Solution for EE E4830 Digital Image Processing Spring 2003, March 13, 2003

P.1.

(a) (20%) Eq. (3.5-20 a,b,c)

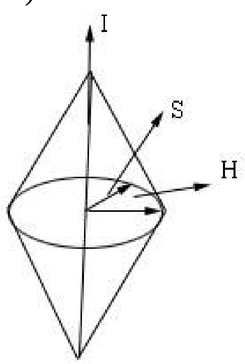
$$\begin{bmatrix} I \\ v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ -1/\sqrt{6} & -1/\sqrt{6} & 2/\sqrt{6} \\ 1/\sqrt{6} & -1/\sqrt{6} & 0 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

$$H = \tan^{-1}\{v_2 / v_1\}$$
$$S = \left(v_1^2 + v_2^2\right)^{1/2}$$

Using the above equations, we get

	Red	Yellow	Blue	Ref. White
I	1/3	2/3	1/3	1
H	$-p/4$	0	0	0
S	$1/\sqrt{3}$	$2/\sqrt{6}$	$2/\sqrt{6}$	0

(b) (14%)



One possible distance measure between colors in HSI is

$$D = a|I_1 - I_2| + b|S_1 - S_2| + g|H_1 - H_2|$$

a, b, g are weights controlling the relative importance among different channels.

Another possible measure is L_2 distance

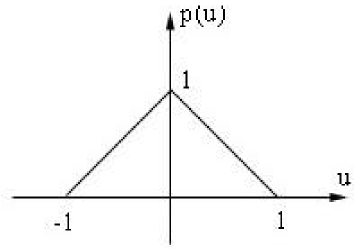
$$D = \left(|I_1 - I_2|^2 + |S_1 - S_2|^2 + |H_1 - H_2|^2\right)^{1/2}$$

but such metrics may not correspond well to perceived distance.

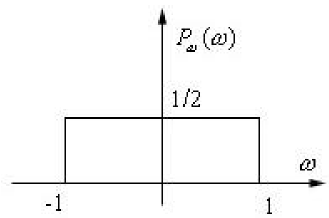
P.2.

(a) (17%)

$$p_u(u) = \begin{cases} 1-|u| & -1 \le u \le 1 \\ 0 & Otherwise \end{cases}$$



We want to design $w = f(u)$ so that pdf of w is as follows:

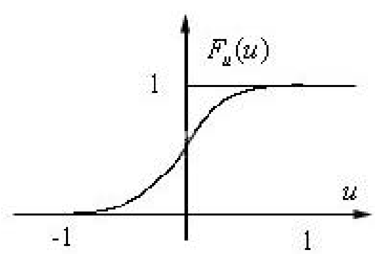


As described in the class notes,
$F_w(w) = F_u(u)$, where F is the cumulative probability distribution function

$$\frac{w - w_{\min}}{w_{\max} - w_{\min}} = F_u(u), \quad w_{\min} \le w \le w_{\max} \qquad (*)$$

Now $w_{\min} = -1,\ w_{\max} = 1$

$$F_u(u) = \int_{-\infty}^{u} p_u(x)dx$$
$$= \begin{cases} (u+1)^2/2 & -1 \le u \le 0 \\ -u^2/2 + u + 1/2 & 0 \le u \le 1 \end{cases}$$

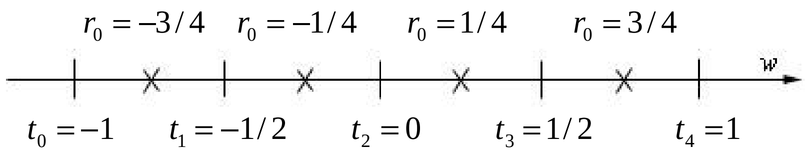


Use equation (*) above,

$$w = 2F_u(u) - 1 = \begin{cases} u^2 + 2u, & -1 \le u \le 0 \\ -u^2 + 2u, & 0 \le u \le 1 \end{cases} \qquad (**)$$

(b) (17%)

Since the pdf of w is uniform, the optimal quantization for w will be the uniform quantizer.

