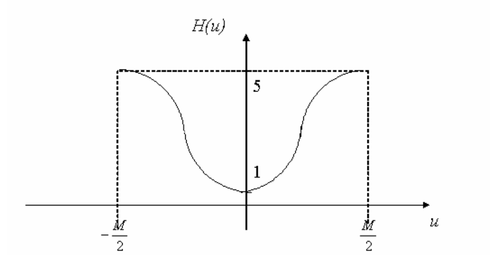
$$DFT(f - \nabla^2 f)$$

$$= \frac{1}{M}\sum_{x=0}^{M-1}[f(x) - (f(x-1) - 2f(x) + f(x+1))]e^{-j\frac{2\pi ux}{M}}$$

$$= F(u)(e^{-j\frac{2\pi u}{M}} + 3 + e^{j\frac{2\pi u}{M}})$$

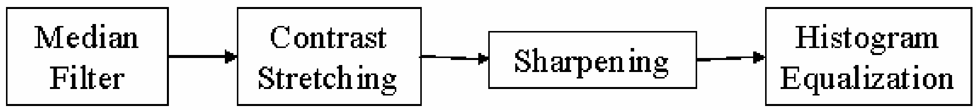
$$= F(u)(3 - 2\cos(\frac{2\pi u}{M}))$$

$$\Rightarrow H(u) = (3 - 2\cos(\frac{2\pi u}{M}))$$



So *H(u)* is a highpass filter.

4.



Median filter: To remove sparse noise.
Contrast Stretching: To remove the brightness problem.
Sharpening: To enhance the edges. Can be unsharped masking or highpasss sharpening.
Histogram equalization: To enhance the quality of the final image.

The order of the abve modules need to be considered carefuly. The first two modules are non-linear processes while the sharpening module is linear. The first module (Median Filter) and the 2nd one (Contrst Stretching) are both point processing and can be switched without causing differences. But the 2nd module and the 3rd one cannot be changed.