

EE E4830 Digital Image Processing, Spring 2003

Midterm Exam

March 13, 2003 Thursday 4:10 pm – 6 pm

Note:

1. Choose 3 out of 4 problems. Each problem carries the same points (34%). If more than 3 problems are answered, the first three on your exam book will be used for grading.
2. Please use only the standard blue color exam book. Only answers written on your exam book will be graded. Please remember to write your name clearly on the cover page.
3. Open books and notes. Use of calculator is OK. But computers and cellular phones are not allowed.
4. Best luck!

P.1 (color space)

Consider the following standard colors in the RGB color space

	Red	Yellow	Blue	Reference White
R_c	1	1	0	1
G_c	0	1	0	1
B_c	0	0	1	1

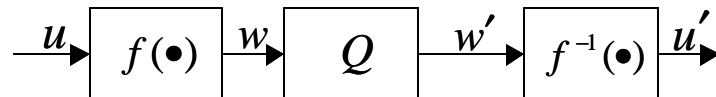
- (a) (20%) Use Equation 3.5-20 to convert the above values to the HSI space
 (b) (14%) Propose a measure for computing the color distance in the HSI space.
 Explain why you think the proposed one is a good measure.

P.2 (Image Quantization)

Consider a random variable, u , whose probability distribution function (pdf) is as follows

$$P_u(u) = \begin{cases} 1 - |u|, & -1 \leq u \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

We want to design a compandor quantizer with the following structure.



- (a) (17%) design a mapping function $f(u)$ so that the pdf of w is uniform over the interval $[-1,1]$
 (b) (17%) Suppose we use a 4-level optimal quantizer to quantize w over the interval $[-1,1]$. What is the mean square error we will have in u' , i.e.,

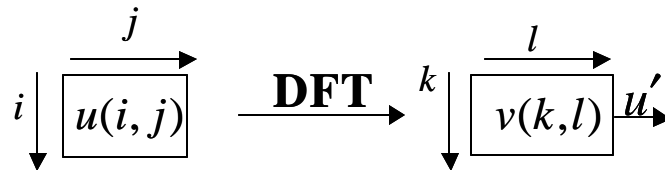
$$\text{MSE}(u) = E[(u-u')^2]$$

(*hint*: compute the decision and reconstruction levels for w and then find the corresponding values for u)

P.3 (Image Transform)

We want to verify the total number of real-value coefficients is unchanged during 2-D DFT.

Consider the standard 2D DFT of an $N \times N$ block (N is even)



If u is real, then we have the following property of conjugate symmetry.

$$v(k, l) = v^*(N - k, N - l)$$

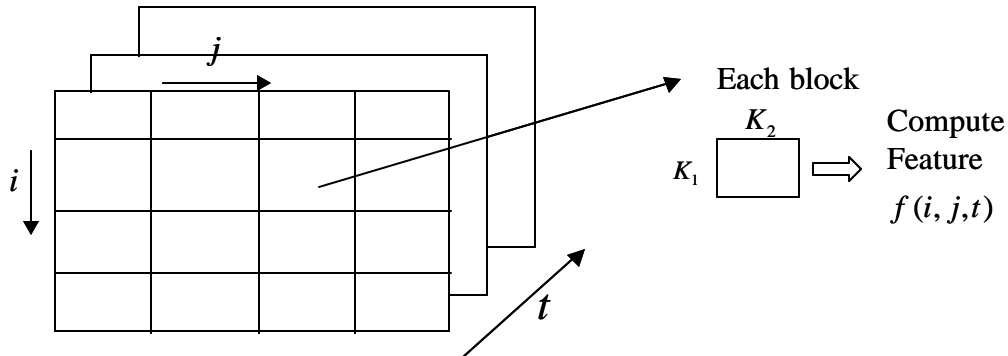
How many independent real-value coefficients are in each of the following regions?

- (a) the first row of $v(k, l)$, i.e., $k=0$
- (b) the first column, i.e., $l=0$
- (c) the middle column (i.e., $k \neq 0, l = N/2$)
- (d) the middle row (i.e., $k = N/2, l \neq 0$)
- (e) four quadrants besides the above regions

Please verify the total number of independent real-value coefficients is N^2 .

P.4 (KL Transform)

Consider multiple image frames in a video sequence shown below



Assume the size of each frame is N_1 pixels by N_2 pixels.

Very often we break each image frame into separate non-overlapping blocks (size: K_1 pixels by K_2 pixels), each of which is used to compute local visual features, such as average colors. Let's denote such a local feature for each block as $f(i, j, t)$, where i and j are indexes in the spatial dimensions, and t is the frame number.

We want to use KLT to exploit the correlations between features at the same spatial locations but different times. This matches the intuition that similar objects may reoccur at the same locations over time. For simplicity, assume the temporal correlation is limited to a finite time window of T frames.

Describe the procedures for deriving the KL transform matrix for this case. Please provide as much details as possible. For example, what are the input and output vectors? What's the pool of empirical data for estimating the covariance matrix? What's the dimension of the KL transform matrix? Provide the mathematical equations for each computation step if necessary. Please use the same notations as those given above in your mathematical equations.