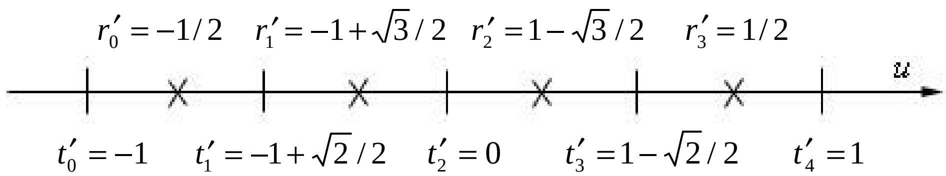


Map t_i & r_i for w to the decision and reconstruction values for u.
Use equation (**) in part (a) to derive the reverse mapping function

$$u = \begin{cases} -1+\sqrt{1+w} & -1 \le w \le 0 \\ 1-\sqrt{1+w} & 0 \le w \le 1 \end{cases}$$

$\therefore$

$$\begin{aligned}
t_0 &= -1 &&\Rightarrow & t_0' &= -1 \\
t_1 &= -1/2 &&\Rightarrow & t_1' &= -1+\sqrt{1/2} \\
t_2 &= 0 &&\Rightarrow & t_2' &= 0 \\
t_3 &= 1/2 &&\Rightarrow & t_3' &= 1-\sqrt{1/2} \\
t_4 &= 1 &&\Rightarrow & t_4' &= 1 \\
r_0 &= -3/4 &&\Rightarrow & r_0' &= -1/2 \\
r_1 &= -1/4 &&\Rightarrow & r_1' &= -1+\sqrt{3}/2 \\
r_2 &= 1/4 &&\Rightarrow & r_2' &= 1-\sqrt{3}/2 \\
r_3 &= 3/4 &&\Rightarrow & r_3' &= 1/2
\end{aligned}$$



$$\begin{aligned}
MSE &= E\{(u-u')^2\} \\
&= \sum_{i=0}^{3} \int_{t_i'}^{t_{i+1}'} (u-r_i')^2\, p_u(u)\,du \\
&= \int_{-1}^{-1+\sqrt{2}/2} (u+1/2)^2 (1+u)\,du + \int_{-1+\sqrt{2}/2}^{0} (u+1-\sqrt{3}/2)^2 (1+u)\,du \\
&\quad + \int_{0}^{1-\sqrt{2}/2} (u-1+\sqrt{3}/2)^2 (1-u)\,du + \int_{1-\sqrt{2}/2}^{1} (u-1/2)^2 (1-u)\,du \\
&= 2\left[\int_{-1}^{-1+\sqrt{2}/2} (u+1/2)^2 (1+u)\,du + \int_{-1+\sqrt{2}/2}^{0} (u+1-\sqrt{3}/2)^2 (1+u)\,du\right] \\
&= 2(0.0071+0.0018) \\
&= 0.0178
\end{aligned}$$

Computation of integral:

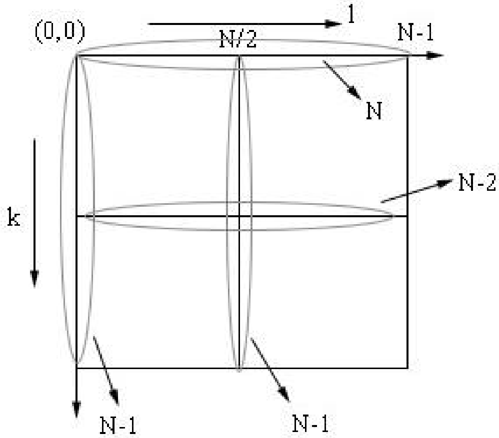
$\int_a^b (u' + c)^2 (1 + u') du'$

Let $u = u' + c,\ u' = u - c$, then

$$\int_{a+c}^{b+c} u^2 (1 - c + u) du = \int_{a+c}^{b+c} u^3 du + (1 - c) \int_{a+c}^{b+c} u^2 du$$

$$= 1/4 \left[(b + c)^4 - (a + c)^4 \right] + (1 - c)/3 \left[(b + c)^3 - (a + c)^3 \right]$$

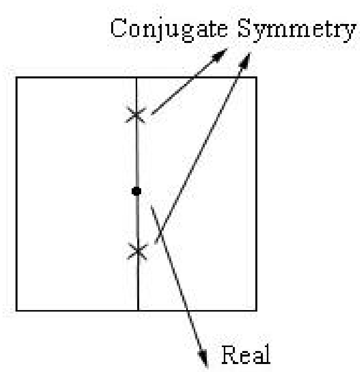
P.3 (34%)



(a) The 1st row is the 1-D DFT of average values of each column. $\Rightarrow$ N real-valued numbers

(b) similar to (a), excluding the origin point $\Rightarrow$ N-1 numbers

(c)

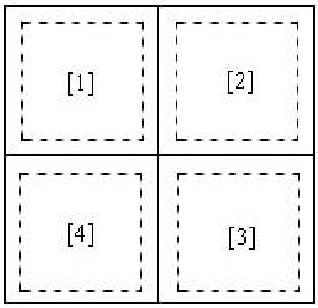


$$v(k, N/2) = v^*(N - k, N/2)$$

$\therefore$ $\begin{array}{l} Total\ \ Nuber = (N/2 - 1) \times 2 + 1 \\ = N - 1 \end{array}$, excluding $v(0, N/2)$

(d) Similar to (c), Total Number = N-2, excluding $v(N/2, 0)$ and $v(N/2, N/2)$

(e)I



$v(k,l)$ in region [1] is conjugate symmetric with $v(k,l)$ in region [3]
Total number of real numbers

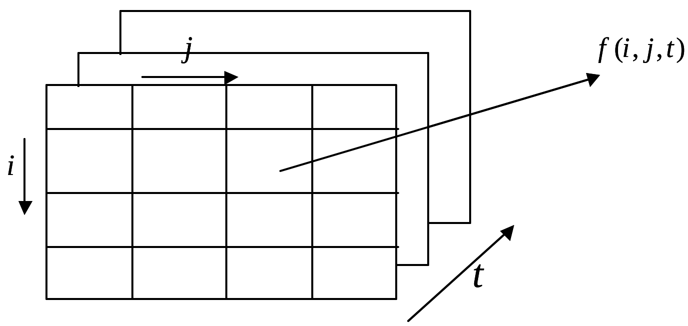
Likewise, there are $=(N/2-1)^2 \times 2$ real numbers in regions [2]+[4]

$\therefore$ If we add up the results in (a) – (e)

Total number of real numbers
$= N+(N-1)+(N-1)+(N-2)+(N/2-1)^2 \times 2+(N/2-1)^2 \times 2$
$= N^2$

This is the same as the total number of pixels in $u(i, j)$.

P.4. (34%)



Since we are interested in the correlation between features at different time, but the same locations, the input vector will be

$$\mathbf{w}_{i,j} = \begin{bmatrix} f(i,j,t_0) \\ f(i,j,t_1) \\ \dots \\ f(i,j,t_{T-1}) \end{bmatrix}, \text{ where T is the time window}$$

From T consecutive image frames, we can get $(N_1/K_1)(N_2/K_2)$ empirical observation of the $\mathbf{u}$ vector.

From the above pool of observation data, we can compute the covariance matrix of $\mathbf{u}$, $R_{\mathbf{u}}$, which is a T by T matrix.

$$R_{\mathbf{u}}(k,l) = \sum_{i=0}^{N_1/K_1-1} \sum_{j=0}^{N_2/K_2-1} \left(f(i,j,t_k) - m_k\right)\left(f(i,j,t_l) - m_k\right)$$

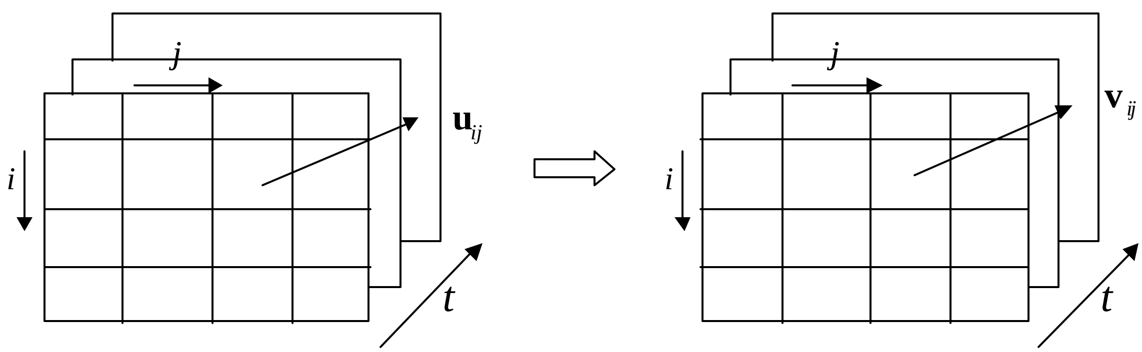
where m_k is the mean of $f(i,j,k)$ over all possible i,j index values.

Find the eigen vectors of $R_{\mathbf{u}}$, $\{\mathbf{f}_0, \mathbf{f}_1, \ldots, \mathbf{f}_{T-1}\}$.

Form the KLT matrix as follows

$$\Phi_{T\times T} = \left[\mathbf{f}_0 \,|\, \mathbf{f}_1 \,|\, \ldots \,|\, \mathbf{f}_{T-1}\right]$$

Now given the original image sequence,



we can use the following KLT transform

$$\mathbf{v}_{ij} = \Phi^{*T} \mathbf{u}_{ij}$$

Here $\mathbf{v}_{ij}$ is a Tx1 vector, Φ is a TxT matrix, and $\mathbf{u}_{ij}$ is a Tx1 vector.
Due to the compaction of energy, some frames of $\mathbf{v}$ near the end can be ignored.